Reasoning with Graphical Models

Class 1: Overview

Rina Dechter

Darwiche chapters 1,3
Dechter-Morgan&claypool book: Chapters 1-2
Pearl chapter 1-2

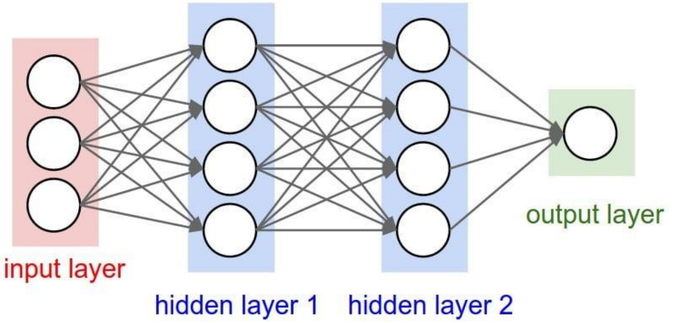
Congressional Breifing: AI at UCI

The Primary AI Challenges

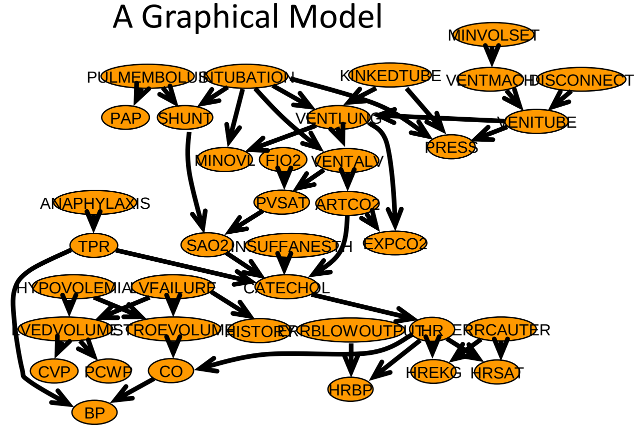
- **Machine Learning** focuses on replicating humans learning

- **Automated reasoning** focuses on replicating how people reason.

A neural network



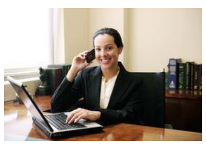
A Graphical Model



Automated Reasoning

Medical Doctor

Lawyer



Policy Maker

Queries:

- Prediction: what will happen?
- Diagnosis: what happened?
- Situation assessment: What is going on?
- Planning, decision making: what to do?

Automated Reasoning

answers

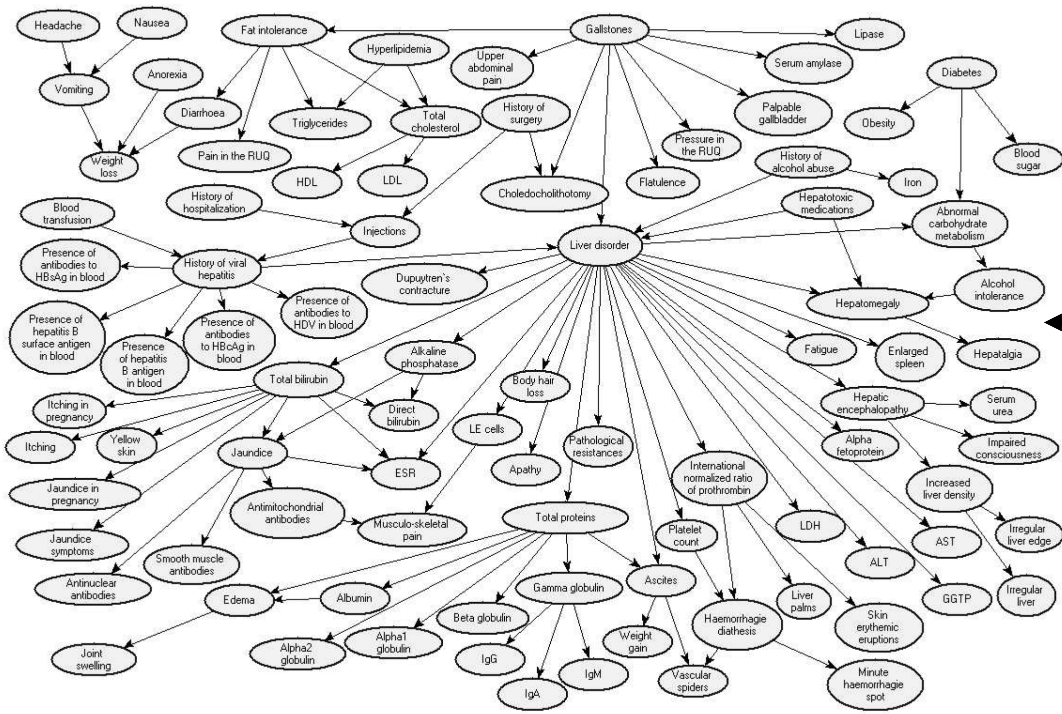
Queries:
- Prediction
- Diagnosis
- Situation assessment
- Planning, decision making

Knowledge is huge, so How to identify what's relevant? → **Graphical Models**

****The field of Automated Reasoning** is focused to a large part on developing general purpose formalisms that enable us to represent knowledge in such a way that we can exploit the relevance relationship quickly.

Graphical Models

Example: diagnosing liver disease (Onisko et al., 1999)



Queries:

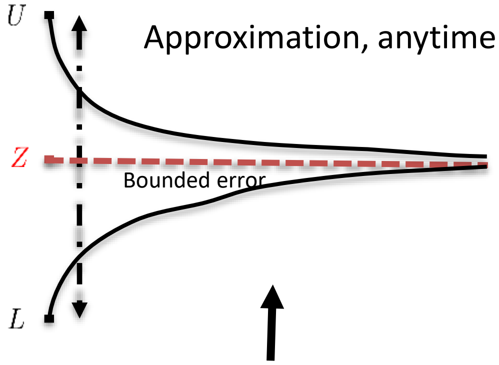
- Prediction
- Diagnosis
- Situation assessment
- Planning, decision making

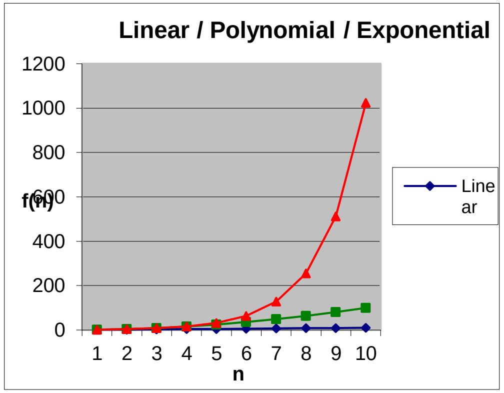
Automated Reasoning:

- Develop methods to answer these questions.
- Learning the models: from experts and data.

Complexity of Automated Reasoning

- Prediction
- Diagnosis
- Planning and scheduling
- Probabilistic Inference
- Explanation
- Decision-making

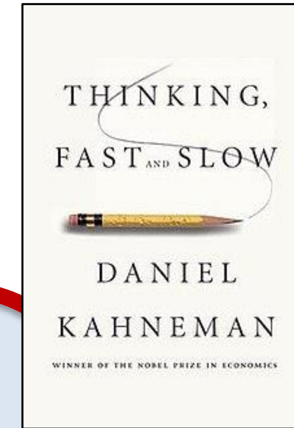


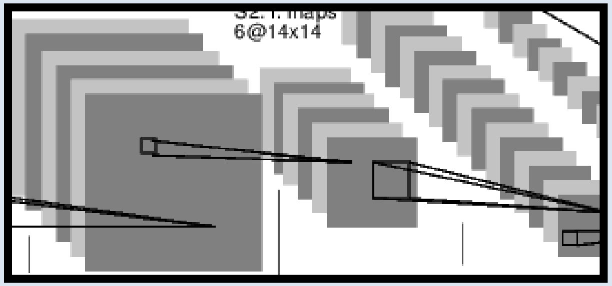


Reasoning is computationally hard

Complexity is exponential

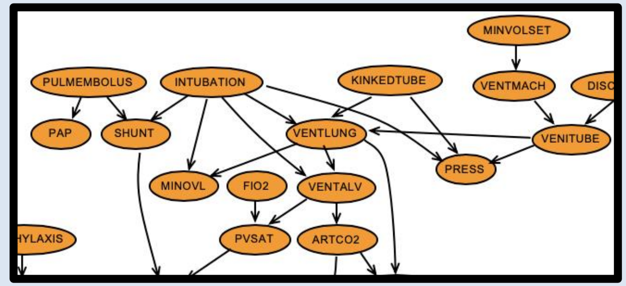
AI Renaissance





- Deep learning
  - Fast predictions
  - “Instinctive”

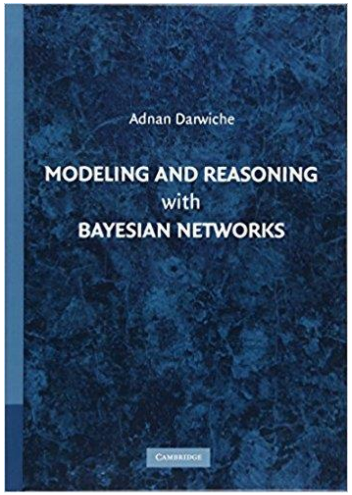
Tools:
Tensorflow, PyTorch, …

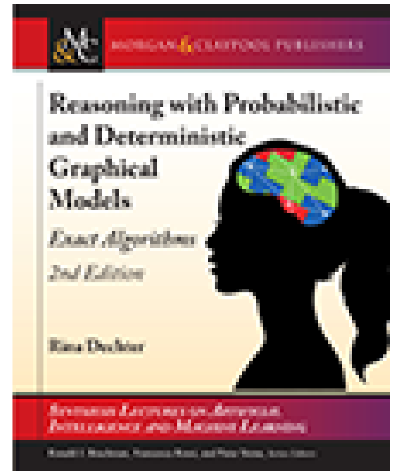


- Probabilistic models
  - Slow reasoning
  - “Logical / deliberative”

