ICS 6A
Solution to Homework Assignment 1 [1]
Winter 2004

1. Rosen, page 16, problem 8.

   a) $r \wedge (-q)$
   b) $p \wedge q \wedge r$
   c) $r \rightarrow p$
   d) $p \wedge (-q) \wedge r$
   e) $(p \wedge q) \rightarrow r$
   f) $r \leftrightarrow (q \vee p)$

2. Rosen, page 17, problem 16.

   a) If you don't send me an e-mail message, then I will not remmember to send you the address.
   b) If you were born in the United States, then you are a citizen of this country.
   c) If you keep your textbook, then it will be a useful reference in your future courses.
   d) If the Red Wings's goalie plays well, then they will win the Stanley Cup.
   e) If you get the job, then you must have the best credentials.
   f) If there is a storm, then the beach erodes.
   g) If you don't have a valid password, then you can not log on to the server.

3. Rosen, page 26, problem 8.

A Demonstration That $[-p \wedge (p \vee q)] \rightarrow q$ is a tautology.

p q	$-p$	$p \vee q$	$-p \wedge (p \vee q)$	$[-p \wedge (p \vee q)] \rightarrow q$
T T	F	T	F	T
T F	F	T	F	T
F T	T	T	T	T
F F	T	F	F	T

Table 1: Page 19-8 a)

[1] Also available at: *http://www.ics.uci.edu/~dechter/ics-6a/winter-2004/*

A Demonstration That $[(p \to q) \land (q \to r)] \to (p \to r)$ is a tautology.					
p q r	$p \to q$	$q \to r$	$(p \to q) \land (q \to r)$	$p \to r$	$[(p \to q) \land (q \to r)] \to (p \to r)$
T T T	T	T	T	T	T
T T F	T	F	F	F	T
T F T	F	T	F	T	T
T F F	F	T	F	F	T
F T T	T	T	T	T	T
F T F	T	F	F	T	T
F F T	T	T	T	T	T
F F F	T	T	T	T	T

Table 2: Page 19-8 b)

A Demonstration That $[p \land (p \to q)] \to q$ is a tautology.			
p q	$p \to q$	$p \land (p \to q)$	$[p \land (p \to q)] \to q$
T T	T	T	T
T F	F	F	T
F T	T	F	T
F F	T	F	T

Table 3: Page 19-8 c)

A Demonstration That $[(p \lor q) \land (p \to r) \land (q \to r)] \to r$ is a tautology.					
p q r	$p \lor q$	$p \to r$	$q \to r$	$(p \lor q) \land (p \to r) \land (q \to r)$	$[(p \lor q) \land (p \to r) \land (q \to r)] \to r$
T T T	T	T	T	T	T
T T F	T	F	F	F	T
T F T	T	T	T	T	T
T F F	T	F	T	F	T
F T T	T	T	T	T	T
F T F	T	T	F	F	T
F F T	F	T	T	F	T
F F F	F	T	T	F	T

Table 4: Page 19-8 d)

4. Rosen, page 26, problem 10.

a)
$$
\begin{aligned}
[-p \wedge (p \vee q)] \rightarrow q &\Leftrightarrow [(-p \wedge p) \vee (-p \wedge q)] \rightarrow q && \text{Distributive laws.}\\
&\Leftrightarrow [F \vee (-p \wedge q)] \rightarrow q && \text{Table 6 on page 18 of Rosen.}\\
&\Leftrightarrow [(-p \wedge q) \vee F] \rightarrow q && \text{Commutative laws.}\\
&\Leftrightarrow (-p \wedge q) \rightarrow q && \text{Identity laws.}\\
&\Leftrightarrow [-(-p \wedge q)] \vee q && \text{Table 6 on page 18 of Rosen.}\\
&\Leftrightarrow [-(-p) \vee -q] \vee q && \text{De Morgan's laws.}\\
&\Leftrightarrow (p \vee -q) \vee q && \text{Double negation law.}\\
&\Leftrightarrow p \vee (-q \vee q) && \text{Associative laws.}\\
&\Leftrightarrow p \vee (q \vee -q) && \text{Commutative laws.}\\
&\Leftrightarrow p \vee T && \text{Table 6 on page 18 of Rosen.}\\
&\Leftrightarrow T && \text{Domination laws.}
\end{aligned}
$$

