CS 295: Causal Reasoning

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The Identification Problem

The Front-Door Criterion, The Do-calculus

Based on Elias Bareinboim slides

Primer, chapter 3, Causality 3.3,3.4,2.5, Biometrica 1995)

Outline

Recapping Backdoor
Computing bd: Inverse probability weighting
Conditional intervention
Front door condition
The do calculus

Frontdoor Criteria

$P(Y_x)$ = P(Y|do(x))

Consider a causal graph G and causal effect $Pr(y_x)$. A set of variables $\mathbf{Z}$ satisfies the frontdoor criteria iff

⋮

If $\mathbf{Z}$ is a frontdoor, then

$$Pr(y_x) = \sum_{\mathbf{z}} Pr(\mathbf{z}|x) \sum_{x'} Pr(y|x', \mathbf{z}) Pr(x')$$

interventional associational

1. $(Y \perp\!\!\!\perp X \mid Z, U_{xy})$
2. $(Z \perp\!\!\!\perp U_{xy} \mid X)$

The Front-door Case

Re-writing the interventional distribution…

$$P(\mathbf{v} \mid do(x)) = \sum_{\mathbf{u}} P(x \mid u_{xy}, u_x) P(z \mid x, u_z) P(y \mid z, u_{xy}, u_y) P(\mathbf{u})$$

$$= \left(\sum_{u_z} P(z \mid x, u_z) P(u_z) \right) \left(\sum_{u_{xy}, u_y} P(y \mid z, u_{xy}, u_y) P(u_{xy}, u_y) \right) \left(\sum_{u_x} P(u_x) \right)$$

$$= P(z \mid x) \sum_{u_{xy}} P(y \mid z, u_{xy}) P(u_{xy})$$

Summing over X

$$= P(z \mid x) \sum_{x', u_{xy}} P(y \mid z, u_{xy}) P(u_{xy} \mid x') P(x')$$

$(Y \perp\!\!\!\perp X \mid Z, U_{xy})$

$$= P(z \mid x) \sum_{x', u_{xy}} P(y \mid z, x', u_{xy}) P(u_{xy} \mid x') P(x')$$

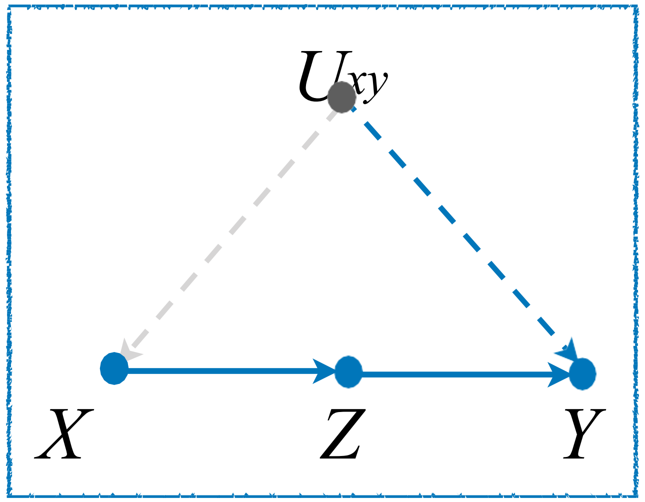
$(U_{xy} \perp\!\!\!\perp Z \mid X)$

$$= P(z \mid x) \sum_{x', u_{xy}} P(y \mid z, x', u_{xy}) P(u_{xy} \mid x', z) P(x')$$

Chain rule and sum out U_{xy}

$$= P(z \mid x) \sum_{x'} \sum_{u_{xy}} P(y, u_{xy} \mid z, x') P(x')$$

Alternative world



$$P(\mathbf{v} \mid do(x)) = P(z \mid x) \sum_{x'} P(y \mid z, x') P(x')$$

$$P(y \mid do(x)) = \sum_{z} P(z \mid x) \sum_{x'} P(y \mid z, x') P(x')$$

These factors can be computed from the observed distribution

The Front Door Criterion

When we cannot block a backdoor path, we may still have a front door path

Consider the century-old debate on the relation between smoking and lung cancer. In the years preceding 1970, the tobacco industry has managed to prevent antismoking legislation by promoting the theory that the observed correlation between smoking and lung cancer could be explained by some sort of carcinogenic genotype that also induces an inborn craving for nicotine.

A graph depicting this example is shown in Figure 3.10(a) This graph does not satisfy

Causal effect not identifiable here

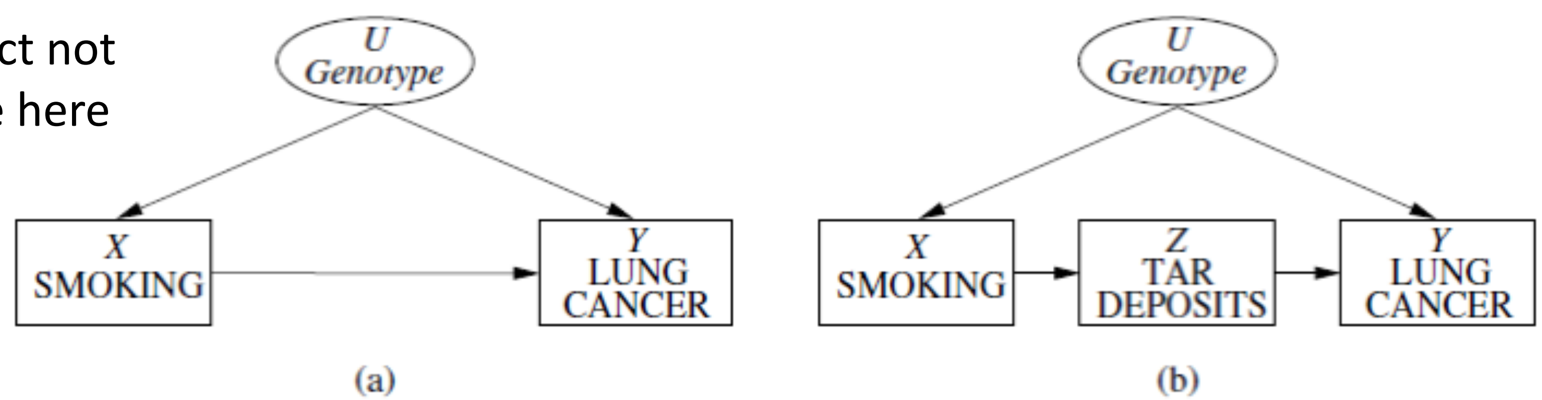


Figure 3.10: A graphical model representing the relationships between smoking (X) and lung cancer (Y), with unobserved confounder (U) and a mediating variable Z

Front Door Condition

We cannot satisfy the backdoor criterion since we cannot measure U. But consider the model in (b).
It does not satisfy the backdoor criterion, but we can measure the tar level, Z, which will allow identifiability of P(Y|do(X)),