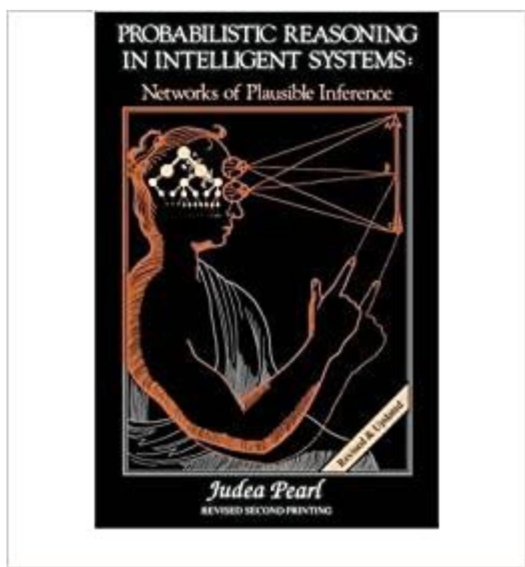
Tools:
Graphical Models,
Probabilistic programming,
Markov Logic, …

Text Books, Outline, Requirements





[Class page]



Probabilistic Graphical models

- Describe structure in large problems
  - Large complex system $F(X)$
  - Made of “smaller”, “local” interactions $f_\alpha(x_\alpha)$
  - Complexity emerges through interdependence

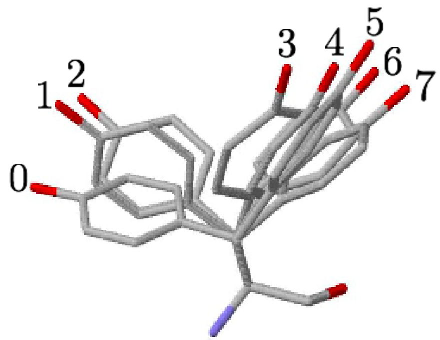
Probabilistic Graphical models

- Describe structure in large problems
  - Large complex system $F(X)$

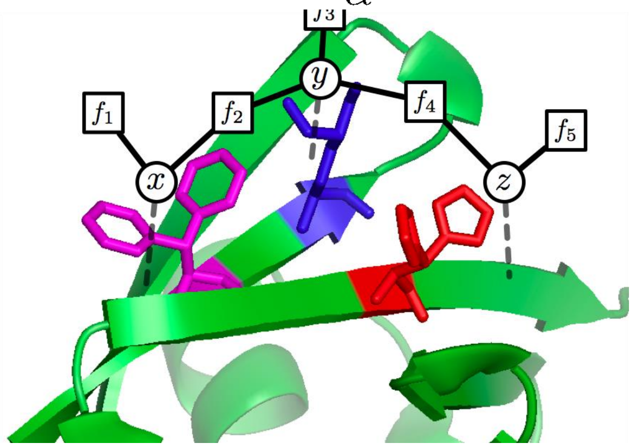
- Protein Structure **prediction**: predicting the 3d structure from given sequences
- CPD: Computational Protein **design** (backbone) algorithms enumerate a combinatorial number of candidate structures to compute the Global Minimum Energy Conformation (GMEC).

-

$$\mathbf{x}^* = \arg\max_{\mathbf{x}} \prod_{\alpha} f_\alpha(\mathbf{x}_\alpha) \qquad f(\mathbf{x}^*) = \max_{\mathbf{x}} \prod_{\alpha} f_\alpha(\mathbf{x}_\alpha)$$



Phenylalanine



[Yanover & Weiss 2002]
[Bruce R. Donald et. Al. 2016]

Probabilistic Graphical models

- Describe structure in large problems
  - Large complex system $F(X)$
  - Made of “smaller”, “local” interactions $f_\alpha(x_\alpha)$
  - Complexity emerges through interdependence
- Examples & Tasks
  - Summation & marginalization

“partition function”

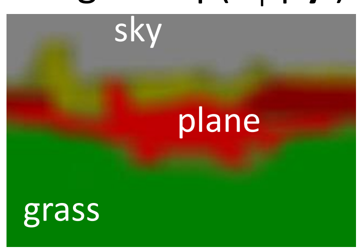
$$p(x_i) = \frac{1}{Z} \sum_{\mathbf{x} \setminus x_i} \prod_\alpha f_\alpha(\mathbf{x}_\alpha) \quad \text{and} \quad Z = \sum_{\mathbf{x}} \prod_\alpha f_\alpha(\mathbf{x}_\alpha)$$

Image segmentation and classification:

Observation **y**

$\Rightarrow$

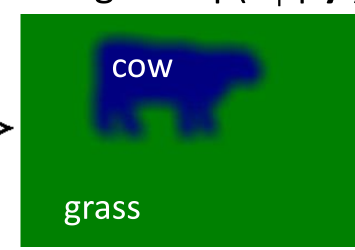
Marginals p(x_i | **y**)



Observation **y**

$\Rightarrow$

Marginals p(x_i | **y**)



e.g., [Plath et al. 2009]

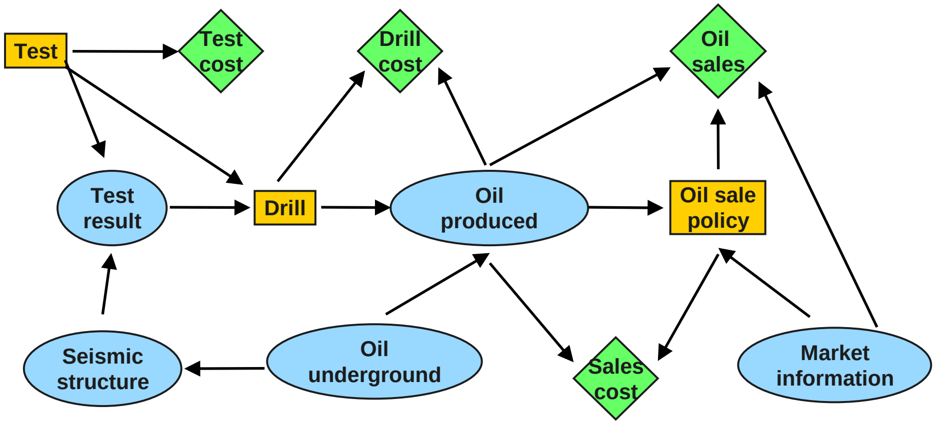
Graphical models

- Describe structure in large problems
  - Large complex system $F(X)$
  - Made of "smaller", "local" interactions $f_\alpha(x_\alpha)$
  - Complexity emerges through interdependence
- Examples & Tasks
  - Mixed inference (marginal MAP, MEU, …)

$$f(\mathbf{x}_M^*) = \max_{\mathbf{x}_M} \sum_{\mathbf{x}_S} \prod_\alpha f_\alpha(\mathbf{x}_\alpha)$$

Influence diagrams &
optimal decision-making

(the "oil wildcatter" problem)

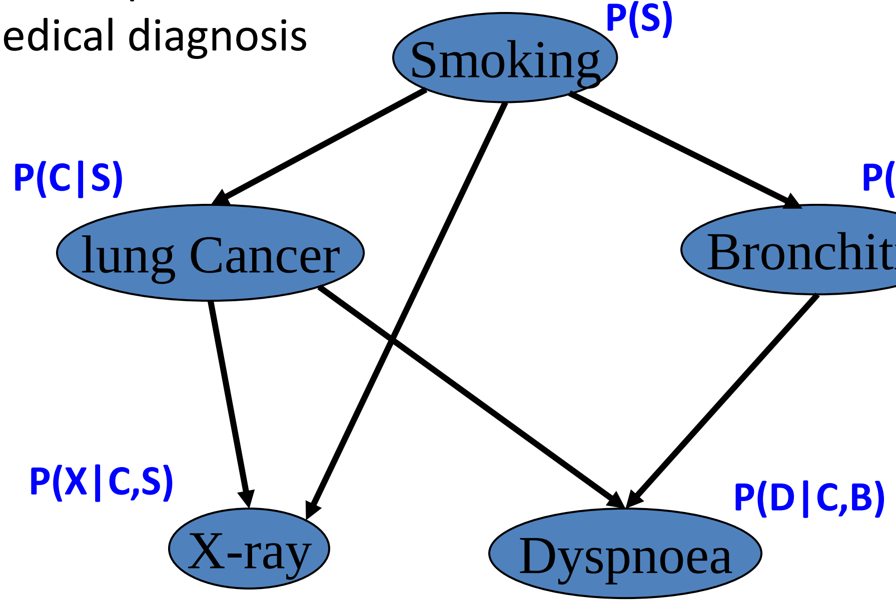


e.g., [Raiffa 1968; Shachter 1986]

In more details…

Bayesian Networks (Pearl 1988)

An early example
From medical diagnosis



BN = (G, Θ)

CPD:

C	B	P(D|C,B)	
0	0	0.1	0.9
0	1	0.7	0.3
1	0	0.8	0.2
1	1	0.9	0.1

P(S, C, B, X, D) = ***P(S) P(C|S) P(B|S) P(X|C,S) P(D|C,B)***

Combination: Product
Marginalization: sum/max

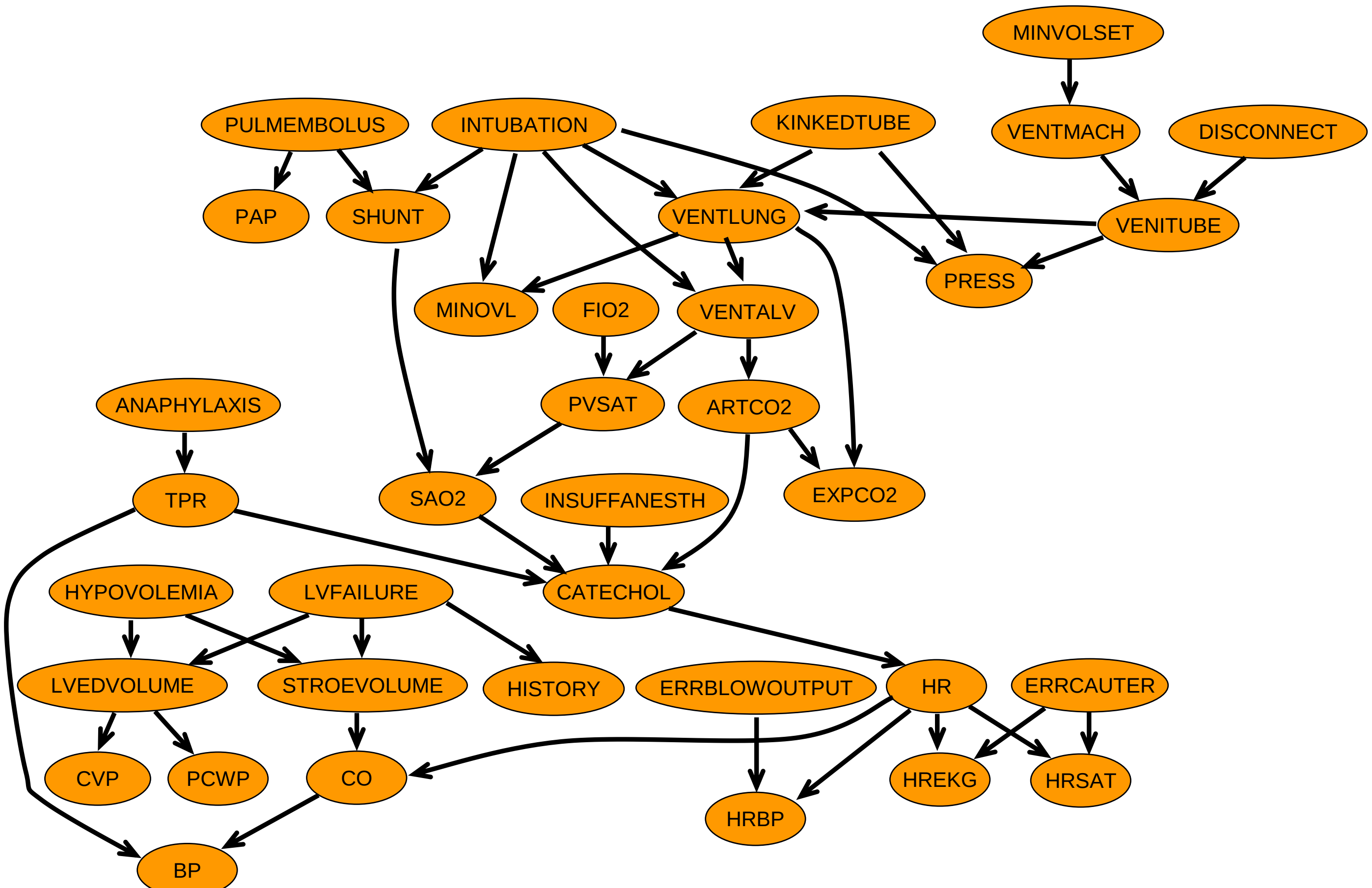
- Posterior marginals, probability of evidence, MPE
- P(D= 0) = $\sum_{S,L,B,X}$ P(S)· P(C|S)· P(B|S)· P(X|C,S)· P(D|C,B

MAP(P)= $max_{S,L,B,X}$ P(S)· P(C|S)· P(B|S)· P(X|C,S)· P(D|C,B)

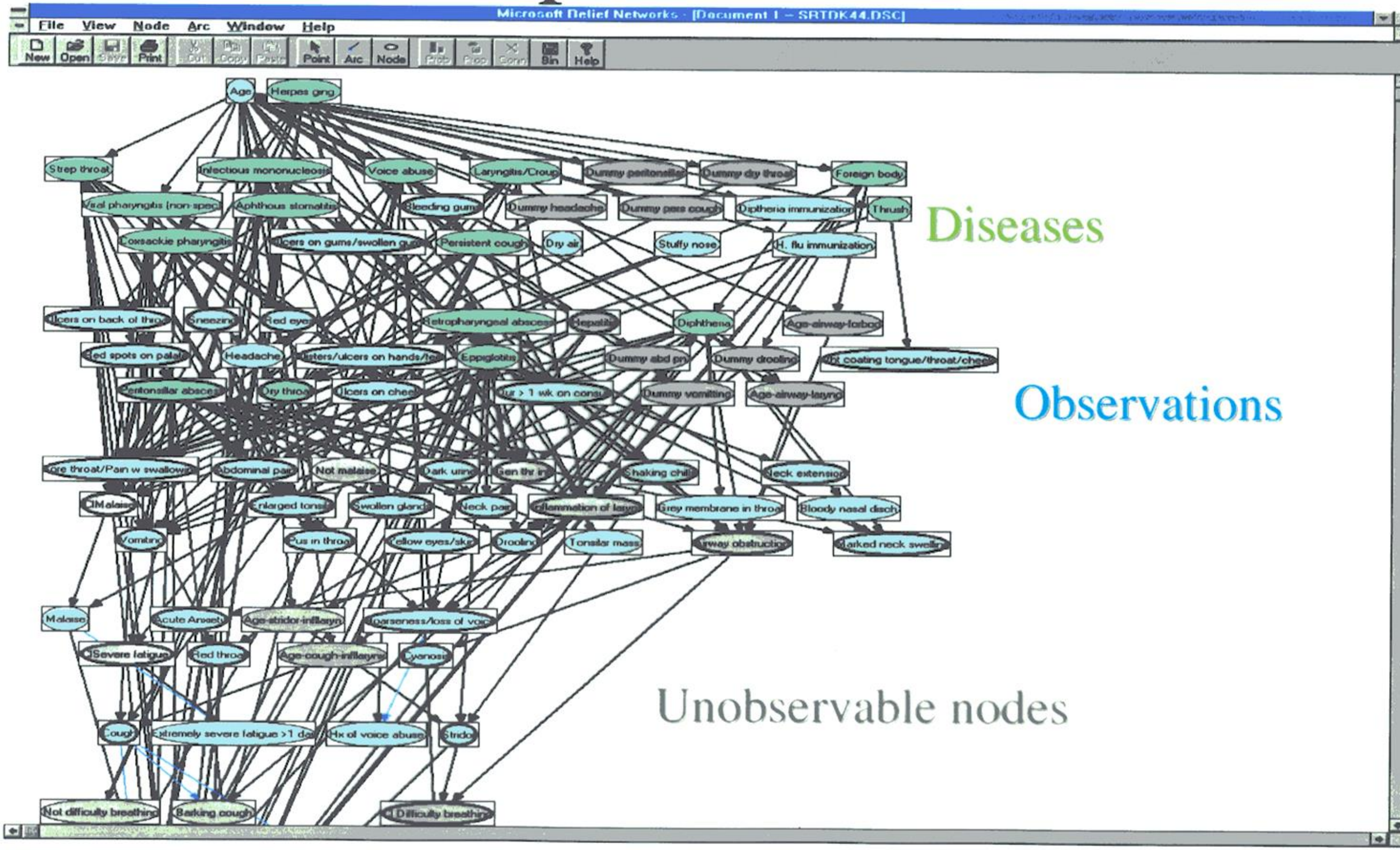
Alarm network [Beinlich et al., 1989]

- Bayes nets: compact representation of large joint distributions

The “alarm” network: 37 variables, 509 parameters (rather than $2^{37} = 10^{11}$!)



Chief Complaint: Sore Throat



KI

Constraint Networks

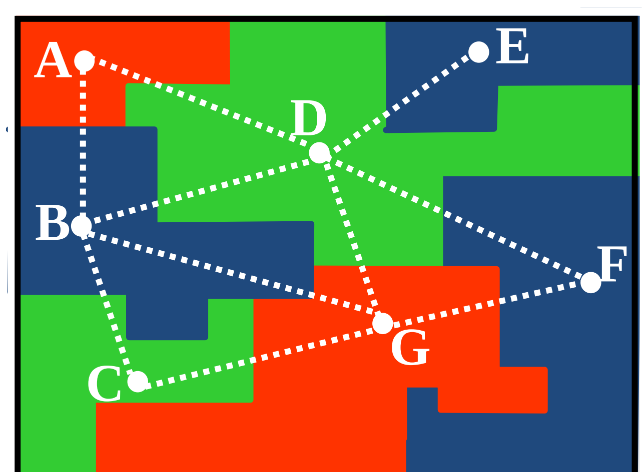
Example: map coloring

Variables - countries (A,B,C,etc.)

