COMPSCI 276
Homework Assignment 4
Fall 2021

Instructor: Rina Dechter Due: Wednesday, November 17

Relevant reading: Dechter chapters 6-7, Darwiche chapter 17

1. (Extra credit, 5 pts.) Read chapters 6-7 in the Dechter's book and provide comments and provide comments on clarity and typos.

2. (35 pts.) This question investigates the AND/OR search space of the network

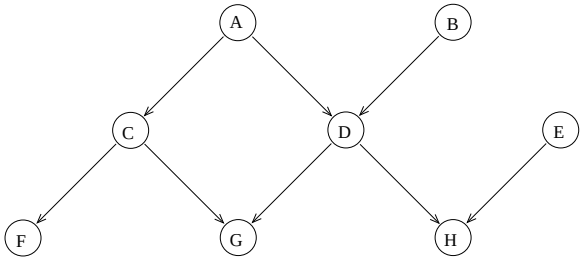

Figure 1: A directed graph

assuming each variable has 2 values in its domain. The CPTs are

a	$p(a)$
0	0.3
1	0.7

b	$p(b)$
0	0.6
1	0.4

e	$p(e)$
0	0.7
1	0.3

y	x	$p(x\|y)$
0	0	0.10
0	1	0.90
1	0	0.30
1	1	0.70

z	y	x	$p(x\|y,z)$
0	0	0	0.25
0	0	1	0.75
0	1	0	0.60
0	1	1	0.40
1	0	0	0.10
1	0	1	0.90
1	1	0	0.20
1	1	1	0.80

The CPTs for G, H and D are identical to the 3-dimensional CPT and the CPTs for C and F are identical to the 2-dimensional CPT.

   (a) (5) Find and present a pseudo tree of this network whose depth is minimal (do the best you can). Call this tree T_1.

   (b) (10) Generate an AND/OR search tree driven by T_1 assuming each variable has at most two values.

   (c) (5) Annotate the arcs with the appropriate weights.

(d) (10) What would be the computational cost of computing the probability of evidence $G = 0$ and $H = 1$ in such a network if you use depth-first search over the AND/OR search tree? Demonstrate the computation (compute the value of each node).

3. (30 pts.) Consider the same graphical model in Figure 1.

   (a) (5) Can the AND/OR search tree be reduced to a smaller AND/OR search graph? If so, demonstrate.

   (b) (5) Provide the OR-based context for every variable in your pseudo-tree.

   (c) (5) Identify all the variables that correspond to dead caches and those that are not.

   (d) (5) Compare the time and space complexity of solving this problem by search with context-based caching vs. bucket-elimination.

   (e) (10) Assume that the CPT $P(x|y,z)$ is changed by making some entries deterministic, as follows: set the probability column of the first 2 lines to 1 and 0 in that order (from top to bottom). Do the same for the last 2 lines (e.g., we make $P(x = 0|y = 1, z = 1) = 0$). Show what would be the changes in the AND/OR search tree as a result.

4. (5 pts.) Prove that if there is a tree-decomposition of G having a treewidth w, then we can create a pseudo-tree whose depth h satisfies $h \leq w \cdot logn$ when n is the number of nodes in the tree.

5. (Extra credit 10 pts.) Given a graph G and an ordering d Show that if we generate an induced-ordered graph along d and then generate a DFS spanning tree over the induced graph starting from the first variable as the root, we get a pseudo-tree.

6. (10, extra credit) (Darwiche question 17.1) Consider a Bayesian network structure with the following edges $A \rightarrow B,\ A \rightarrow C,\ and\ A \rightarrow D$. Compute the ML parameter estimates for this structure given the following data set in Figure 2.

7. (10) Darwiche question 17.2 described also in Figure 3. Optional: Use also GenIE to learn the network.

8. (Extra credit) Darwiche question 17.6.

9. (optional) Darwiche question 17.14.

Case	*A*	*B*	*C*	*D*
1	*T*	*F*	*F*	*F*
2	*T*	*F*	*F*	*T*
3	*F*	*F*	*T*	*F*
4	*T*	*T*	*F*	*T*
5	*F*	*F*	*T*	*T*
6	*F*	*T*	*T*	*F*
7	*F*	*T*	*T*	*T*
8	*T*	*F*	*F*	*T*
9	*F*	*F*	*T*	*F*
10	*T*	*T*	*T*	*T*

Figure 2: A dataset

Exercise 17.2. *Consider a Bayesian network structure with edges* $A \longrightarrow B$, $B \longrightarrow C$, *and the following data set:*

Case	*A*	*B*	*C*
1	*T*	*F*	*F*
2	*T*	*F*	*F*
3	*F*	*F*	*T*
4	*T*	*F*	*F*

Compute parameters estimates that have a maximum likelihood given the above data set. Are such estimates unique? If not, how many ML estimates do we have in this case?

Figure 3: question 17.2