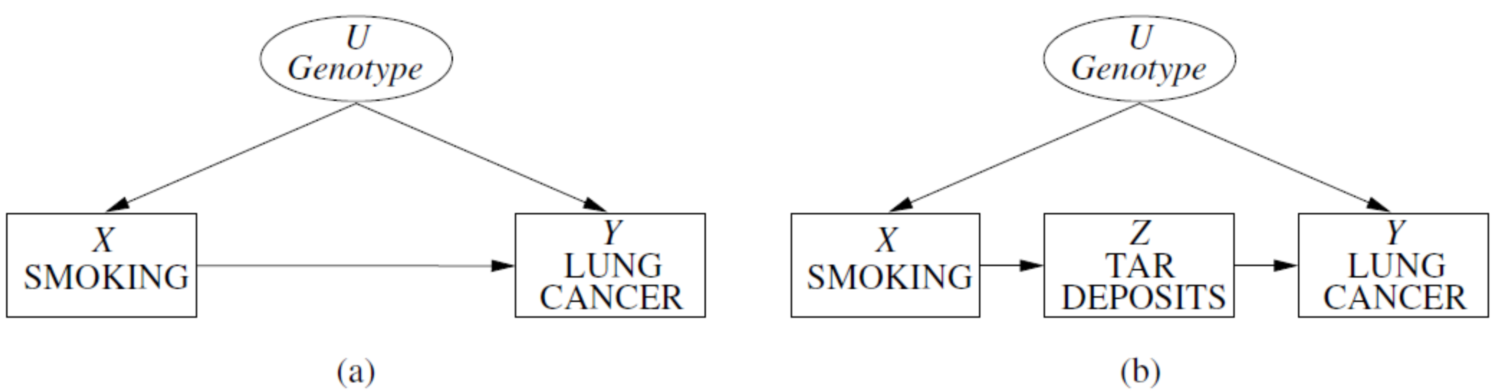


Figure 3.10: A graphical model representing the relationships between smoking (X) and lung cancer (Y), with unobserved confounder (U) and a mediating variable Z

Example (Front-door)

Table 3.1: A hypothetical dataset of randomly selected samples showing the percentage of cancer cases for smokers and nonsmokers in each tar category (numbers in thousands)

	Tar 400		**No tar 400**		**All subjects 800**	
	[illegible]	[illegible]kers	Smokers *20*	Nonsmokers *380*	Smokers *400*	Nonsmokers *400*
No cancer	323 (85%)	1 (5%)	18 (90%)	38 (10%)	341 (85%)	39 (9.75%)
Cancer	57 (15%)	19 (95%)	2 (10%)	342 (90%)	59 (15%)	361 (90.25%)

Tobaco industry:
Only 15% of smoker developed cancer while 90% from the non-smoker

Antismoke lobbyist:
If you smoke you have 95% tar vs no smokers (380/400 vs 20/400)

Table 3.2 Reorganization of the dataset of Table 3.1 showing the percentage of cancer cases in each smoking-tar category (number in thousands)

	SMOKERS 400		**NON-SMOKERS 400**		**ALL SUBJECTS 800**	
	Tar *380*	No tar *20*	Tar *20*	No tar *380*	Tar *400*	No tar *400*
No cancer	323 (85%)	18 (90%)	1 (5%)	38 (10%)	324 (81%)	56 (19%)
Cancer	57 (15%)	2 (10%)	19 (95%)	342 (90%)	76 (9%)	344 (81%)

If you have more tar, you increase the chance of cancer in both smoker (from 10% to 15%) and non-smokers (from 90% To 95%).

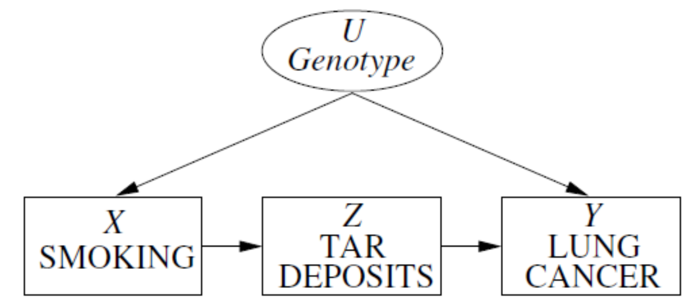
Front-door Condition

The graph of Figure 3.10(b) enables us to decide between these two groups of statisticians. First, we note that the effect of X on Z is identifiable, since there is no backdoor path from X to Z. Thus, we can immediately write

$$P(Z = z|do(X = x)) = P(Z = z|X = x) \tag{3.12}$$

Next we note that the effect of Z on Y is also identifiable, since the backdoor path from Z to Y, namely $Z \leftarrow X \leftarrow U \rightarrow Y$, can be blocked by conditioning on X. Thus we can write

$$P(Y = y|do(Z = z)) = \sum_x P(Y = y|Z = z, X = x) \text{ P(x)} \tag{3.13}$$



(b)

We are now going to chain together the two partial effects to obtain the overall effect of X on Y. The reasoning goes as follows: If nature chooses to assign Z the value z, then the probability of Y would be $P(Y = y|do(Z = z))$. But the probability that nature would choose to do that, given that we choose to set X at x, is $P(Z = z|do(X = x))$. Therefore, summing over all states z of Z we have

$$P(Y = y|do(X = x)) = \sum_z P(Y = y|do(Z = z))P(Z = z|do(X = x)) \tag{3.14}$$

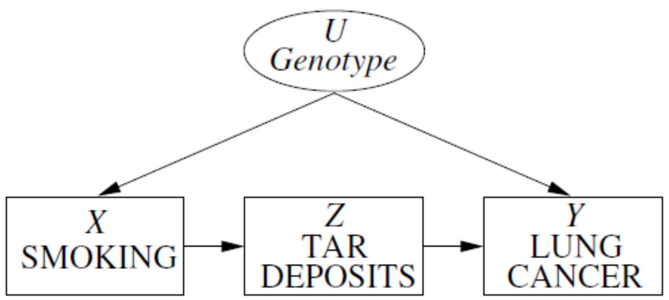
The terms on the right hand side of (3.14) were evaluated in (3.12) and (3.13), and we can substitute them to obtain a *do*-free expression for $P(Y = y|do(X = x))$. We also distinguish between the x that appears in (3.12) and the one that appears in (3.13), the latter of which is merely an index of summation and might as well be denoted x'. The final expression we have is

$$P(Y = y|do(X = x)) =$$
$$\textstyle\sum_z \sum_{x'} P(Y = y|Z = z, X = x')P(X = x')P(Z = z|X = x) \tag{3.15}$$

Equation (3.15) is known as the *front-door formula*.

Front-door Condition

Definition 3.4.1 (Front-Door)
A set of variables Z is said to satisfy the front-door *criterion relative t*
variables (X,Y) if



(b)

1. *Z intercepts all directed paths from X to Y.*
2. *There is no unblocked backdoor path from X to Z.*
3. *All backdoor paths from Z to Y are blocked by X.*

Theorem 3.4.1 (Front-Door Adjustment)
If Z satisfies the front-door criterion relative to (X,Y) and if $P(x,z) > 0$, then the causal effect of X on Y is identifiable and is given by the formula

$$P(y|do(x)) = \sum_z P(z|x) \sum_{x'} P(y|x',z)P(x'). \tag{3.16}$$

Outline

Recapping Backdoor
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Conditional intervention
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Rules of Do-Calculus

Theorem 3.4.1. The following transformations are valid for any do-distribution induced by a causal model M:

Rule 1: Adding/removing Observations

$$P(y|do(x),z,w)=P(y|do(x),w) \quad \text{if} \quad (Z \perp\!\!\!\perp Y \mid W)_{G_{\overline{X}}}$$

Rule 2: Action/observation exchange

$$P(y|do(x),do(z),w)=P(y|do(x),z,w) \quad \text{if} \quad (Z \perp\!\!\!\perp Y \mid X, W)_{G_{\overline{X}\underline{Z}}}$$

Rule 3: Adding/removing Actions

$$P(y|do(x),do(z),w)=P(y|do(x),w) \quad \text{if} \quad (Z \perp\!\!\!\perp Y \mid X, W)_{G_{\overline{X}\overline{Z(W)}}}$$