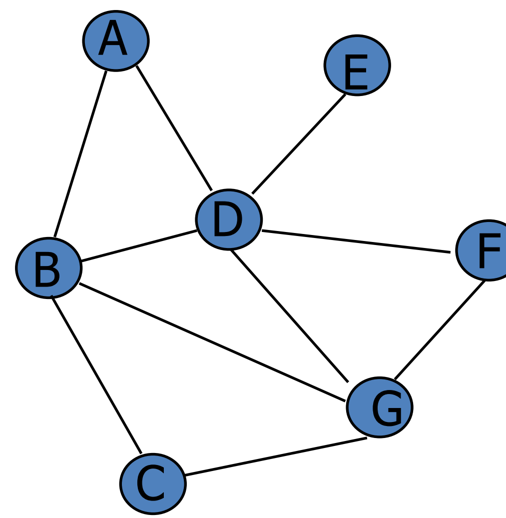
Values - colors (red, green, blue)

Constraints: **A ≠ B, A ≠ D, D ≠ E,** *etc.*

A	B
red	green
red	yellow
green	red
green	yellow
yellow	green
yellow	red



Constraint graph



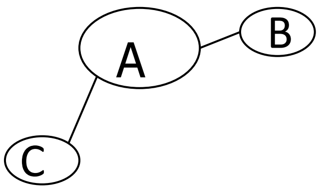
Propositional Reasoning

Example: party problem

- If Alex goes, then Becky goes: $A \rightarrow B$
- If Chris goes, then Alex goes: $C \rightarrow A$

(Alex goes = A, Becky goes = B, Chris goes = C, Alex goes = A)

- **Question:**
  Is it possible that Chris goes to the party but Becky does not?



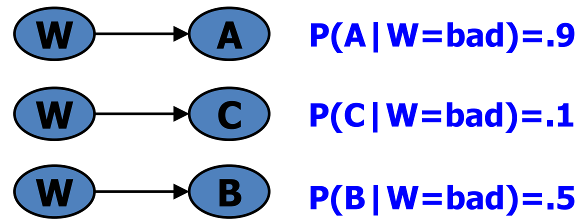
Is the *propositional theory*

$\varphi = \{A \rightarrow B, C \rightarrow A, \neg \mathbf{B}, \mathbf{C}\}$ satisfiable?

Probabilistic reasoning (directed)

Party example: the weather effect

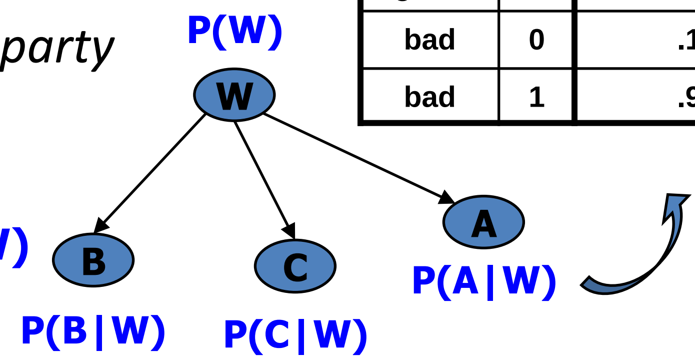
- Alex is-likely-to-go in bad weather
- Chris rarely-goes in bad weather
- Becky is indifferent but unpredictable



Questions:

- *Given bad weather, which group of individuals is most likely to show up at the party?*
- *What is the probability that Chris goes to the party but Becky does not?*

W	A	P(A|W)
good	0	.01
good	1	.99
bad	0	.1
bad	1	.9



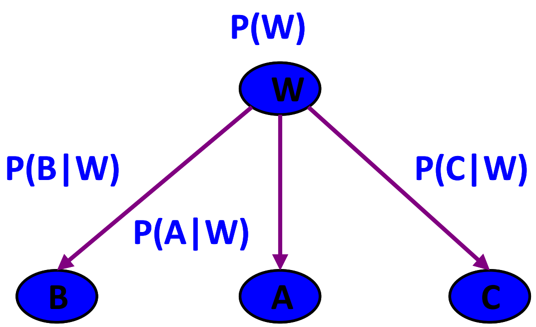
$P(W,A,C,B) = P(B|W) \cdot P(C|W) \cdot P(A|W) \cdot P(W)$

$P(A,C,B|W=bad) = 0.9 \cdot 0.1 \cdot 0.5$

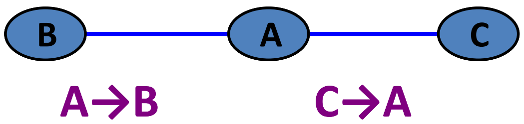
Mixed Probabilistic and Deterministic networks

Alex is-likely-to-go in bad weather
Chris rarely-goes in bad weather
Becky is indifferent but unpredictable

PN



CN



Query:
Is it likely that Chris goes to the party if Becky does not but the weather is bad?

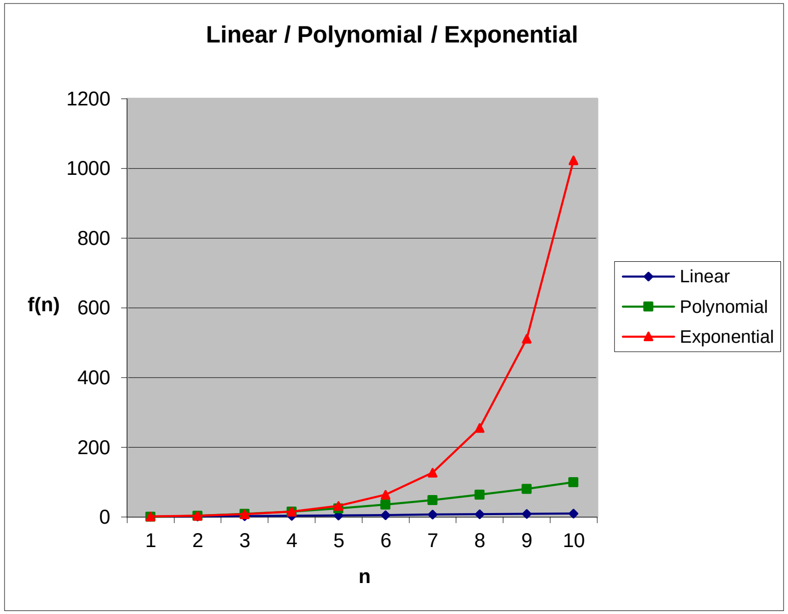
$$P(C, \neg B \mid w = bad, A \rightarrow B, C \rightarrow A)$$

Example domains for graphical models

- Natural Language processing
  - Information extraction, semantic parsing, translation, topic models, …
- Computer vision
  - Object recognition, scene analysis, segmentation, tracking, …
- Computational biology
  - Pedigree analysis, protein folding and binding, sequence matching, …
- Networks
  - Webpage link analysis, social networks, communications, citations, ….
- Robotics
  - Planning & decision making

Complexity of Reasoning Tasks

- Constraint satisfaction
- Counting solutions
- Combinatorial optimization
- Belief updating
- Most probable explanation
- Decision-theoretic planning



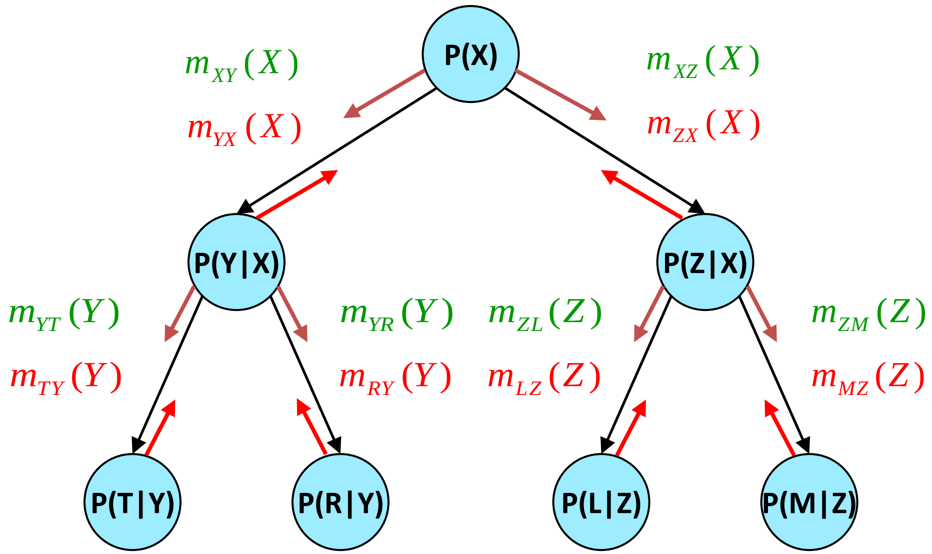
Reasoning is computationally hard

Complexity is
Time and space(memory)

Tree-solving is easy

Belief updating (sum-prod)

CSP – consistency (projection-join)



MPE (max-prod)

