# Causal Inference
# ICS 295 (Winter 2023)
# Homework 1

Due: Thurseday, January 26, 2023

Notice that Problems 1 and 2, and questions 5(a,c), have a (*). This means that you are encouraged to do these questions, but I will not grade them closely. As such, they are allotted a representative low score and will often be scored in a pass/fail manner, sometimes based solely on the attempt.

**Problem 1. (*) [1 point]**

*(*)* Read Primer chapters 1 and 2 and Chapter 1 of Book of Why (W). In a few sentences (not more than 4) discuss how these two sources (P and W) relate.

**Problem 2. *(*)* Study question 2.4.1 in the Primer [4 points]**

Figure 2.9 in the Primer book represents a causal graph from which the error terms have been deleted. Assume that all those errors are mutually independent.

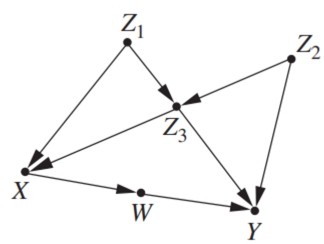



**Figure 2.9** A causal graph used in study question 2.4.1, all $U$ terms (not shown) are assumed independent

**(a)** [1 point] *(*)* For each pair of nonadjacent nodes in the graph, determined whether they are independent conditional on all other variables.

**(b)** [1 point] *(*)* For every variable $V$in the graph find a minimal set of nodes that renders $V$ independent of all other variables in the graph.

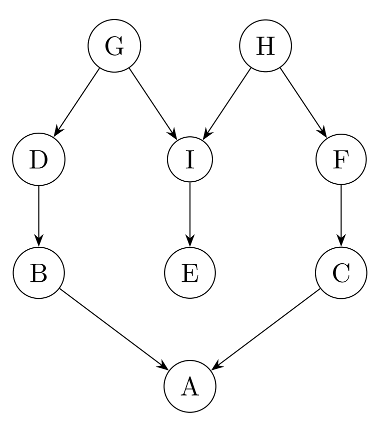
**(c)** [1 point] *(*)* Suppose we wish to estimate the value of $Y$ from measurements taken on all other variables in the model. Find the smallest set of variables that would yield as good an estimate of $Y$as when we measured all variables.

**(d)** [1 point] *(*)* Repeat question (e) assuming that we wish to estimate the value of $Z_2$.

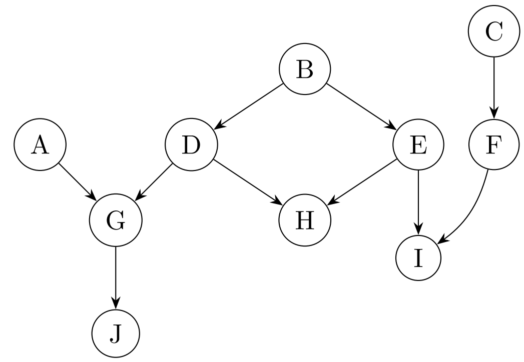
**Problem 3. D-Separation [20 points]**

For the two graphs below, determine which variables are d-connected to $A$ given $S$

**(a)** [10 points] $S = \{B\}$

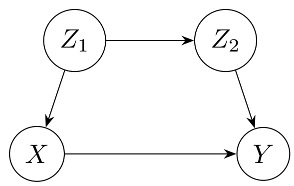


**(b)** [10 points] $S = \{J\}$




**Problem 4. Query Estimation** [**40 points**]

Consider the following graphical model:

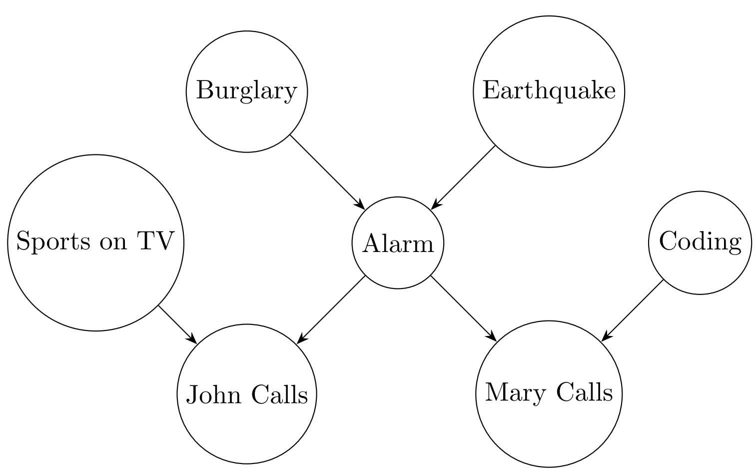



The target query is $Q = \sum_{Z_1} P(y \mid x, z_1) P(z_1)$.

**(a)** [20 points] Is $Q = P(y \mid x)$? Justify your answer.

**(b)** [20 points] Suppose that only $P(X, Y, Z_2)$ is give as input. Is $Q$ estimable? If so, show how to do it. Otherwise, explain why is that the case.

**Problem 5. Graphical Models, (parts (a) and (c) optional)** [**25 points**]




| P(B=1) | P(E=1) | P(S=1) | P(C=1) |
|---|---|---|---|
| 0.02 | 0.01 | 0.80 | 0.15 |

| B | E | $P(A=1 \vert BE)$ | A | C | $P(M=1 \vert AC)$ | A | S | $P(J=1 \vert AS)$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0.01 | 0 | 0 | 0.05 | 0 | 0 | 0 |
| 0 | 1 | 0.3 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0.9 | 1 | 0 | 0.85 | 1 | 0 | 0.97 |
| 1 | 1 | 0.98 | 1 | 1 | 0.15 | 1 | 1 | 0.1 |

**(a)** [1 point] *(*)* List all d-separation statements that hold assuming that John Calls (i.e., $J = 1$).

**(b)** [20 points] Construct a Bayesian network structure with nodes Burglary, Earthquake, JohnCall,MaryCall, SportsOnTV, Coding (ie, node Alarm is removed), which is a minimal I-map for the marginal distribution over these variables defined by the above network. Be sure to get all dependencies that remain from the original network! In particular note that there must be an arrow between $J$ and $M$ since now there is no set that could separate them. Depending on the direction of this arrow we need to add another one. In the example above after adding $J \to M$ it seems that $(S \perp\!\!\!\perp M \mid J)_G$ but this is not the case in $P$, hence we need an arrow $S \to M$ so that $G$ is a valid I-map of $P$.

**(c)** [4 points] *(*)* Compute the given probabilities:

**(i)** [1 point] *(*)* $P(M = 1)$

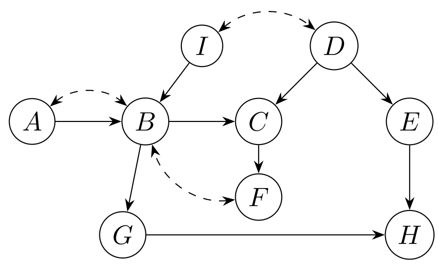
**(ii)** [1 point] *(*)* $P(J = 1|C = 0)$

**(iii)** [1 point] *(*)* $P(E = 1|M = 1, B = 0)$

**(iv)** [1 point] *(*)* $P(M = 1|B = 1, J = 0)$

**Problem 6. d-Separation [16 points]**

**(a)** Consider the following causal diagram:




For each case find a set that, when conditioned, on d-separates the given pair of (sets) variables

**(i)** [4 points] $A$ and $F$

**(ii)** [4 points] $A$ and $C$

**(iii)** [4 points] $D$ and $\{F, H\}$

**(iv)** [4 points] $I$ and $H$