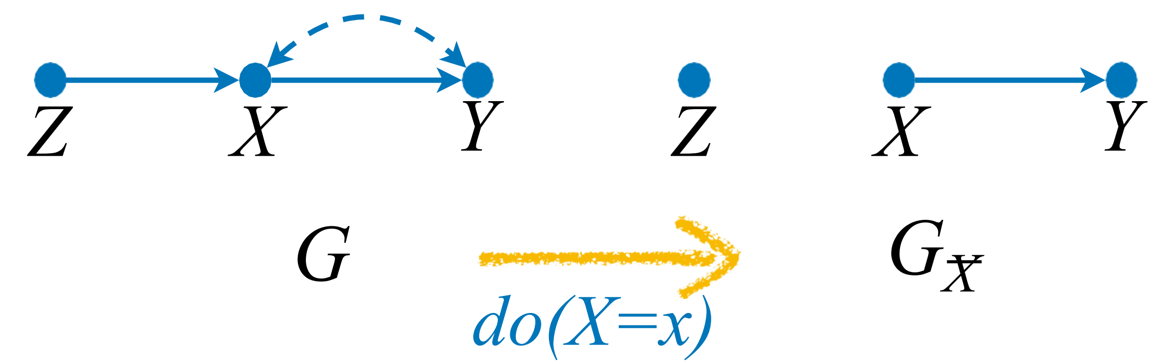
where $Z(W)$ is the set of Z-nodes that are not ancestors of any W-node in $G_{\overline{X}}$.

Insight 1: Adding/removing Observations

- Adding/removing observations

In the original model, Z and Y may be not separable, e.g.:

$$(Z \not\perp\!\!\!\perp Y),\ (Z \not\perp\!\!\!\perp Y \mid X)$$



However, in the the do(X)-world (model M_x), Y and Z are d-separated, that is,

$$(Z \perp\!\!\!\perp Y)_{G_{\overline{X}}} \implies P(y|do(x),z)=P(y|do(x))$$

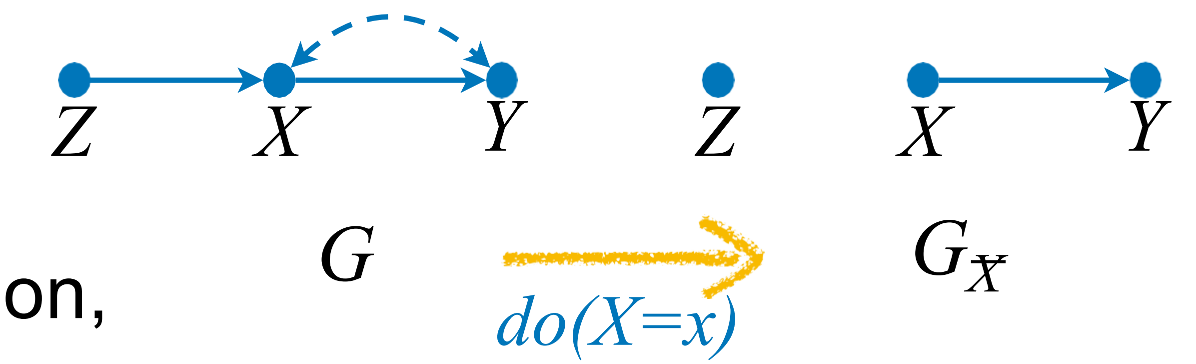
Let's verify this equality!

Try it yourself

Insight 1: Adding/removing Observations

- Adding/removing observations

$P(y|do(x),z)=P(y|do(x))$?



First, let's write the interventional distribution,

$$P(\mathbf{v} \mid do(x))$$

$$= \sum_{\mathbf{u}} P(z \mid u_z) P(y \mid x, u_y, u_{xy}) P(\mathbf{u})$$

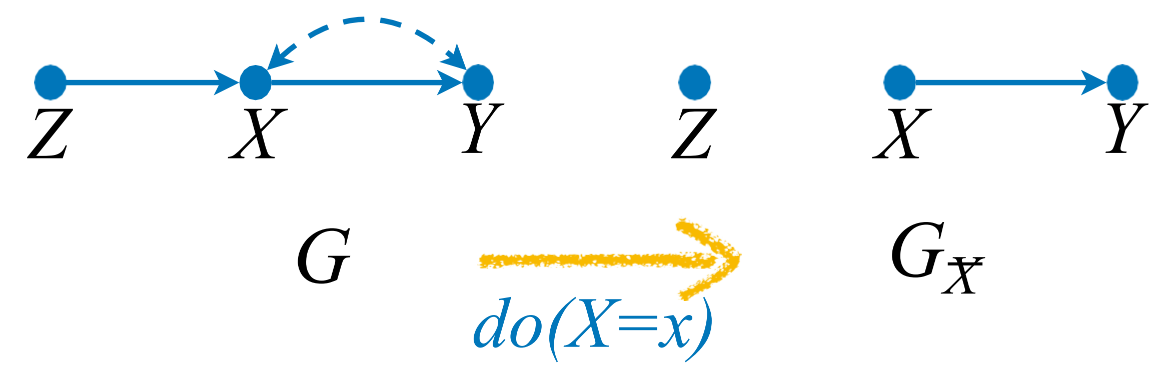
$$= P(z) \sum_{u_{xy}} P(y \mid x, u_{xy}) P(u_{xy})$$

From here we can already see that
P(Y|do(X) = to the sum expression since we can just
Sum over Z to get it.

Insight 1: Adding/removing Observations

- Adding/removing observations

$P(y|do(x),z)=P(y|do(x))$?



And, let's rewrite the conditional effects,

$$P(y \mid do(x), z) = \frac{P(y, z \mid do(x))}{P(z \mid do(x))}$$

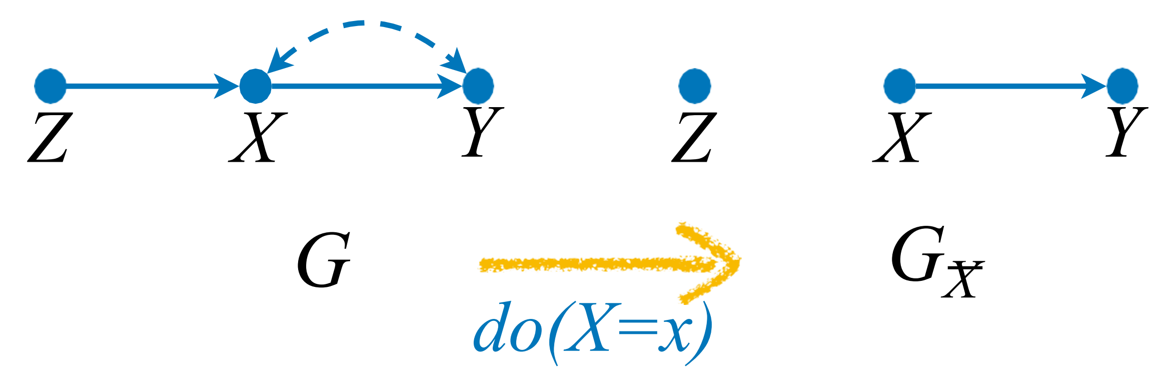
$$P(y, z \mid do(x)) = P(z) \sum_{u_{xy}} P(y \mid x, u_{xy}) P(u_{xy})$$

$$P(z \mid do(x)) = \sum_{y} P(z) \sum_{u_{xy}} P(y \mid x, u_{xy}) P(u_{xy}) = P(z)$$

Insight 1: Adding/removing Observations

- Adding/removing observations

$P(y|do(x),z)=P(y|do(x))$?