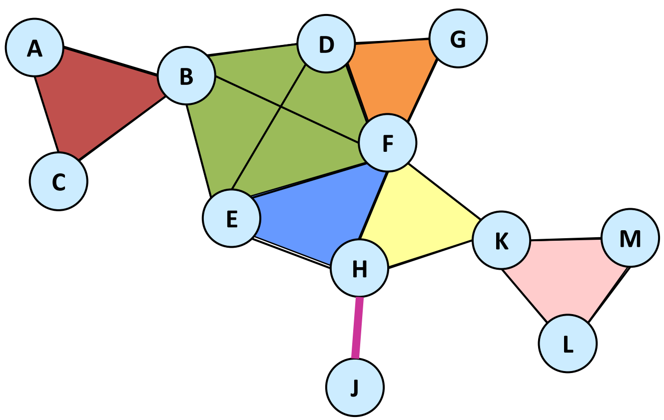
#CSP (sum-prod)

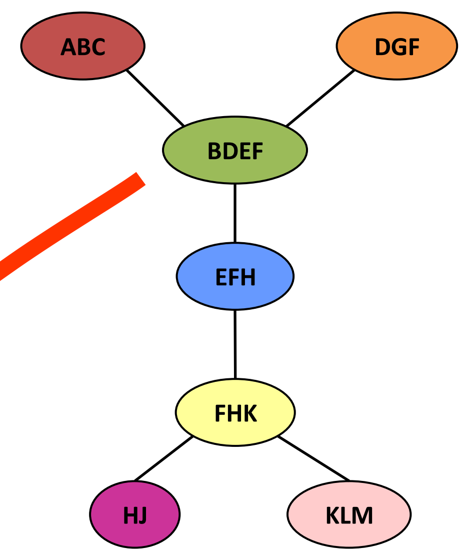
Trees are processed in linear time and memory

Transforming into a Tree

- **By Inference (thinking)**
  - Transform into a single, equivalent tree of sub-problems
- **By Conditioning (guessing)**
  - Transform into many tree-like sub-problems.

Inference and Treewidth





Inference algorithm:

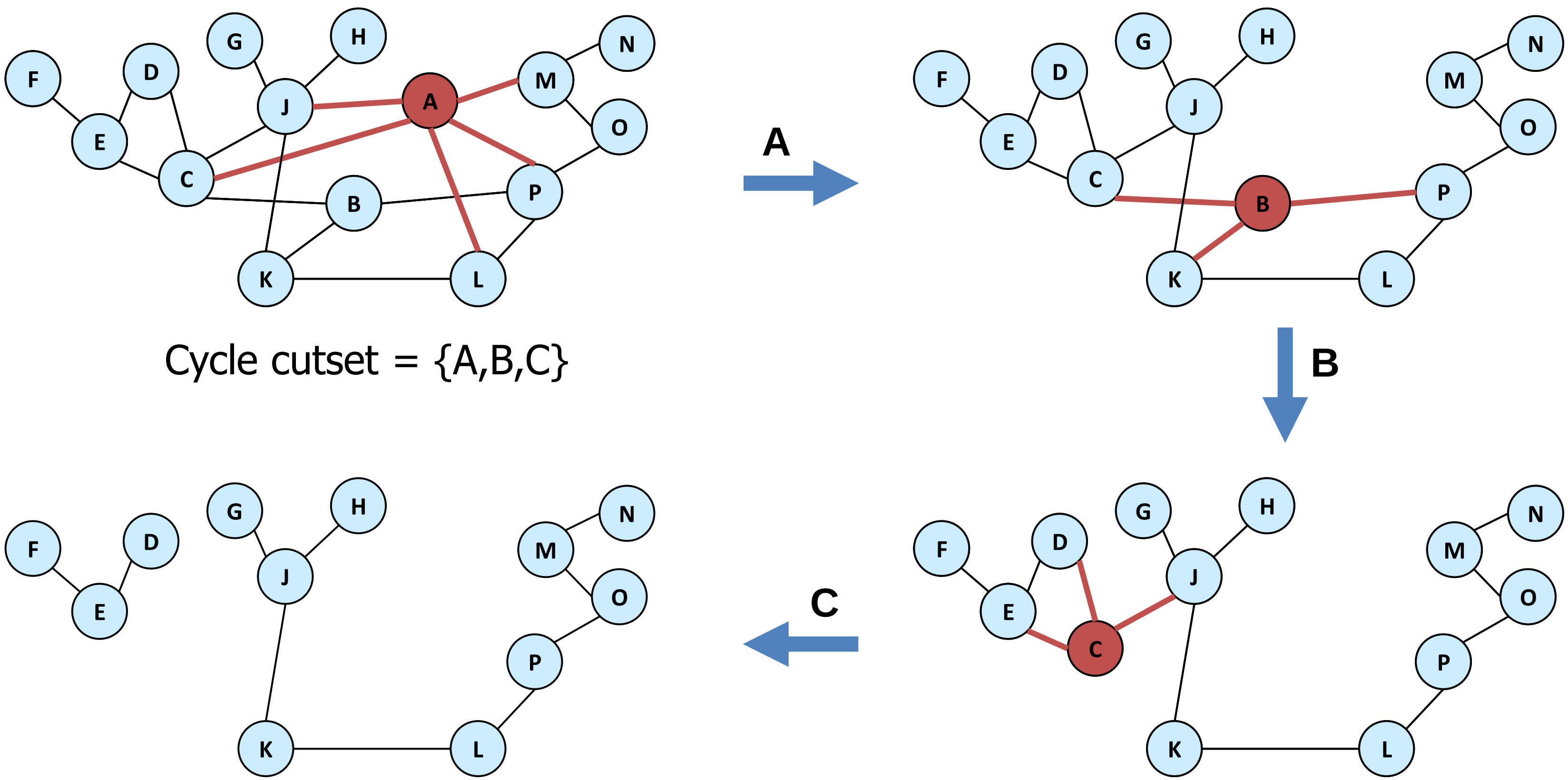
Time: exp(tree-width)

Space: exp(tree-width)