b)
$$
\begin{aligned}
&[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r) && \\
&\Leftrightarrow [(-p \vee q) \wedge (-q \vee r)] \rightarrow (-p \vee r) && \text{Table 6 on page 18 of Rosen.}\\
&\Leftrightarrow -[(-p \vee q) \wedge (-q \vee r)] \vee (-p \vee r) && \text{Table 6 on page 18 of Rosen.}\\
&\Leftrightarrow [-(-p \vee q) \vee -(-q \vee r)] \vee (-p \vee r) && \text{De Morgan's laws.}\\
&\Leftrightarrow [-p \vee -(-p \vee q)] \vee [r \vee -(-q \vee r)] && \text{Associative and Commutative laws.}\\
&\Leftrightarrow -[p \wedge (-p \vee q)] \vee -[-r \wedge (-q \vee r)] && \text{De Morgan's laws.}\\
&\Leftrightarrow -[(p \wedge -p) \vee (p \wedge q)] \vee -[(-r \wedge -q) \vee (-r \wedge r)] && \text{Distributive laws.}\\
&\Leftrightarrow -[F \vee (p \wedge q)] \vee -[(-r \wedge -q) \vee F] && \text{Table 6 on page 18 of Rosen.}\\
&\Leftrightarrow -(p \wedge q) \vee -(-r \wedge -q) && \text{Identity laws.}\\
&\Leftrightarrow (-p \vee -q) \vee (r \vee q) && \text{De Morgan's laws.}\\
&\Leftrightarrow (-p \vee r) \vee (q \vee -q) && \text{Associative and Commutative laws.}\\
&\Leftrightarrow (-p \vee r) \vee T && \text{Table 6 on page 18 of Rosen.}\\
&\Leftrightarrow T && \text{Domination laws.}
\end{aligned}
$$

c)
$$
\begin{aligned}
[p \wedge (p \rightarrow q)] \rightarrow q &\Leftrightarrow [p \wedge (-p \vee q)] \rightarrow q && \text{Table 6 on page 18 of Rosen.}\\
&\Leftrightarrow [(p \wedge -p) \vee (p \wedge q)] \rightarrow q && \text{Distributive laws.}\\
&\Leftrightarrow [F \vee (p \wedge q)] \rightarrow q && \text{Table 6 on page 18 of Rosen.}\\
&\Leftrightarrow [(p \wedge q) \vee F] \rightarrow q && \text{Commutative laws.}\\
&\Leftrightarrow (p \wedge q) \rightarrow q && \text{Identity laws.}\\
&\Leftrightarrow [-(p \wedge q)] \vee q && \text{Table 6 on page 18 of Rosen.}\\
&\Leftrightarrow (-p \vee -q) \vee q && \text{De Morgan's laws.}\\
&\Leftrightarrow -p \vee (-q \vee q) && \text{Associative laws.}\\
&\Leftrightarrow -p \vee T && \text{Table 6 on page 18 of Rosen.}\\
&\Leftrightarrow T && \text{Domination laws.}
\end{aligned}
$$

d)
$$
\begin{aligned}
&[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r && \\
&\Leftrightarrow [(p \vee q) \wedge (-p \vee r) \wedge (-q \vee r)] \rightarrow r && \text{Table 6 on page 18 of Rosen.}\\
&\Leftrightarrow -[(p \vee q) \wedge (-p \vee r) \wedge (-q \vee r)] \vee r && \text{Table 6 on page 18 of Rosen.}\\
&\Leftrightarrow -(p \vee q) \vee -(-p \vee r) \vee -(-q \vee r) \vee r && \text{De Morgan's laws.}\\
&\Leftrightarrow (-p \wedge -q) \vee (p \wedge -r) \vee (q \wedge -r) \vee r && \text{De Morgan's laws.}
\end{aligned}
$$