Substituting the factors back…

$$P(y \mid do(x), z) = \frac{P(z)\sum_{u_{xy}} P(y \mid x, u_{xy})P(u_{xy})}{P(z)}$$

$$= \sum_{u_{xy}} P(y \mid x, u_{xy})P(u_{xy})$$

$$= \sum_{z} P(z) \sum_{u_{xy}} P(y \mid x, u_{xy})P(u_{xy})$$

$$= \sum_{z} P(\mathbf{v} \mid do(x)) = P(y \mid do(x))$$

$(Z \perp\!\!\!\perp Y)_{G_{\overline{X}}}$ 👍 👍

Rules of Do-Calculus

Theorem 3.4.1. The following transformations are valid for any do-distribution induced by a causal model M:

Rule 1: Adding/removing Observations

$$P(y|do(x),z,w)=P(y|do(x),w) \quad \text{if} \quad (Z \perp\!\!\!\perp Y \mid W)_{G_{\overline{X}}}$$

Rule 2: Action/observation exchange

$$P(y|do(x),do(z),w)=P(y|do(x),z,w) \quad \text{if} \quad (Z \perp\!\!\!\perp Y \mid X, W)_{G_{\overline{X}\underline{Z}}}$$

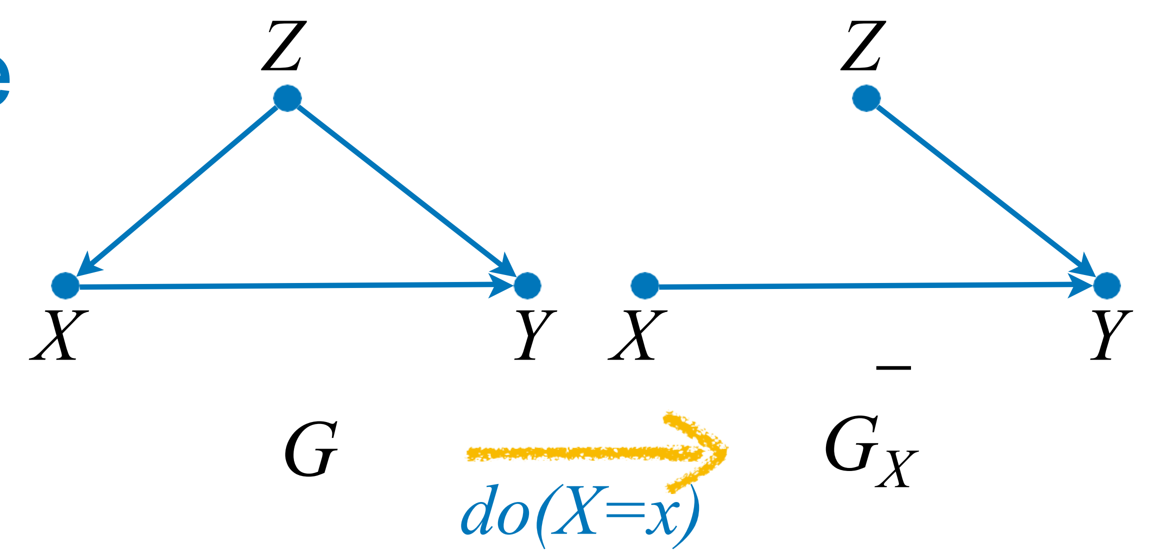
Rule 3: Adding/removing Actions

$$P(y|do(x),do(z),w)=P(y|do(x),w) \quad \text{if} \quad (Z \perp\!\!\!\perp Y \mid X, W)_{G_{\overline{X}\overline{Z(W)}}}$$

where $Z(W)$ is the set of Z-nodes that are not ancestors of any W-node in $G_{\overline{X}}$.

Insight 2: Action/Observation Exchange

- Action/Observation Exchange



After observing Z, variable Y reacts to X in the same way, with and without intervention.

Note that given Z, Y is correlated with X only through causal paths, hence, $see(X=x)$ will be equiv. to $do(X=x)$.

Idea. If Z blocks all bd-paths w.r.t (X, Y), then cond. on Z, all the remaining association is equal to the causation.

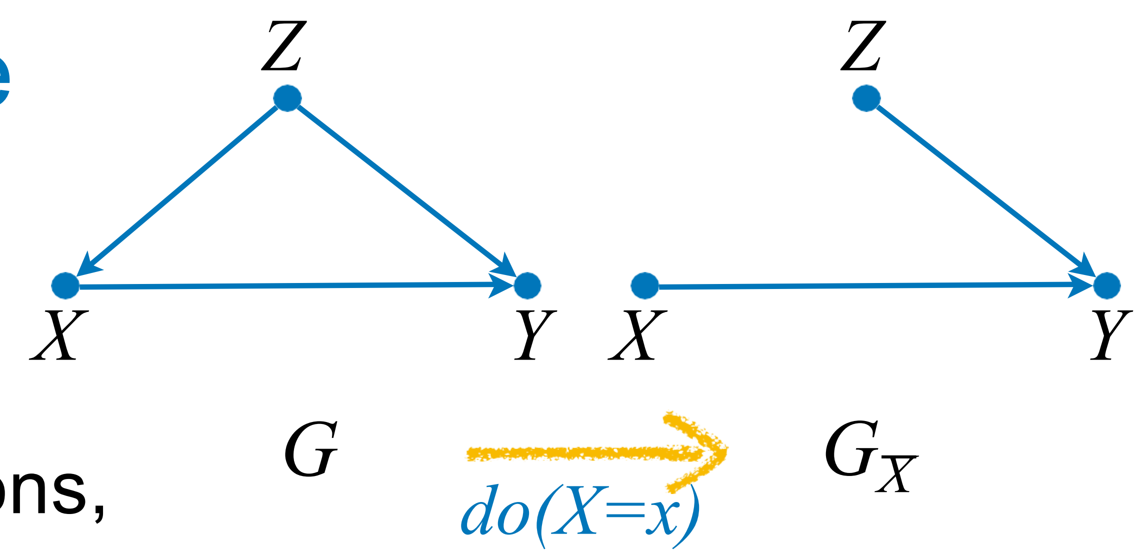
$$(Y \perp\!\!\!\perp X \mid Z)_{G_{\underline{X}}} \quad \Longrightarrow \quad P(y|do(x),z)=P(y|x,z)$$

Let's verify this equality!

Insight 2: Action/Observation Exchange

- Action/Observation Exchange

$P(y|do(x),z)=P(y|x,z)$?



