Figure 1: Probability distributions for problem 3.

x	y	z	$p(x, y, z)$
0	0	0	0.12
0	0	1	0.18
0	1	0	0.04
0	1	1	0.16
1	0	0	0.09
1	0	1	0.21
1	1	0	0.02
1	1	1	0.18

Rina Dechter, Spring-2018
Assigned: April 18
Due: May 2nd

COMPSCI 276: Reasoning with Graphical Models, Problem Set 2

Relevant reading: Darwiche chapter 4 and Dechter1 chapter 4 and 5.
Please indicate clearly the question numbers that you answer.

1. (Extra credit, 5 pts) Read chapter 4 and 5 in Dechter1 and provide comments on clarity and typos.

2. **(20 pts)** (Darwiche 3.13) There are three urns labeled one, two and three. The urns contain, respectively, three white and three black balls, four white and two black balls, and one white and two black balls. An experiment consists of selecting an urn at random, then drawing a ball from it.

   (a) Define the set of worlds that correspond to the various outcomes of this experiments. Assume you have two variables U with values 1; 2; 3 and C with values black and white.

   (b) Define the joint probability distribution over the set of possible worlds identified above.

   (c) Find the probability of selecting urn 2 and drawing a black ball.

   (d) Find the probability of drawing a black ball.

   (e) Find the conditional probability that urn 2 was selected, given that a black ball was drawn.

   (f) Find the probability of selecting urn 1 or a white ball.

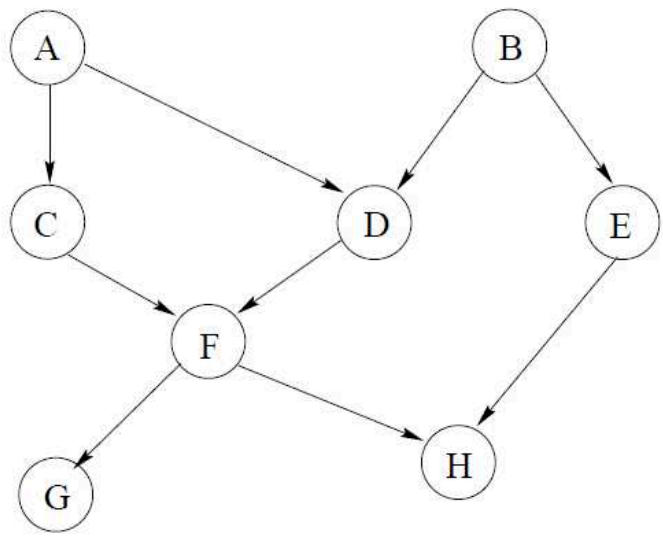
3. **(10 pts)** Use the joint-probability distribution in the table of Figure 1 to compute the following conditional probability for all values of x y and z.

   (a) $p(x|y,z)$

   (b) $p(y|x,z)$

   (c) $p(z|x,y)$

4. **(20 pts)** (Darwiche, exercise 4.1) Consider the DAG (Figure 4.14):



A	$\Pr(a)$
1	.2
0	.8

B	$\Pr(b)$
1	.7
0	.3

B	E	$\Pr(e\|b)$
1	1	.1
1	0	.9
0	1	.9
0	0	.1

A	B	D	$\Pr(d\|a,b)$
1	1	1	.5
1	1	0	.5
1	0	1	.6
1	0	0	.4
0	1	1	.1
0	1	0	.9
0	0	1	.8
0	0	0	.2

Figure 4.14: A Bayesian network with some of its CPTs.

   (a) List the Markovian assumptions asserted by the DAG.

   (b) Express $P(a,b,c,d,e,f,g,h)$ in terms of network parameters.

(c) Compute $P(A = 0, B = 0)$ and $P(E = 1|A = 1)$. Justify your answers.

(d) True or false? Why?

- $dsep(A, BH, E)$
- $dsep(G, D, E)$
- $dsep(AB, F, GH)$

5. **(Optional)** Consider a set of four variables $\{X, Y, Z, W\}$, which are related by:

$$I(X, \phi, Y) \text{ and } I(X, \{Y, W\}, Z).$$

Find the minimal list of independencies generated by the above two, satisfying each of the following conditions separately.

(a) The symmetry property.
(b) The symmetry and decomposition properties.
(c) The semigraphoid properties. (axioms 3.6a-3.6d)
(d) The graphoid properties. (axioms 3.6a-3.6e)

6. **(15 pt)** Referring to the directed graph in Figure 2, determine whether or not each of the following Probabilistic independencies is true using the D-separation criterion.

(a) $I(E, \phi, G)$.
(b) $I(C, \phi, D)$.
(c) $I(C, G, D)$.
(d) $I(B, A, C)$.
(e) $I(\{C, D\}, \phi, E)$.
(f) $I(F, A, \{E, H\})$.