$\Leftrightarrow (-p \wedge -q) \vee (p \wedge -r) \vee [r \vee (q \wedge -r)]$	Associative and Commutative laws.
$\Leftrightarrow (-p \wedge -q) \vee (p \wedge -r) \vee [(r \vee q) \wedge (r \vee -r)]$	Distributive laws.
$\Leftrightarrow (-p \wedge -q) \vee (p \wedge -r) \vee [(r \vee q) \wedge T]$	Table 6 on page 18 of Rosen.
$\Leftrightarrow (-p \wedge -q) \vee (p \wedge -r) \vee (r \vee q)$	Identity laws.
$\Leftrightarrow [q \vee (-p \wedge -q)] \vee [r \vee (p \wedge -r)]$	Associative and Commutative laws.
$\Leftrightarrow [(q \vee -p) \wedge (q \vee -q)] \vee [(r \vee p) \wedge (r \vee -r)]$	Distributive laws.
$\Leftrightarrow [(q \vee -p) \wedge T] \vee [(r \vee p) \wedge T]$	Table 6 on page 18 of Rosen.
$\Leftrightarrow (q \vee -p) \vee (r \vee p)$	Identity laws.
$\Leftrightarrow (q \vee r) \vee (p \vee -p)$	Associative and Commutative laws.
$\Leftrightarrow (q \vee r) \vee T$	Table 6 on page 18 of Rosen.
$\Leftrightarrow T$	Domination laws.

5. Rosen, page 41, problem 16.

6. Rosen, page 41, problem 22.

7. Rosen, page 42, problem 34.

8. Rosen, page 43, problem 48.

9. Rosen, page 51, problem 4.

   a) $\exists x \exists y P(x, y)$: There is a student in your class who has taken a computer science course at your school.
   b) $\exists x \forall y P(x, y)$: There is a student in your class who has taken all the computer science courses at your school.
   c) $\forall x \exists y P(x, y)$: For every student, there is a computer science course at your school such that the student took the class.
   d) $\exists y \forall x P(x, y)$: There is a computer science course such that every student in your class has taken it.
   e) $\forall y \exists x P(x, y)$: For every computer science course y at your school, there is a student in your class such that took your class.
   f) $\forall x \forall y P(x, y)$: For every student x in your class, for every computer science course y at your school, x has taken y. In other words, every student in your class has taken all of the computer science courses at your school.

10. Rosen, page 52, problem 8.

    a) $\exists x \exists y Q(x, y)$
    b) $-\exists x \exists y Q(x, y)$
    c) $\exists x (Q(x, Jeopardy) \wedge Q(x, Wheel\ of\ Fortune))$
    d) $\forall y \exists x Q(x, y)$
    e) $\exists x_1 \exists x_2 (x_1 \neq x_2 \wedge Q(x_1, Jeopardy) \wedge Q(x_2, Jeopardy))$

11. Rosen, page 52, problem 12.

    a) $-I(Jerry)$
    b) $-C(Rachel, Chelsea)$

c) $-C(Jan, Sharon)$
d) $-\exists x C(x, Bob)$
e) $\forall x(C(Sanjay, x) \leftrightarrow x \neq Joseph)$
f) $\exists x - I(x)$
g) $-\forall x I(x)$
h) $\exists x \forall y(I(y) \leftrightarrow y = x)$
i) $\exists x \forall y(-I(y) \leftrightarrow y = x)$
j) $\forall x(I(x) \rightarrow (\exists y(y \neq x \wedge C(x, y))))$
k) $\exists x(I(x) \wedge (\forall y(y \neq x \rightarrow -C(x, y))))$
l) $\exists x \exists y(x \neq y \wedge -C(x, y))$
m) $\exists x \forall y C(x, y)$
n) $\exists x \exists y(x \neq y \wedge (-\exists z(C(x, z) \wedge C(y, z))))$
o) $\exists x \exists y(x \neq y \wedge \forall z((z \neq x \wedge z \neq y) \rightarrow (C(x, z) \vee C(y, z))))$