First, let's write the interventional distributions,

$$P(y,z \mid do(x)) = \sum_{\mathbf{u}} P(z \mid u_z) P(y \mid x, z, u_y) P(\mathbf{u}) = P(z) P(y \mid x, z)$$

$$P(z \mid do(x)) = \sum_{y} P(z) P(y \mid x, z) = P(z)$$

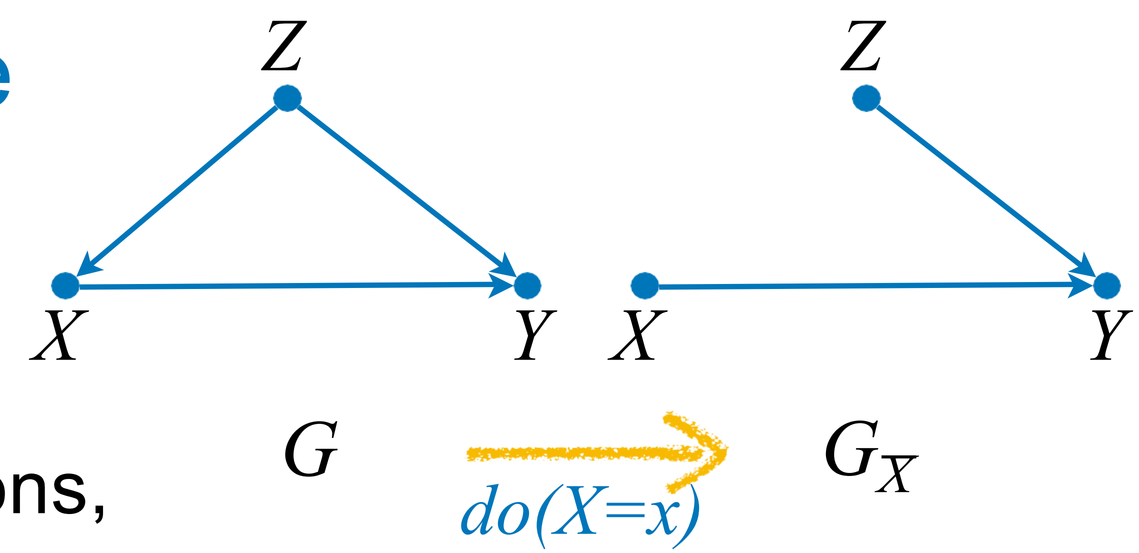
$$P(y \mid do(x), z) = \frac{P(z, y \mid do(x))}{P(z \mid do(x))} = \frac{P(z) P(y \mid x, z)}{P(z)} = P(y \mid x, z)$$

$$(Y \perp\!\!\!\perp X \mid Z)_{G_{\underline{X}}}$$

Insight 2: Action/Observation Exchange

- Action/Observation Exchange

$P(y|do(x),z)=P(y|x,z)$?



First, let's write the interventional distributions,

$$P(y,z \mid do(x)) = \sum_{\mathbf{u}} P(z \mid u_z)P(y \mid x, z, u_y)P(\mathbf{u}) \qquad P(z \mid do(x)) = \sum_y P(z)P(y \mid x, z)$$

$$= P(z)P(y \mid x, z) \qquad = P(z)$$

Looks familiar?

BD perhaps?

$$P(y \mid do(x), z) = \frac{P(z,y \mid do(x))}{P(z \mid do(x))} = \frac{P(z)P(y \mid x, z)}{P(z)} = P(y \mid x, z)$$

$$(Y \perp\!\!\!\perp X \mid Z)_{G_{\underline{X}}}$$ 👍

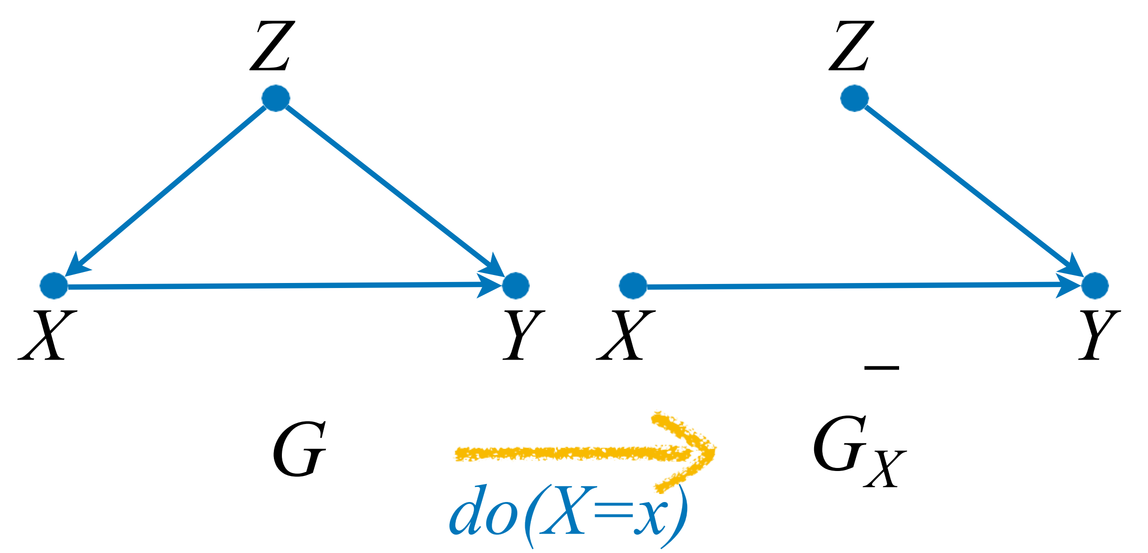
Insight 2: Action/Observation Exchange

- Action/Observation Exchange

Great, but what about the equality

$$P(y|do(x)) = P(y|x)?$$

$$(Y \perp\!\!\!\perp X)_{G_{\underline{X}}}$$



Let's compare left and right-hand sides:

$$P(y \mid do(x)) = \sum_z \sum_{\mathbf{u}} P(y \mid x, z, u_y) P(z \mid u_z) P(\mathbf{u})$$

$$= \sum_z P(y \mid x, z) P(z)$$

$$P(y \mid x) = \sum_z P(y \mid x, z) P(z \mid x)$$

Almost any model compatible with this causal graph, $P(y|x)$ *and* $P(y \mid do(x))$ will **not** be equal since $P(z) \neq P(z \mid x)$ almost surely.

Rules of Do-Calculus

Theorem 3.4.1. The following transformations are valid for any do-distribution induced by a causal model M:

Rule 1: Adding/removing Observations

$$P(y|do(x),z,w)=P(y|do(x),w) \quad \text{if} \quad (Z \perp\!\!\!\perp Y \mid W)_{G_{\overline{X}}}$$

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Rule 3: Adding/removing Actions

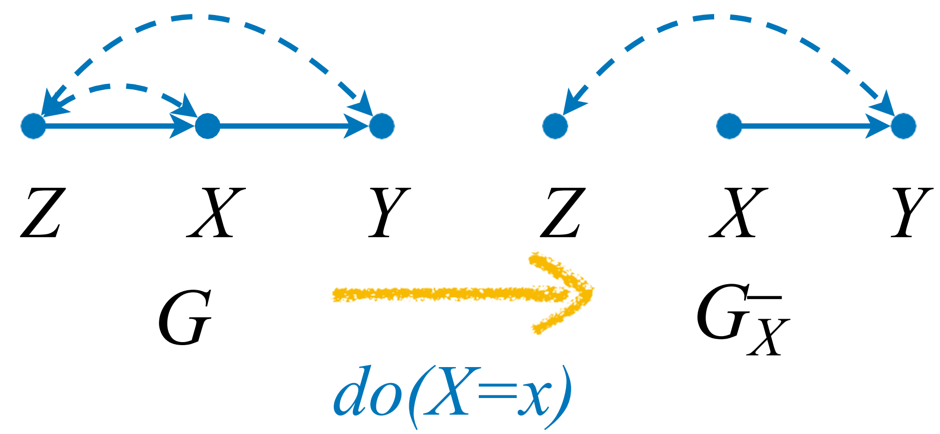
$$P(y|do(x),do(z),w)=P(y|do(x),w) \quad \text{if} \quad (Z \perp\!\!\!\perp Y \mid X, W)_{G_{\overline{X}\overline{Z(W)}}}$$

where $Z(W)$ is the set of Z-nodes that are not ancestors of any W-node in $G_{\overline{X}}$.