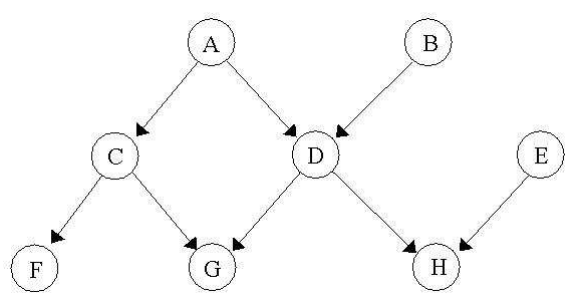
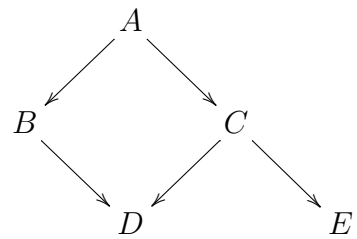


Figure 2: A directed graph.

7. **(15 pt)** (Question 4.14 in Darwiche book.) Suppose that the DAG



is a P-map of some distribution Pr. Construct a minimal I-map G' for Pr using each of the following variable orders:

(a) A, D, B, C, E

(b) A, B, C, D, E

(c) E, D, C, B, A

8. **(extra credit, 10 pt), Darwiche 4.24** Prove that the d-separation is equivalent to regular separation in an the ancestral graph. Namely that Z d-separates X from Y if in the moral graph that includes X, Y, Z and their ancestors Z separates X from Y.

9. **(30 pts)** Consider the Bayesian network in Figure 3.

   (a) Apply BE-bel to obtain the:

      i. marginal probability of variable F.

      ii. marginal probability of variable G

      iii. joint marginal probability of variables F and G.

   For each part show the schematic computation over buckets (you can take advantage of shared computation). The computation itself can be done manually or using any software tool of your choice.

   (b) Suppose now that the evidence $\{D = 0, C = 1\}$ has been observed. Apply BE-bel to obtain the probability of evidence. You can do calculation by hand or use any software tool to solve the computational parts of this question.

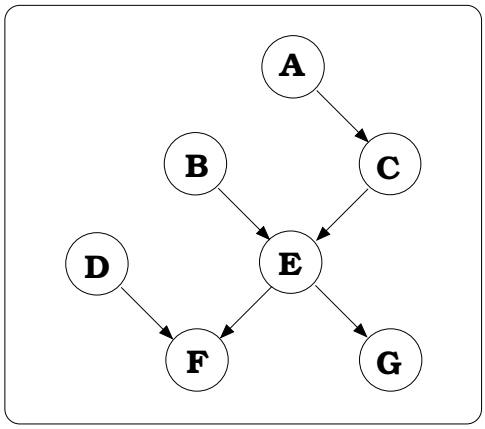
   (c) Explain how BE-mpe can find the most probable explanation (mpe) given the evidence F=0. Demonstrate your computation. Again, you can do calculation by hand or use any software tool to solve the computational parts of this question.

10. **(30 pts)** Given the directed graph G in Figure 2,

   (a) Compute the induced-graph along ordering: $d_1 = F, C, A, G, D, H, E, B$ and the induced-width for each variable. What is its induced-width of G?

   (b) Use min-induced width to compute an ordering, called d_2, and its ordered graph. Compute the induced-width along d_2.

   (c) What is the induced width of G? Explain your answer.



a	$p(a)$
0	0.3
1	0.7

b	$p(b)$
0	0.6
1	0.4

d	$p(d)$
0	0.7
1	0.3

a	c	$p(c\|a)$
0	0	0.15
0	1	0.85
1	0	0.25
1	1	0.75

e	g	$p(g\|e)$
0	0	0.10
0	1	0.90
1	0	0.30
1	1	0.70

b	c	e	$p(e\|b,c)$
0	0	0	0.40
0	0	1	0.60
0	1	0	0.45
0	1	1	0.55
1	0	0	0.60
1	0	1	0.40
1	1	0	0.30
1	1	1	0.70

d	e	f	$p(f\|d,e)$
0	0	0	0.25
0	0	1	0.75
0	1	0	0.60
0	1	1	0.40
1	0	0	0.10
1	0	1	0.90
1	1	0	0.20
1	1	1	0.80

Figure 3:

(d) Apply BE-bel along the ordering d_1 and show the λ functions created, their placement and the expressions for deriving the functions. (optional: do the same for ordering d_2)

(e) Now show the bucket-tree associated with the ordering $d_1 = F, C, A, G, D, H, E, B$ and display all the massages (πs and λs) along the tree.

(f) Assuming you performed all the computation without any evidence. How can you extract the marginal probability of D? Explain.

(g) Assume you compute the beliefs using join-tree clustering. What would be the time and space complexity. Explain.

11. **(20 pts)** (revised question 4.5 in Pearl's book) Consider the network in Figure 4.

(a) What is the dual graph of this network

(b) Find a join-tree representation for the network and show how you would compute $Bel(D_1|M_2 = false, M_3 = true, M_4 = false)$ schematically (demonstrating the type of messages that would be passed).

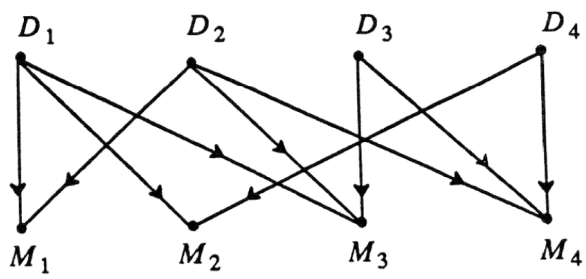


Figure 4: A two layer network