COMPSCI 276
Homework Assignment 2
Fall 2021

Instructor: Rina Dechter Due:Wednesday, October 27

Relevant reading: (Darwiche chapter 5,6. Dechter chapter 4).

1. (extra credit, 5 points). Provide comments on chapter 4 in Dechter's book.

2. (Darwiche 5.4) (15 pts) We have two sensors that are meant to detect extreme temperature, which occurs 20% of the time. The sensors have identical specifications, with a false positive rate of 1% and a false negative rate of 3%. If the power is o (dead battery), the sensors will read negative regardless of the temperature. Suppose now that we have two sensor kits: Kit A where both sensors receive power from the same battery, and Kit B where they receive power from independent batteries. Assuming that each battery has a 0.9 probability of power availability, what is the probability of extreme temperature given each of the following scenarios:

   (a) two negative sensor readings

   (b) The two sensor readings are positive?

   (c) One sensor reads positive while the other reads negative.

   Answer the previous questions with respect to each kit.

3. (Darwiche 5.6) (10 pts) Lisa is given a fair coin C1 and asked to flip it eight times in a row. Lisa also has a biased coin C2, with a probability 0.8 of landing heads. All we know is that Lisa flipped the fair coin initially, but we believe that she intends to switch to the biased coin, and that she tends to be 10% successful in performing the switch. Suppose that we observe the outcome of the eight coin flips and want to find out whether Lisa managed to perform a coin switch and when.

   (a) Describe a Bayesian network and a corresponding query that solves this problem.

   (b) What is the solution to this problem assuming the flips came out as follows:

      i. tails, tails, tails, heads, heads, heads, heads, heads.

      ii. tails, tails, heads, heads, heads, heads, heads, heads

4. (30 pts) Consider the Bayesian network in Figure 1.

   (a) (20 pts)Apply BE-bel to obtain the:

      i. marginal probability of variable F.

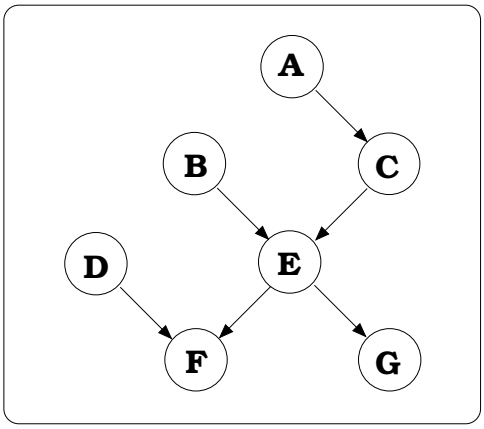
      ii. marginal probability of variable G

      iii. joint marginal probability of variables F and G.

For each part show the schematic computation over buckets (you can take advantage of shared computation). The computation itself can be done manually or using any software tool of your choice.

(b) (5 pts) Suppose now that the evidence $\{D = 0, C = 1\}$ has been observed. Apply BE-bel to obtain the probability of evidence. You can do calculation by hand or use any software tool to solve the computational parts of this question. Show the schematic computation over buckets.

(c) (5 pts) Explain how BE-mpe can find the most probable explanation (mpe) given the evidence F=0. Demonstrate your computation. Again, you can do calculation by hand or use any software tool to solve the computational parts of this question.



a	$p(a)$
0	0.3
1	0.7

b	$p(b)$
0	0.6
1	0.4

d	$p(d)$
0	0.7
1	0.3

a	c	$p(c\|a)$
0	0	0.15
0	1	0.85
1	0	0.25
1	1	0.75

e	g	$p(g\|e)$
0	0	0.10
0	1	0.90
1	0	0.30
1	1	0.70

b	c	e	$p(e\|b,c)$
0	0	0	0.40
0	0	1	0.60
0	1	0	0.45
0	1	1	0.55
1	0	0	0.60
1	0	1	0.40
1	1	0	0.30
1	1	1	0.70

d	e	f	$p(f\|d,e)$
0	0	0	0.25
0	0	1	0.75
0	1	0	0.60
0	1	1	0.40
1	0	0	0.10
1	0	1	0.90
1	1	0	0.20
1	1	1	0.80

Figure 1:

5. (10 pts) (Question 6.5 in Darwiche): Consider a network N that has a single root X and n leaves $Y_1, ..., Y_n$ where N contains edges $X \to Y_i$, for all i from 1 to n.

   (a) What is the width of the order $Y_1, ..., Y_n, X$ with respect to N?

   (b) What is the width of the order $X, Y_1, ..., Y_n$ with respect to N?

6. (5 pts) (Question 6.8 in Darwiche) What is the width of order A, B, C, D, E, F, G, H with respect to the network in Figure 2?

7. (25 pts) Given the directed graph G in Figure 3,

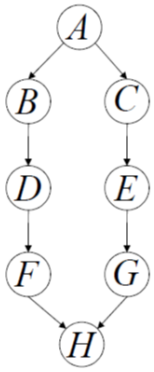


Figure 6.11: A Bayesian network structure.

Figure 2: A Bayesian network

(a) Compute the induced-graph along ordering: $d_1 = F, C, A, G, D, H, E, B$ and the induced-width for each variable. What is G's induced-width along d_1?

(b) Use min-induced width (MIW) to compute an ordering, called d_2. Show its ordered graph. Compute the induced-width along d_2.

(c) What is the induced width of the graph G? Explain your answer.

(d) Apply BE-bel along the ordering d_1 and show the λ functions created, their placement and the expressions for deriving the functions. (optional: do the same for ordering d_2)

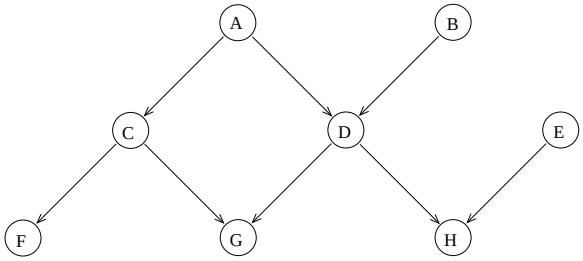


Figure 3: A directed graph