treewidth = 4 - 1 = 3

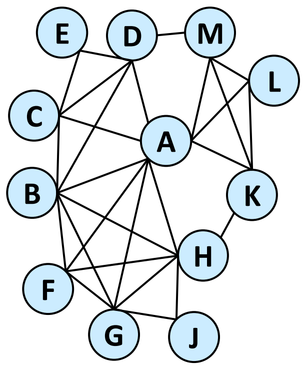
treewidth = (maximum cluster size) - 1

Conditioning and Cycle cutset

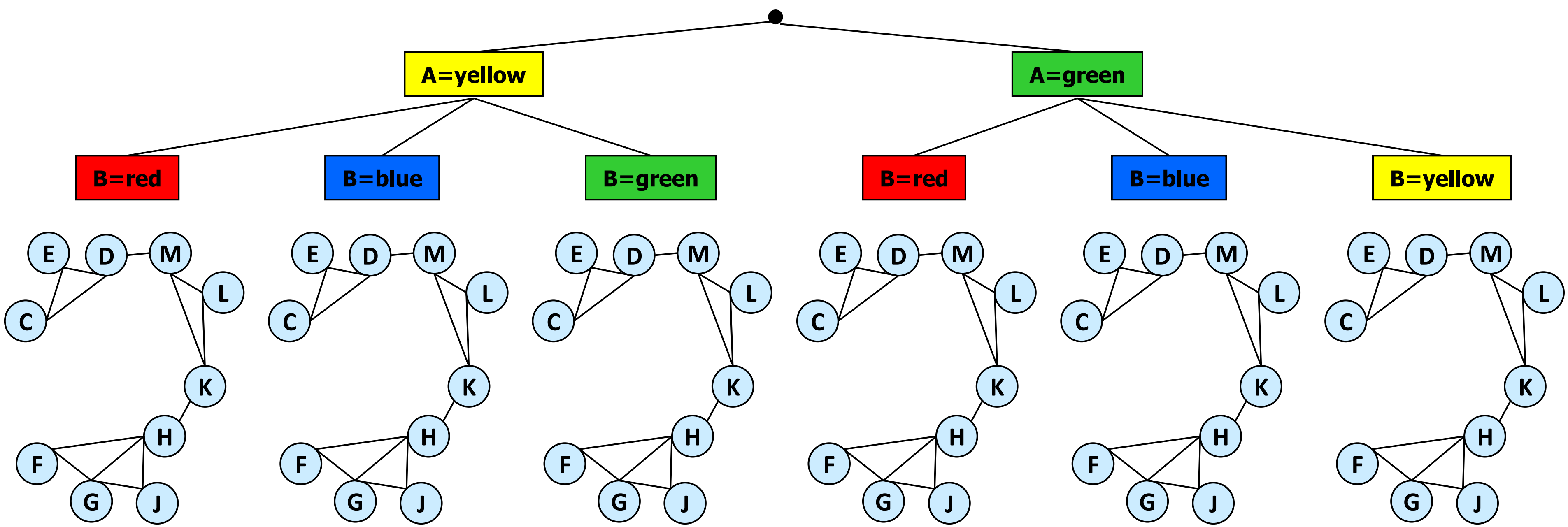


Search over the Cutset

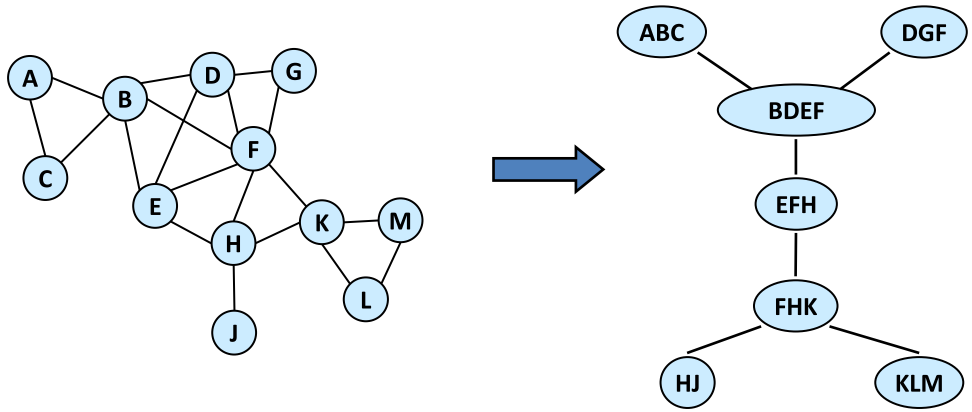
Graph Coloring problem



- Inference may require too much memory
- **Condition** on some of the variables

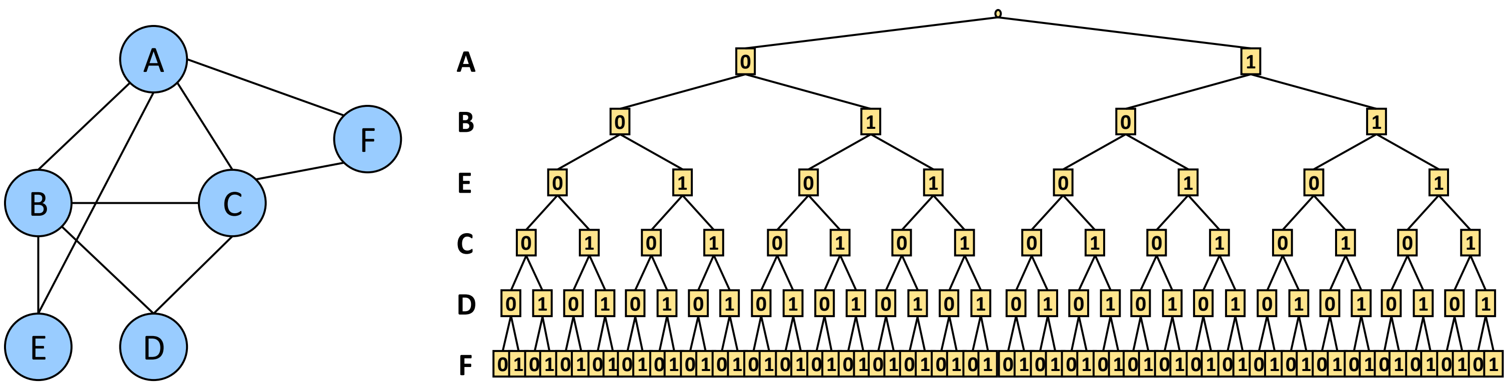


Bird's-eye View of Exact Algorithms



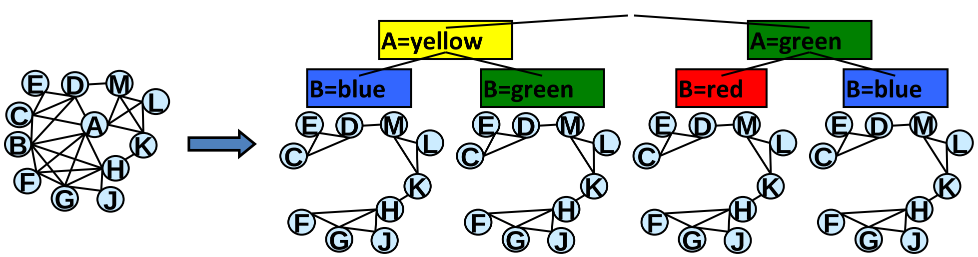
Inference

exp(w*) time/space



Search

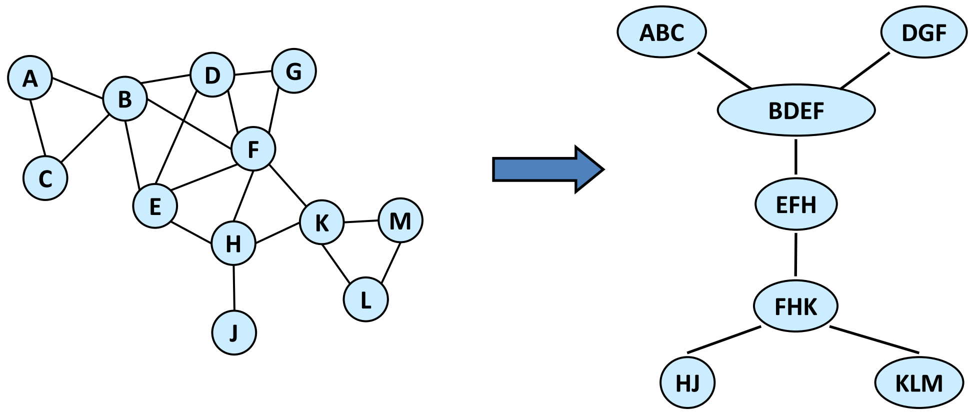
Exp(w*) time
O(w*) space



Search+inference:
Space: exp(q)
Time: exp(q+c(q))

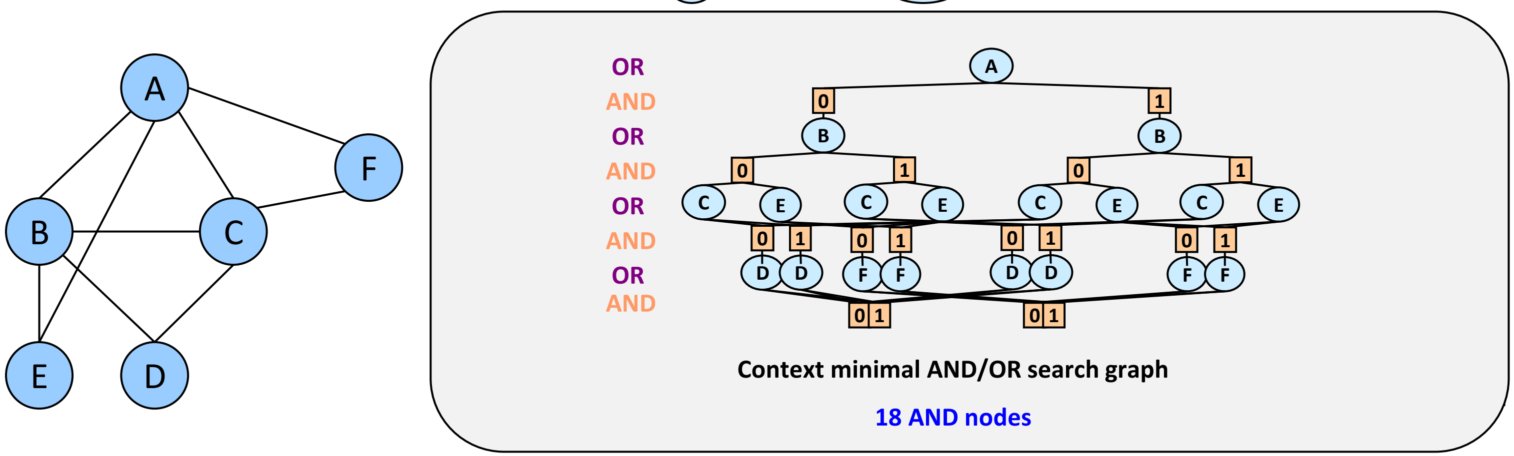
q: user controlled

Bird's-eye View of Exact Algorithms



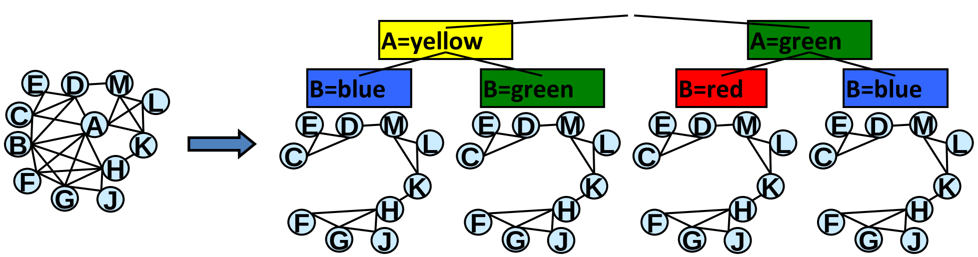
Inference

exp(w*) time/space



Search

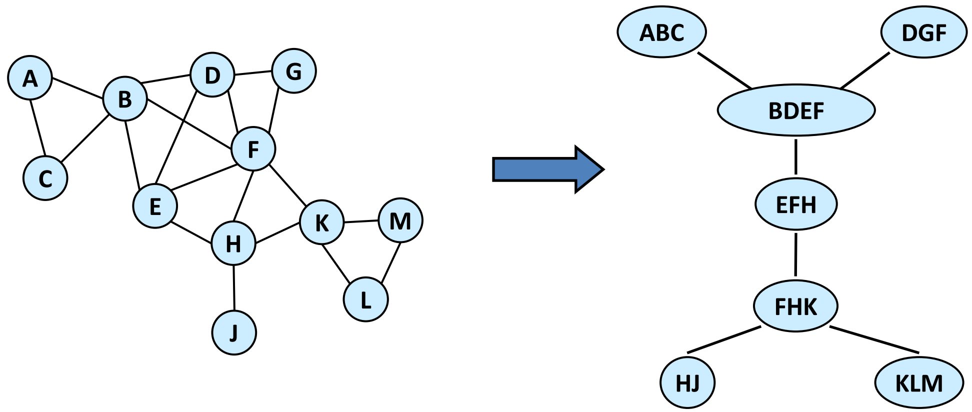
Exp(w*) time
O(w*) space



Search+inference:
Space: exp(q)
Time: exp(q+c(q))

