CS295 Causal Reasoning Paper Presentation

[ACM] Detecting Latent Heterogeneity (Pearl, 2015)

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**Main References**

1. Estimating Heterogeneous Treatment Effects with Observational Data (Yu Xie, Jennie E. Brand, Ben Jann, 2012)

2. Effects of Treatment on the Treated: Identification and Generalization (Ilya Shpitser and Judea Pearl, 2009)

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**Paper Presentation:** Detecting Latent Heterogeneity (Pearl, 2015), Hao-Che Hsu, 05/17/2021
Heterogeneity

• Treatment might affect different experimental subjects in different ways.
  ▶ Individuals respond differently to treatment or intervention.
  ▶ Idiosyncratic groups in the population.

• Why do we care?
  ▶ vaccine is uniformly beneficial
  ▶ program evaluations
  ▶ bias causal estimates
Heterogeneity

How do we detect heterogeneity?
Program evaluation

- job training program

- randomized experiment: training → get a job

- Study (a year later): hiring rate among the trained is even higher

- eligible and enrolled: smarter, more resourceful, more socially connected
  ▶ would have found a job regardless of training

- population is not homogeneous

- informed → enroll (little benefit) / uninformed(weak) → not aggressively recruited
Assess the Degree of Heterogeneity

1. Covariate-specific methods

2. Compare the treated and the untreated
Covariate-induced Heterogeneity

• Does having this characteristic respond differently from those not having it?

• The unbiased estimator for the counterfactuals: \( \frac{1}{N} \sum_{i=1}^{N} (Y_{1i} - Y_{0i}) \)

• Comparing the covariates \((C')\):

\[
D(c_i, c_j) = \text{abs} \left[ E(Y_1 - Y_0 | C = c_i) - E(Y_1 - Y_0 | C = c_j) \right]
\]

\[ E(Y_1 - Y_0 | C = c_0) = E(Y | X = 1, C = c_0) - E(Y | X = 0, C = c_0) \]

• Heterogeneity lowerbound: \( \text{LB}_{\text{heterogeneity}} = \max_{\{c_i, c_j\} \in C} D(c_i, c_j) \)

• Does set \( C \) satisfy the backdoor criterion?
Unconfoundedness
Conditional independence assumption

- Also known as
  - Statistics: ignorability
  - Parametric: selection on observables
    - (Heckman and Robb, 1984)
  - Missing data: missing at random
  - Causal DAG: backdoor criterion

- \((Y_1, Y_0) \perp X|C\)

- \(Y = \beta_0 + \alpha X + C'\beta + \epsilon\), where \(X \perp \epsilon|C\)
  - \(X\) is exogenous

- Children from poor families are selected into the program \(\Rightarrow\) not able to compete
Identify $c$-specific effect (of $X$ on $Y$)

$C$ is admissible

- Identification: $E(Y_1 - Y_0 | C = c) = E(Y | X = 1, C = c) - E(Y | X = 0, C = c)$
Identify $c$-specific effect

$(C \cup S)$ is admissible

- Adjust $S$:

$$E(Y_1 - Y_0 | C = c)$$

$$= \sum_s \left[ E(Y | X = 1, S = s, C = c) - E(Y | X = 0, S = s, C = c) \right] \cdot P(s|c)$$
Identify $c$-specific effect

$C$ is not admissible (identifiable)

• Frontdoor adjustment
Identify $c$-specific effect

$C$ is not admissible (not identifiable)
Statistically Indistinguishable

- Treatments: $U_1$ and $U_2$
- Covariates: $S$ and $C$
- Outcomes: $X$ and $Y$

**Identifiable**

- Graph shows direct paths from $S$ to $C$, $C$ to $X$, and $C$ to $Y$
- Graph shows direct paths from $S$ to $U_1$ and $U_2$

**Not Identifiable**

- Graph lacks direct paths from $S$ to $C$, $C$ to $X$, or $C$ to $Y$
- Graph lacks direct paths from $S$ to $U_1$ and $U_2$

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Paper Presentation: Detecting Latent Heterogeneity (Pearl, 2015), Hao-Che Hsu, 05/17/2021
Latent Heterogeneity

- What if when heterogeneity is not presented in any covariates?

- If there is heterogeneity, would the causal effect on the treated group vs the control/untreated group the same?

- Noticed that in RCT:
  - The treated is just as bad as the control if the were not assigned treatment.
  - If the control were given treatment they will be as good as the treated.