Insight 3: Adding/Removing Actions

- Adding/Removing Actions

If there is no causal path from X to Z, then an intervention on X will have no effect on Z.



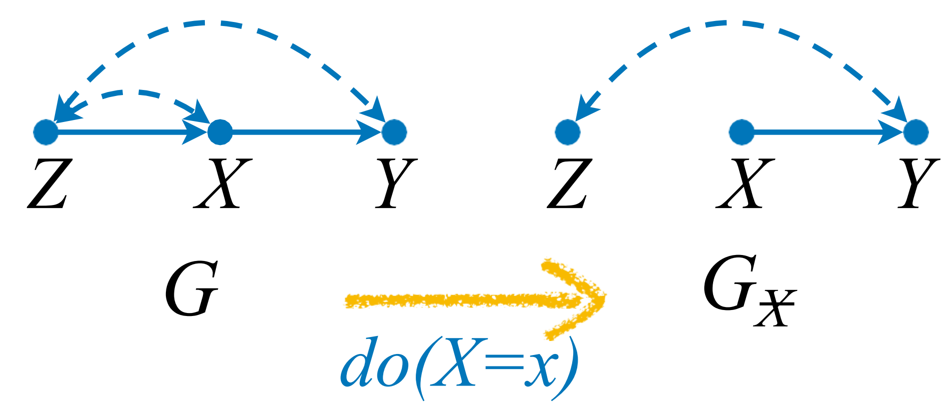
$$(Z \perp\!\!\!\perp X)_{G_{\overline{X}}} \implies P(z|do(x)) = P(z)$$

Let's verify this equality!

Insight 3: Adding/Removing Actions

- Adding/Removing Actions

$$P(z \mid do(x)) = P(z)\ ?$$



$$P(z \mid do(x)) = \sum_{y} P(\mathbf{v} \mid do(x))$$

$$= \sum_{y} \sum_{u_{zy},u_{zx}} P(z \mid u_{zy}, u_{zx}) P(y \mid x, u_{zy}) P(u_{zy}, u_{zx})$$

$$= \sum_{u_{zy},u_{zx}} P(z \mid u_{zy}, u_{zx}) P(u_{zy}, u_{zx})$$

$$= P(z) \qquad (Z \perp\!\!\!\perp X)_{G_{\overline{X}}}$$

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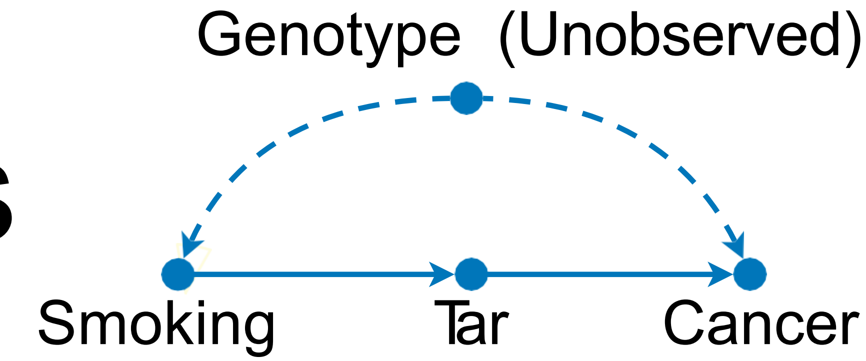
Properties of Do-Calculus

Theorem (soundness and completeness of do-calculus for causal identifiability from P(v)).

The causal quantity $Q = P(y|do(x))$ is identifiable from $P(v)$ and G if and only if there exists a sequence of application of the rules of do-calculus and the probability axioms that reduces Q into a do-free expression.

Syntactic goal: Re-express original Q without do()!

Derivation in Do-Calculus



$$P(c \mid do(s)) = \sum_t P(c \mid do(s), t) P(t \mid do(s))$$ Probability Axioms

$$= \sum_t P(c \mid do(s), do(t)) P(t \mid do(s))$$ Rule 2 $(T \perp\!\!\!\perp C \mid S)_{G_{\underline{T}}}$

$$= \sum_t P(c \mid do(t)) P(t \mid do(s))$$ Rule 3 $(S \perp\!\!\!\perp C \mid T)_{G_{\overline{C,T}}}$

$$= \sum_t P(c \mid do(t)) P(t \mid s)$$ Rule 2 $(S \perp\!\!\!\perp T)_{G_{\underline{S}}}$

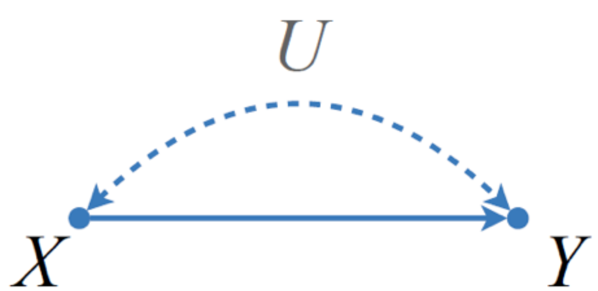
$$= \sum_t \sum_{s'} P(c \mid do(t), s') P(s' \mid do(t)) P(t \mid s)$$ Probability Axioms

$$= \sum_t \sum_{s'} P(c \mid t, s') P(s' \mid do(t)) P(t \mid s)$$ Rule 2 $(T \perp\!\!\!\perp C \mid S)_{G_{\underline{T}}}$

$$= \sum_t \sum_{s'} P(c \mid t, s') P(s') P(t \mid s)$$ Rule 3 $(T \perp\!\!\!\perp S)_{G_{\overline{T}}}$

Example. Non-identifiable Effect

- Let M be a model compatible with G and inducing an observational distribution $P(\mathbf{v})$:



$$M = \begin{cases} X \leftarrow U \\ Y \leftarrow f_Y(X, U) \end{cases}$$

$U \sim P(u) \quad P(x, y) > 0$

$$M^{(1)} = \begin{cases} X \leftarrow U_{xy} \\ Y \leftarrow \begin{cases} f_Y(X, U) & \textbf{if } U = X \\ 0 & \textbf{otherwise} \end{cases} \end{cases}$$

$$M^{(2)} = \begin{cases} X \leftarrow U_{xy} \\ Y \leftarrow \begin{cases} f_Y(X, U) & \textbf{if } U = X \\ 1 & \textbf{otherwise} \end{cases} \end{cases}$$

$P^{(1)}(U) = P^{(2)}(U) = P(U)$

- Without intervention, U is always equal to X in both models, hence Y always outputs $f_Y(X, U)$ and $P^{(1)}(\mathbf{v}) = P^{(2)}(\mathbf{v}) = P(\mathbf{v})$.

$$\begin{aligned} P^{(i)}(x, y) &= \sum_u P^{(i)}(x \mid u) P^{(i)}(y \mid x, u) P(u) \\ &= P^{(i)}(y \mid x, U = x) P(U = x) \\ &= P(y \mid x) P(x) \\ &= P(x, y) \end{aligned}$$