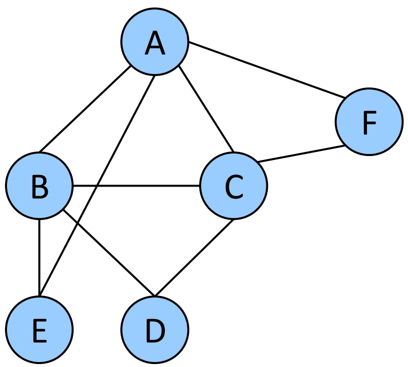
q: user controlled

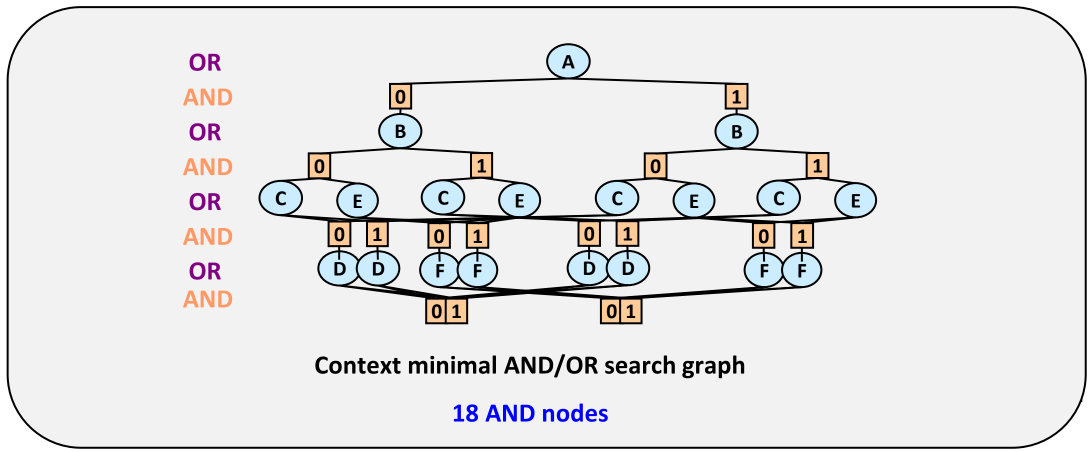
Bird's-eye View of Approximate Algorithms



Inference

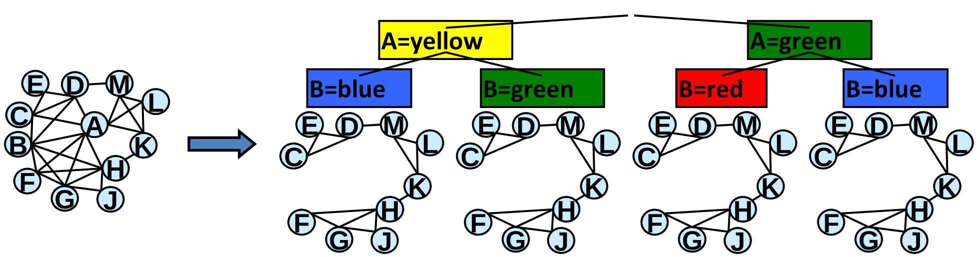
Bounded Inference





Search

Sampling



Search + inference:

Sampling + bounded inference

Examples: Common Sense Reasoning

- **Figuring time on a plane☹Sep 2021**): window view show night time, yest arrival in an hour to an bright afternoon. How would night become a day all of a sudden?
- **Zebra on Pajama**: (7:30 pm): I told Susannah: you have a nice pajama, but it was just a dress. Why jump to that conclusion?: 1. because time is night time. 2. certain designs look like pajama.
- **Cars going out of a parking lot:** You enter a parking lot which is quite full (UCI), you see a car coming : you think ah… now there is a space (vacated), OR… there is no space and this guy is looking and leaving to another parking lot. What other clues can we have?
- **Robot gets out at a wrong level:** A robot goes down the elevator. stops at 2nd floor instead of ground floor. It steps out and should immediately recognize not being in the right level, and go back inside.
- **Turing quotes**
  - If machines will not be allowed to be fallible they cannot be intelligent
  - (Mathematicians are wrong from time to time so a machine should also be allowed)

Why Uncertainty?

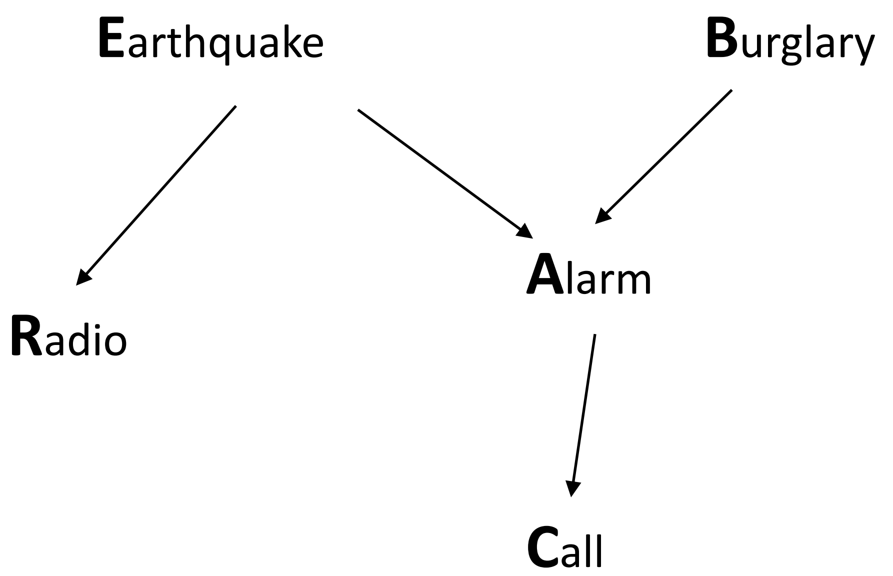
- AI goal: to have a declarative, model-based, framework that allows computer system to reason.
- People reason with partial information
- Sources of uncertainty:
  - Limitation in observing the world: e.g., a physician see symptoms and not exactly what goes in the body when he performs diagnosis. Observations are noisy (test results are inaccurate)
  - Limitation in modeling the world,
  - maybe the world is not deterministic.

Why/What/How Uncertainty?

- Why Uncertainty?
  - Answer: It is abundant
- What formalism to use?
  - Answer: Probability theory
- How to overcome exponential representation?
  - Answer: Graphs, graphs, graphs… to capture irrelevance, independence, causality

Basics of Probabilistic Calculus (Chapter 3)

The Burglary Example



Degrees of Belief

- Assign a degree of belief or probability in $[0, 1]$ to each world ω and denote it by $\Pr(\omega)$.
- The belief in, or probability of, a sentence α:

$$\Pr(\alpha) \stackrel{def}{=} \sum_{\omega \models \alpha} \Pr(\omega).$$

world	Earthquake	Burglary	Alarm	$\Pr(.)$
ω_1	true	true	true	.0190
ω_2	true	true	false	.0010
ω_3	true	false	true	.0560
ω_4	true	false	false	.0240
ω_5	false	true	true	.1620
ω_6	false	true	false	.0180
ω_7	false	false	true	.0072
ω_8	false	false	false	.7128

Properties of Beliefs

- A bound on the belief in any sentence:

$$0 \leq \Pr(\alpha) \leq 1 \qquad \text{for any sentence } \alpha.$$

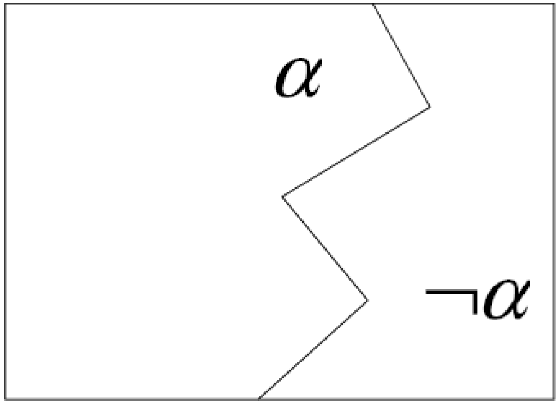
- A baseline for inconsistent sentences:

$$\Pr(\alpha) = 0 \qquad \text{when } \alpha \text{ is inconsistent.}$$

- A baseline for valid sentences:

$$\Pr(\alpha) = 1 \qquad \text{when } \alpha \text{ is valid.}$$

Properties of Beliefs



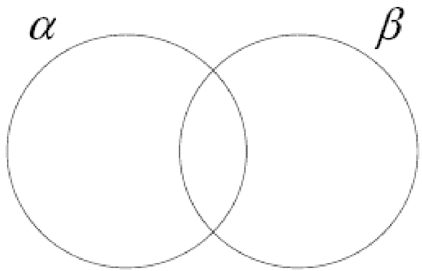
- The belief in a sentence given the belief in its negation:

$$\Pr(\alpha) + \Pr(\neg\alpha) = 1.$$

Example

$$\begin{aligned} \Pr(\text{Burglary}) &= \Pr(\omega_1) + \Pr(\omega_2) + \Pr(\omega_5) + \Pr(\omega_6) = .2 \\ \Pr(\neg\text{Burglary}) &= \Pr(\omega_3) + \Pr(\omega_4) + \Pr(\omega_7) + \Pr(\omega_8) = .8 \end{aligned}$$

