Causal Discovery

Adapted from CAUSALITY[1] chapter 2

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Motivation

I would rather discover one cause than gain the kingdom of Persia.
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2. What assumptions would allow us to infer causal models from these clues?
3. Will the inferred models be useful?

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Yes?

But humans do it!

Temporal Knowledge

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Thought experiment

Let A and B be dependent. Let $t_A < t_B$. Does $A \to B$?

No!

- A is rooster crowing, B is sun going up
- A is barometer falling, B is it raining outside

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- Some patterns *suggest* causality

Framework

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A causal model is a pair $M = \langle D, \Theta_D \rangle$ consisting of a causal structure D and a set of parameters Θ_D compatible with D. The parameters Θ_D assign a function $x_i = f_i(pa_i, u_i)$ for each X_i in D and where U_i is a random disturbance distributed according to $P(u_i)$ independently of all other u.

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One latent structure $L = \langle D, O \rangle$ is preferred to another $L' = \langle D', O \rangle$ ($L \preceq L'$) if and only if D' can mimic D over O. Two latent structures are equal ($L \equiv L'$) if and only if $L \preceq L'$ and $L \succeq L'$

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A latent structure L is minimal with respect to a class $\mathcal{L}$ of latent structures if and only if there is no member of $\mathcal{L}$ that is strictly preferred to L.

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A variable A has a causal influence on variable B if and only if there exists a directed path from A to B om every consistent minimal latent structure.

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Given the following observations over variables $\{A, B, C, D\}$:

- A is independent of B
- D is independent of $\{A, B\}$ given C
- There is no other independence

What is the minimal causal structure?

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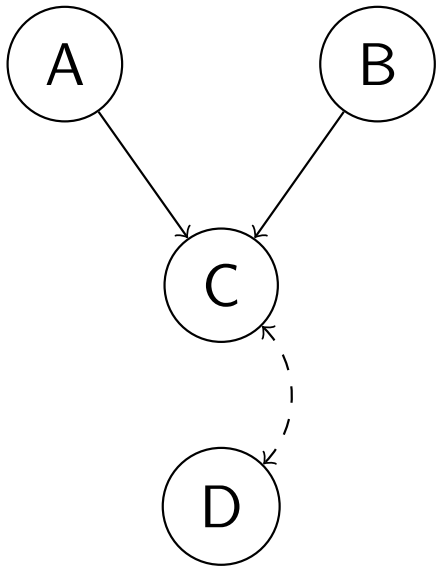

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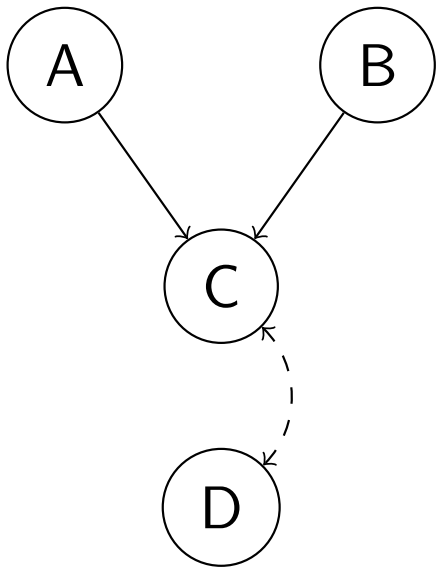