$1[x = u]$

Both models induce the same graph G and have the same $P(\mathbf{v})$

Example. Non-identifiable Effect

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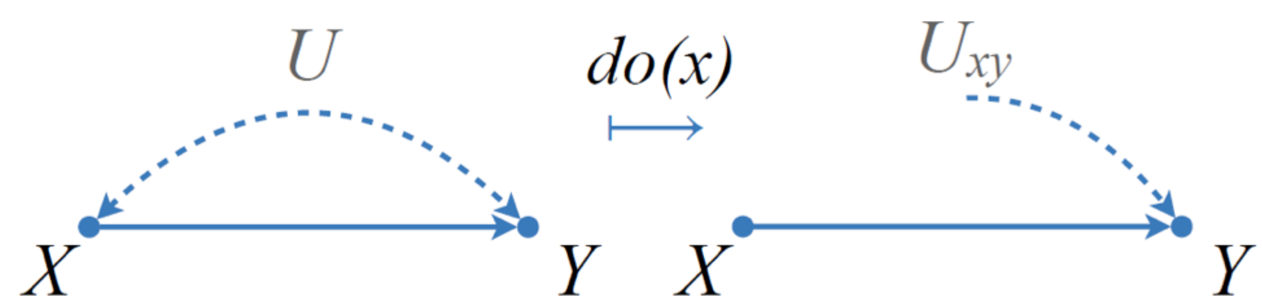


$$M_x^{(1)} = \begin{cases} X \leftarrow \cancel{U_{xy}}\ x \\ Y \leftarrow \begin{cases} f_Y(X,U) & \textbf{if } U = X \\ 0 & \textbf{otherwise} \end{cases} \end{cases}$$

$$M_x^{(2)} = \begin{cases} X \leftarrow \cancel{U_{xy}}\ x \\ Y \leftarrow \begin{cases} f_Y(X,U) & \textbf{if } U = X \\ 1 & \textbf{otherwise} \end{cases} \end{cases}$$

$$P^{(1)}(U) = P^{(2)}(U) = P(U)$$

- Under intervention $do(X{=}x)$, U and X do not need to match, hence $M_x^{(1)}$ and $M_x^{(2)}$ will output $Y{=}1$ with different probability:

$$
\begin{aligned}
&P^{(i)}(y \mid do(x)) \\
&= \sum_u P^{(i)}(y \mid x, u) P(u) \\
&= P^{(i)}(y \mid x, U = x) P(U = x) \\
&\quad + P^{(i)}(y \mid x, U \neq x) P(U \neq x) \\
&= P(y \mid x) P(x) + 1[i = 1](1 - P(x))
\end{aligned}
$$

0 in $M_x^{(1)}$, 1 in $M_x^{(2)}$

Even though both models induce the same graph G and have the same $P(\mathbf{v})$, the causal effect $P^{(1)}(y|do(x)) \neq P^{(2)}(y|do(x))$!

Non-identifiability Machinery

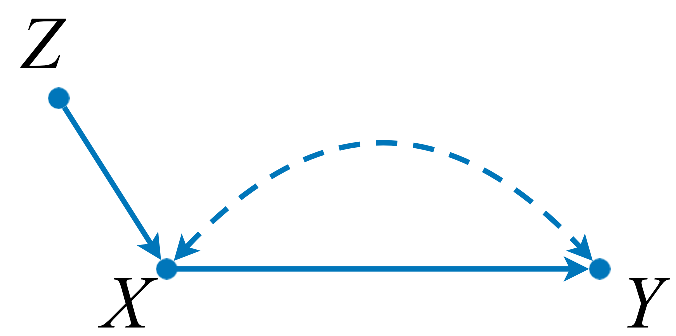
Lemma (Graph-subgraph ID (Tian and Pearl, 2002))

- If $Q = P(y \mid do(x))$ is not identifiable in G, then Q is not identifiable in the graph resulting from adding a directed or bidirected edge to G.
- Converse. If $Q = P(y|do(x))$ is identifiable in G, Q is still identifiable in the graph resulting from removing a directed or bidirected edge from G.

Non-identifiability Machinery

- Proof idea. Suppose M_1, M_2 induce the same $P(\mathbf{v})$ but differ in $P(y|do(x))$. Construct two new models M_1', M_2' with any $P(z)$ and let $P_i'(x|z,u_{xy})=P_i(x|u_{xy}),\ i=1,2$.

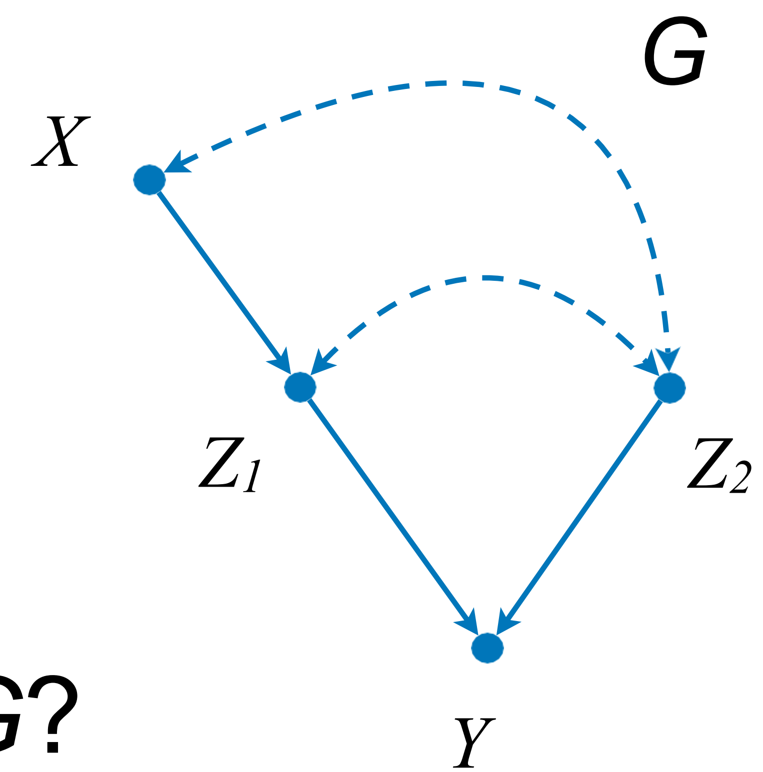
  This construction entails $P_1'(y|do(x))\neq P_2'(y|do(x))$.



Question: Do all non-ID models look like the bow graph?

Non-identifiability Puzzle

- Is P(y | do(x)) identifiable from *G*?
  - Is *G* of bow-shape?



- Is P(y | do(x), z2) identifiable from *G*?
- Is P(y | do(x, z2)) identifiable from *G*?

P(Y|do(x) is not identifiable
But when conditioning on Z_1, or Z_2 they are.
So, computing the effect of a joint intervention can be easier than
Their individual interventions.