Properties of Beliefs



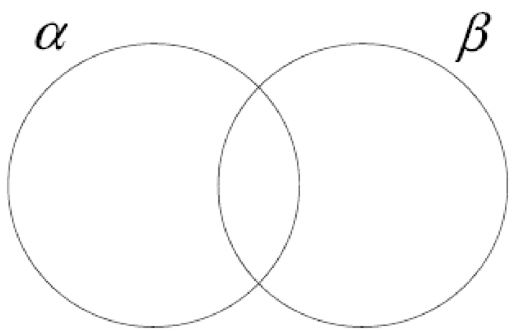
- The belief in a disjunction:

$$\Pr(\alpha \vee \beta) = \Pr(\alpha) + \Pr(\beta) - \Pr(\alpha \wedge \beta).$$

- Example:

$$\begin{aligned}
\Pr(\text{Earthquake}) &= \Pr(\omega_1) + \Pr(\omega_2) + \Pr(\omega_3) + \Pr(\omega_4) = .1 \\
\Pr(\text{Burglary}) &= \Pr(\omega_1) + \Pr(\omega_2) + \Pr(\omega_5) + \Pr(\omega_6) = .2 \\
\Pr(\text{Earthquake} \wedge \text{Burglary}) &= \Pr(\omega_1) + \Pr(\omega_2) = .02 \\
\Pr(\text{Earthquake} \vee \text{Burglary}) &= .1 + .2 - .02 = .28
\end{aligned}$$

Properties of Beliefs



- The belief in a disjunction:

  $\Pr(\alpha \vee \beta) = \Pr(\alpha) + \Pr(\beta)$ when α and β are mutually exclusive.

Entropy

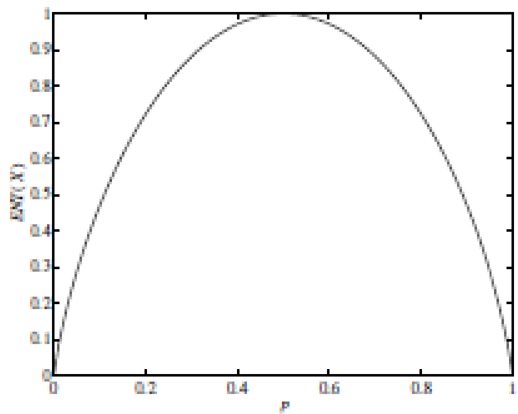
Quantify uncertainty about a variable X using the notion of entropy:

$$\mathrm{ENT}(X) \stackrel{def}{=} -\sum_{x} \Pr(x) \log_2 \Pr(x),$$

where $0 \log 0 = 0$ by convention.

	Earthquake	Burglary	Alarm
true	.1	.2	.2442
false	.9	.8	.7558
ENT(.)	.469	.722	.802

Entropy



- The entropy for a binary variable X and varying $p = \Pr(X)$.
- Entropy is non-negative.
- When $p = 0$ or $p = 1$, the entropy of X is zero and at a minimum, indicating no uncertainty about the value of X.
- When $p = \frac{1}{2}$, we have $\Pr(X) = \Pr(\neg X)$ and the entropy is at a maximum (indicating complete uncertainty).

Bayes Conditioning

Alpha and beta are events

Closed form for Bayes conditioning:

$$\Pr(\alpha|\beta) = \frac{\Pr(\alpha \wedge \beta)}{\Pr(\beta)}.$$

Defined only when $\Pr(\beta) \neq 0$.

Degrees of Belief

world	Earthquake	Burglary	Alarm	$\Pr(.)$
ω_1	true	true	true	.0190
ω_2	true	true	false	.0010
ω_3	true	false	true	.0560
ω_4	true	false	false	.0240
ω_5	false	true	true	.1620
ω_6	false	true	false	.0180
ω_7	false	false	true	.0072
ω_8	false	false	false	.7128

$$
\begin{aligned}
\Pr(\text{Earthquake}) &= \Pr(\omega_1) + \Pr(\omega_2) + \Pr(\omega_3) + \Pr(\omega_4) = .1 \\
\Pr(\text{Burglary}) &= .2 \\
\Pr(\neg\text{Burglary}) &= .8 \\
\Pr(\text{Alarm}) &= .2442
\end{aligned}
$$

Belief Change

Burglary is independent of Earthquake

Conditioning on evidence Earthquake:

$$\begin{aligned}\Pr(\text{Burglary}) &= .2\\ \Pr(\text{Burglary}|\text{Earthquake}) &= .2\\ \\ \Pr(\text{Alarm}) &= .2442\\ \Pr(\text{Alarm}|\text{Earthquake}) &\approx .75 \uparrow\end{aligned}$$

The belief in Burglary is not changed, but the belief in Alarm increases.

Belief Change

Earthquake is independent of burglary

Conditioning on evidence Burglary:

$$\begin{aligned} \Pr(\text{Alarm}) &= .2442 \\ \Pr(\text{Alarm}|\text{Burglary}) &\approx .905 \uparrow \\ \\ \Pr(\text{Earthquake}) &= .1 \\ \Pr(\text{Earthquake}|\text{Burglary}) &= .1 \end{aligned}$$

The belief in Alarm increases in this case, but the belief in Earthquake stays the same.

Belief Change

The belief in Burglary increases when accepting the evidence Alarm. How would such a belief change further upon obtaining more evidence?

- Confirming that an Earthquake took place:

$$\begin{aligned} \Pr(\text{Burglary}|\text{Alarm}) &\approx .741 \\ \Pr(\text{Burglary}|\text{Alarm} \wedge \text{Earthquake}) &\approx .253 \downarrow \end{aligned}$$

  We now have an explanation of Alarm.

- Confirming that there was no Earthquake:

$$\begin{aligned} \Pr(\text{Burglary}|\text{Alarm}) &\approx .741 \\ \Pr(\text{Burglary}|\text{Alarm} \wedge \neg\text{Earthquake}) &\approx .957 \uparrow \end{aligned}$$

  New evidence will further establish burglary as an explanation.

Conditional Independence

$\Pr$ finds α conditionally independent of β given γ iff

$$\Pr(\alpha|\beta \wedge \gamma) = \Pr(\alpha|\gamma) \quad \text{or } \Pr(\beta \wedge \gamma) = 0.$$

Another definition

$$\Pr(\alpha \wedge \beta|\gamma) = \Pr(\alpha|\gamma)\Pr(\beta|\gamma) \quad \text{or } \Pr(\gamma) = 0.$$

Variable Independence

$\Pr$ finds **X** independent of **Y** given **Z**, denoted $I_{\Pr}(\mathbf{X}, \mathbf{Z}, \mathbf{Y})$, means that $\Pr$ finds **x** independent of **y** given **z** for all instantiations **x**, **y** and **z**.

Example

$\mathbf{X} = \{A, B\}$, $\mathbf{Y} = \{C\}$ and $\mathbf{Z} = \{D, E\}$, where A, B, C, D and E are all propositional variables. The statement $I_{\Pr}(\mathbf{X}, \mathbf{Z}, \mathbf{Y})$ is then a compact notation for a number of statements about independence:

$A \wedge B$ is independent of C given $D \wedge E$;

$A \wedge \neg B$ is independent of C given $D \wedge E$;

$\vdots$

$\neg A \wedge \neg B$ is independent of $\neg C$ given $\neg D \wedge \neg E$;

That is, $I_{\Pr}(\mathbf{X}, \mathbf{Z}, \mathbf{Y})$ is a compact notation for $4 \times 2 \times 4 = 32$ independence statements of the above form.

Further Properties of Beliefs

Chain rule

$$\begin{aligned}&\Pr(\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n)\\ &= \Pr(\alpha_1 | \alpha_2 \wedge \ldots \wedge \alpha_n)\Pr(\alpha_2 | \alpha_3 \wedge \ldots \wedge \alpha_n) \ldots \Pr(\alpha_n).\end{aligned}$$

Case analysis (law of total probability)

$$\Pr(\alpha) = \sum_{i=1}^{n} \Pr(\alpha \wedge \beta_i),$$

where the events $\beta_1, \ldots, \beta_n$ are mutually exclusive and exhaustive.

Further Properties of Beliefs

Another version of case analysis

$$\Pr(\alpha) = \sum_{i=1}^{n} \Pr(\alpha|\beta_i)\Pr(\beta_i),$$

where the events $\beta_1, \ldots, \beta_n$ are mutually exclusive and exhaustive.

Two simple and useful forms of case analysis are these:

$$\begin{aligned} \Pr(\alpha) &= \Pr(\alpha \wedge \beta) + \Pr(\alpha \wedge \neg\beta) \\ \Pr(\alpha) &= \Pr(\alpha|\beta)\Pr(\beta) + \Pr(\alpha|\neg\beta)\Pr(\neg\beta). \end{aligned}$$

The main value of case analysis is that, in many situations, computing our beliefs in the cases is easier than computing our beliefs in α. We shall see many examples of this phenomena in later chapters.

Further Properties of Beliefs

Bayes rule

$$\Pr(\alpha|\beta) = \frac{\Pr(\beta|\alpha)\Pr(\alpha)}{\Pr(\beta)}.$$

- Classical usage: α is perceived to be a cause of β.
- Example: α is a disease and β is a symptom–
- Assess our belief in the cause given the effect.
- Belief in an effect given its cause, $\Pr(\beta|\alpha)$, is usually more readily available than the belief in a cause given one of its effects, $\Pr(\alpha|\beta)$.

End of slides