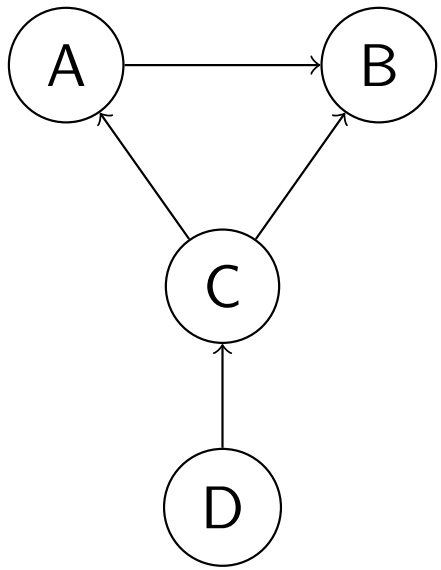

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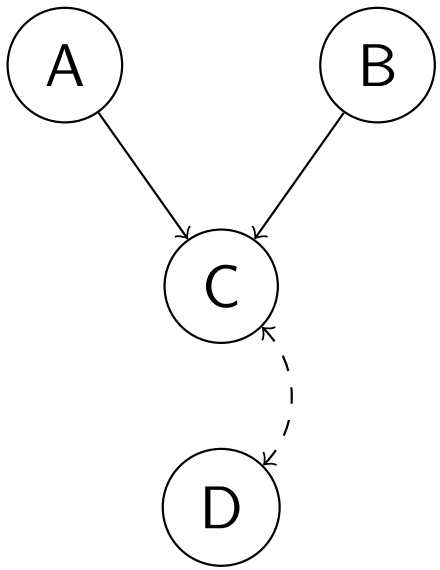

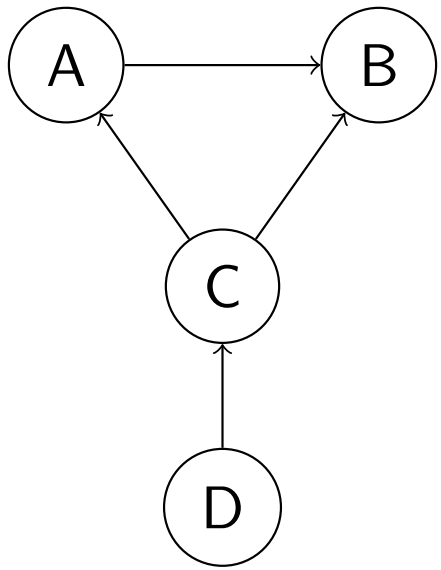

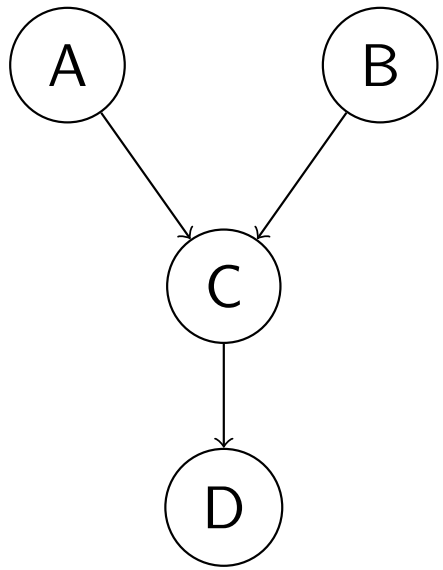

Example II

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- A is independent of B
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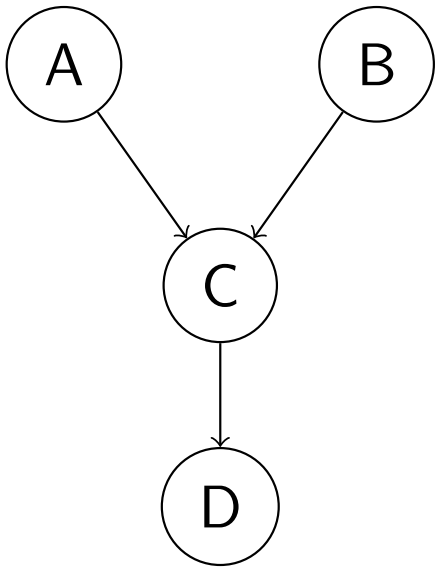
- Unique minimal structure?



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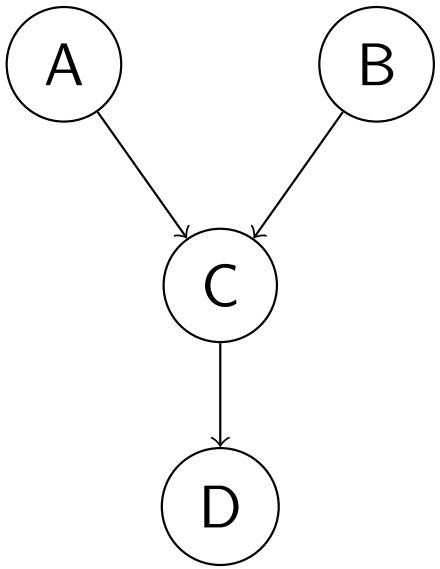