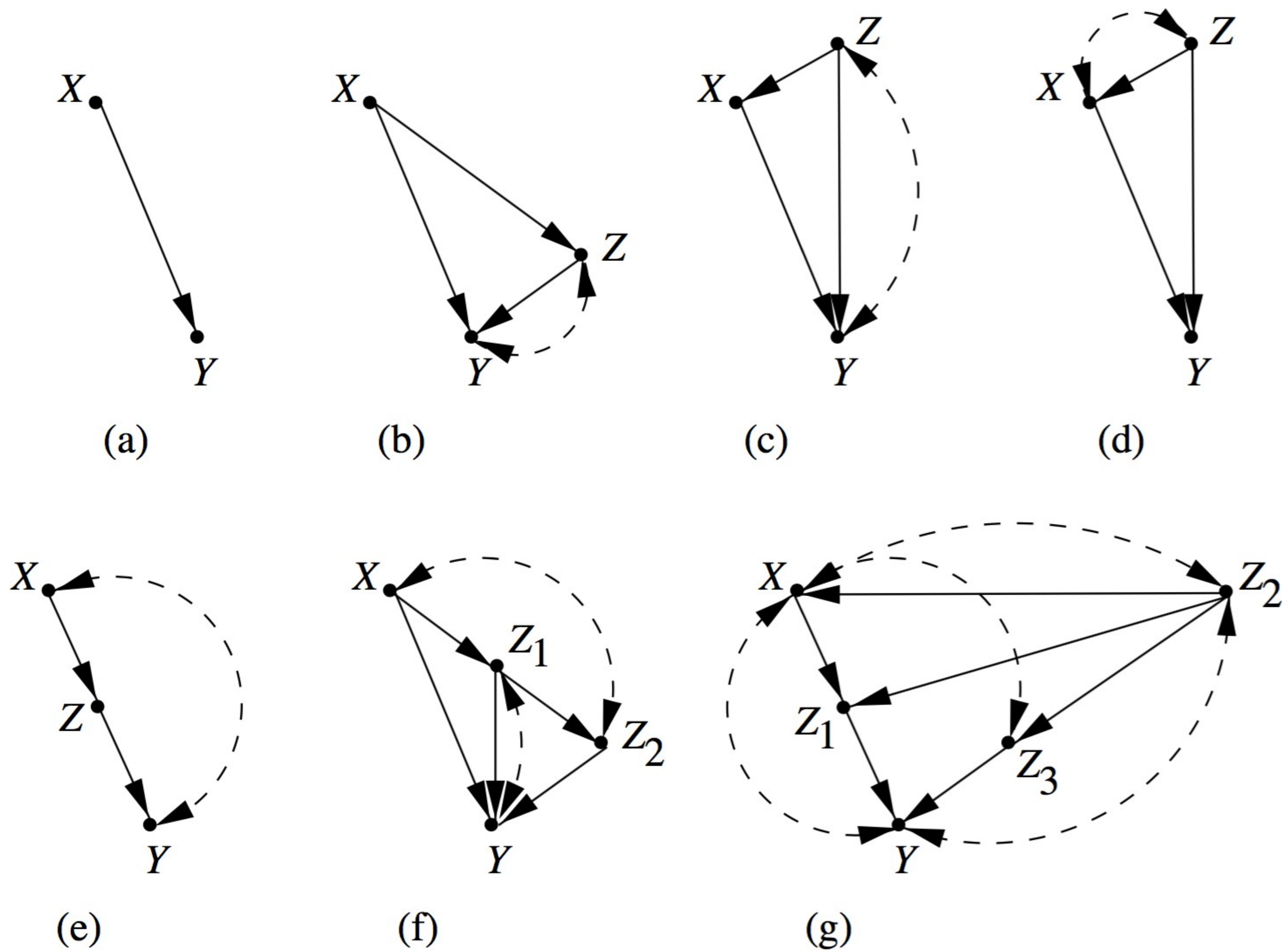
[C] sec 35.

Non-Identifiability Criterion

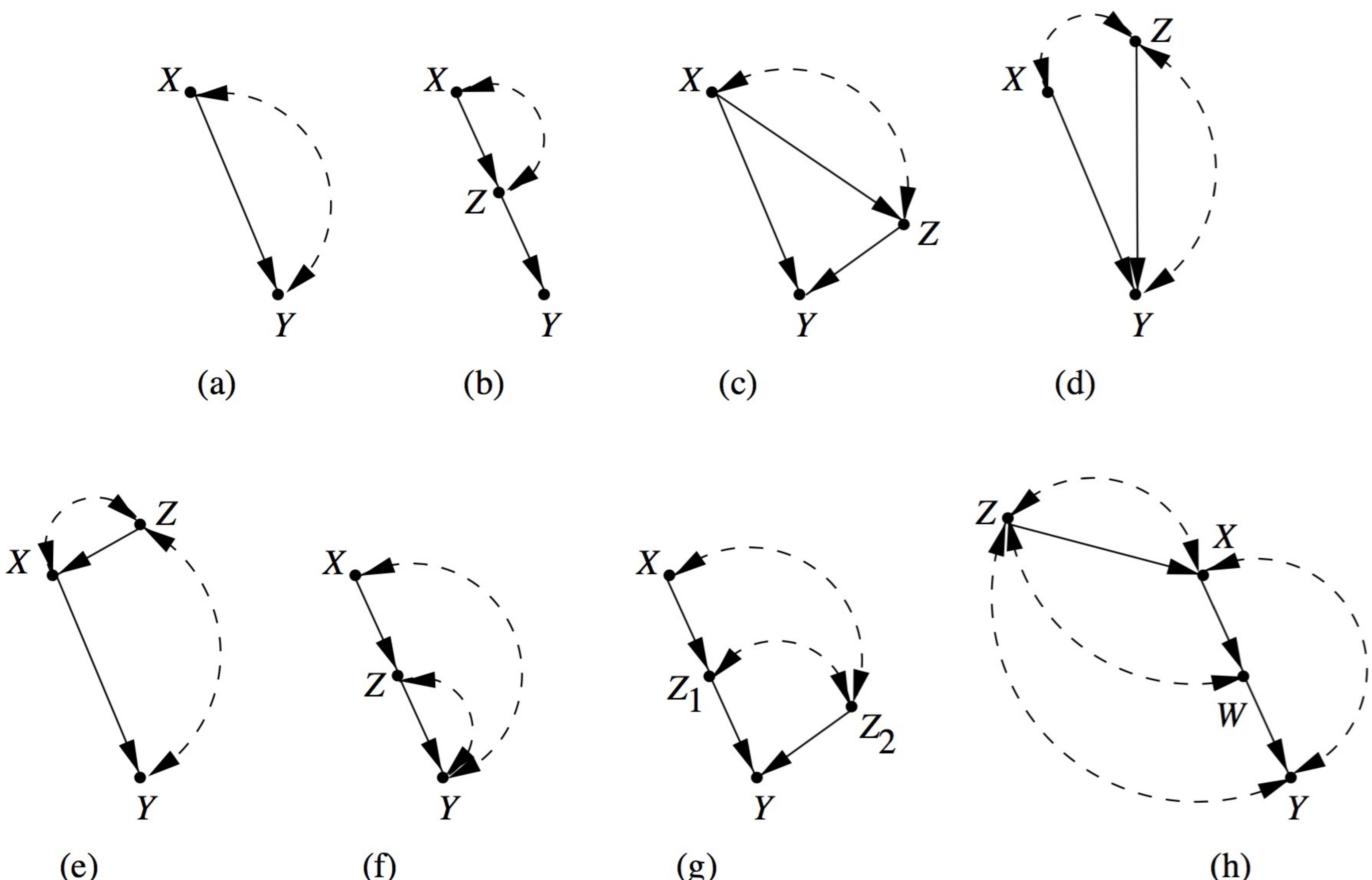
Theorem (Graphical criterion for non-identifiability of joint interventional distributions (Tian, 2002)).

If there is a bidirected path connecting X to any of its children in G, then $P(\mathbf{v}|do(x))$ is not identifiable from $P(\mathbf{v})$ and G.

Some Identifiable Graphs



Some Non-Identifiable Graphs



Summary

- The do-calculus provides a syntactical characterization to the problem of policy evaluation for atomic interventions.
- The problem of confounding and identification is essentially solved, non-parametrically.
- Simpson's Paradox is mathematized and dissolved.
- Applications are pervasive in the social and health sciences as well as in statistics, machine learning, and artificial intelligence.