- There must be bias in the ATE.
ATE Decomposition

$p(X = 1)$: proportion treated, $p(X = 0)$: proportion untreated (control)

\[
\text{ATE} = E(Y_1 - Y_0)
= E(Y_1|X = 1)p(X = 1) + E(Y_1|X = 0)[1 - p(X = 1)] - E(Y_0|X = 0)p(X = 0) - E(Y_0|X = 1)[1 - p(X = 0)]
= \left[ E(Y_1|X = 1) - E(Y_0|X = 0) \right]
\]

ATE from observed RCT data

\[
- \left[ E(Y_0|X = 1) - E(Y_0|X = 0) \right]
\]
pretreatment heterogeneity bias (Type-I bias, baseline bias, selection bias)

\[
- \left\{ \left[ E(Y_1|X = 1) - E(Y_0|X = 1) \right] - \left[ E(Y_1|X = 0) - E(Y_0|X = 0) \right] \right\} \cdot p(X = 0)
\]
proportion untreated

Unweighted treatment-effect heterogeneity bias (Type-II bias, variable-effect bias)

- There is no $C$ (covariates) involved in the decomposition.
The Bias

- **Pretreatment heterogeneity bias (selection bias)**
  - Output difference of two groups if neither receives treatment
  - If the control were given treatment they will be as good as the treated
  - Source: fixed effect and covariates
  - Fixed by controlling covariates
  - e.g. Head Start program select poor family children

- **Treatment-effect heterogeneity bias (ATT − ATU)·p(X=0)**
  - Difference in the average treatment effect of two groups
  - If there's no heterogeneity, this term vanishes
  - Source: variable effect
  - Can't be controlled for by covariates
  - e.g. attending college is selective
Fixed Effect & Variable Effect

- Model structural equations:
  
  \[ y = \beta x + \gamma z + \delta xz + \epsilon_y \]
  
  \[ x = \alpha z + \epsilon_x \]
  
  \[ z = \epsilon_z \]
Detecting Latent Heterogeneity

Goal: Identify ATT and ATU.
Three ways of detection

1. Randomized trails with binary treatments

2. Covariate adjustment

3. Instrumental variables
1. Randomized Trials with Binary Treatments

Before randomization...

- Pretreatment heterogeneity bias (selection bias)
  
  \[ E(Y_0|X = 1) - E(Y_0|X = 0) = \frac{[E(Y_0) - E(Y_0|X = 0)]}{p(X = 1)} \]

- Unweighted Treatment-effect heterogeneity bias
  
  \[ \text{ATT} - \text{ATU} = E(Y_1|X = 1) - \left[ E(Y_0) - E(Y_0|X = 0)[1 - p(X = 1)] \right] - \frac{E(Y_1) - E(Y_1|X = 1)p(X = 1)}{1 - p} - E(0|X = 0) \]

  \[ = \frac{[E(Y_1|X = 1) - E(Y_1)]}{1 - p} + \frac{p}{p} \]

  - not population heterogeneity, but the het. that has preferential selection to treatment
  - None zero?

- Estimate \( E(Y_0) = E(Y|X = 0) \) and \( E(Y_1) = E(Y|X = 1) \) empirically in RCT.
2. Detecting Heterogeneity Through Adjustments

If backdoor criterion holds for some set $Z$

Adjustment formula and variation:

- $E(Y_0) = \sum_z E(Y|X = 0, z) \cdot p(z)$

- $E(Y_0|X = 1) = \sum_z E(Y|X = 0, z) \cdot p(z|X = 1)$
2. Detecting Heterogeneity Through Adjustments

**Theorem**

\( p(Y_x = y | X = x') \) is identifiable in \( G \) iff \( p(y|w, do(x)) \) is identifiable in \( G' \) which from \( G \), adds a new node \( W \) with the same set of parents as \( X \) and no children.

\[
p(y|w, do(x)) = \frac{p(y, w|do(x))}{p(w)} = \frac{\sum z p(y|z, x)p(w, z)}{p(w)} = \sum z p(y|z, x)p(z|w) \bigg|_{w=x'}
\]
2. Detecting Heterogeneity Through Adjustments

$Z$ of covariates is an admissible set

- $E(Y_a|X = b) = p(y|b, do(a)) = \sum_z E(Y|X = a, z) \cdot p(z|X = b)$

- ATT-ATU

$$=E(Y_1 - Y_0|X = 1) - E(Y_1 - Y_0|X = 0)$$
$$=E(Y_1|X = 1) - E(Y_0|X = 1) - E(Y_1|X = 0) + E(Y_0|X = 0)$$
$$=\sum_z [E(Y|X = 1, z) - E(Y|X = 0, z)] [p(z|X = 1) - p(z|X = 0)]$$
3. Detecting Heterogeneity Through Mediating Instruments Z

The frontdoor criterion

- set $Z$ intercept all directed paths from $X$ to $Y$

- $ATT - ATU = \sum_Z [E(Y|X = 1, z) - E(Y|X = 0, z)] [p(z|X = 1) - p(z|X = 0)]$