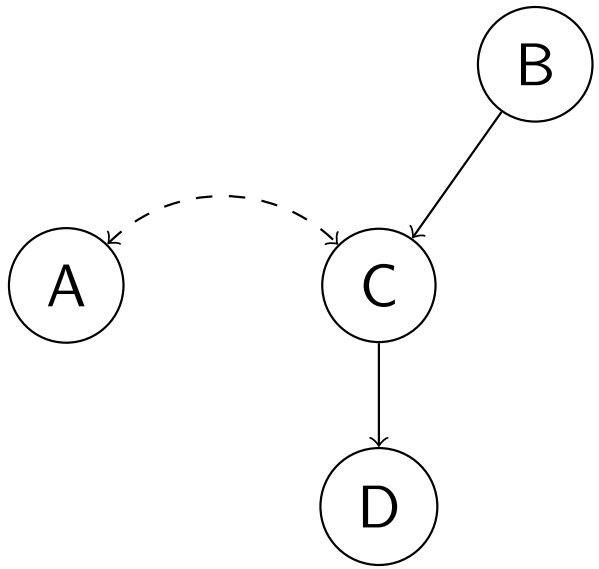

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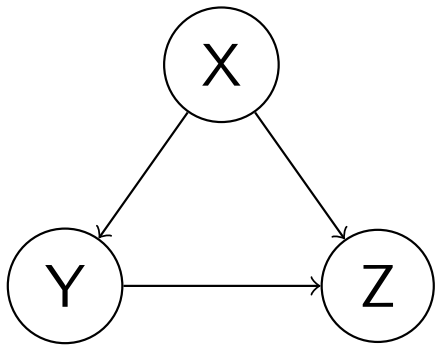

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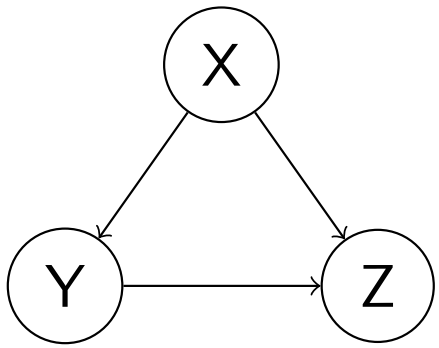

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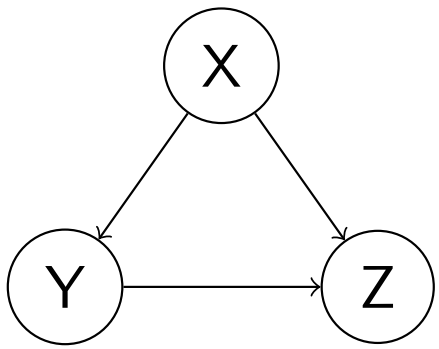

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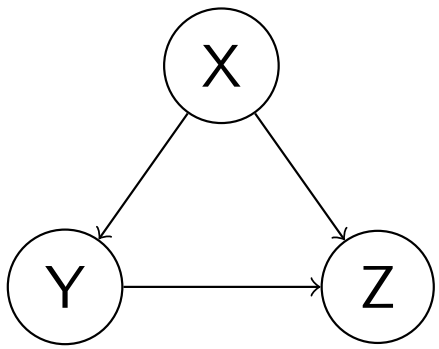

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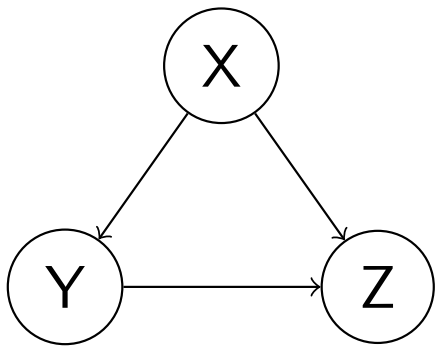

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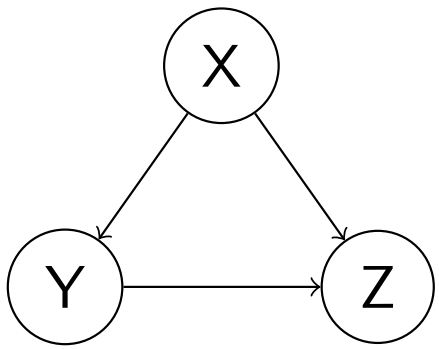

Homework Problem

- Recall:

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Homework

Provide a structural causal model with the above dependencies.
Describe why this model would not be assumed to be the structural causal model using the minimality and stability criteria.

Table of Contents

References I


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