**Motivation and Contribution**

- **IDs are a powerful formalism for reasoning with sequential decision-making problems under uncertainties**
  - Involve random (or chance) variables, decision variables and utility functions
- **Task:** find the maximum expected utility (MEU) and the corresponding optimal policy
- **Notoriously difficult to solve exactly in practice**
- **Recent work focused on bounding the MEU**
  - E.g., information relaxation, reformulation to Marginal MAP, partitioning over join-trees
- **Contribution:**
  - Revisit multi-operator cluster DAG (MCDAG) decompositions for influence diagrams
  - Partitioning-based (mini-bucket) approximation for MCDAGs to upper bound the MEU
  - Apply cost-shifting to tighten the upper bounds further
- In practice, we obtain significantly tighter bounds (by several orders of magnitude) than existing schemes

**Influence Diagrams**

\[
MEU = \max_{D_0} \sum_{C_1} \max_{D_1} \sum_{C_2} \max_{D_2} \sum_{C_3} \max_{D_4} P(C_1 | C_3) \cdot P(C_2 | C_4) \cdot u(D_0) + u(D_1, C_1, D_2) + u(D_2, C_2) + u(D_3, C_3, D_4)
\]

**Variable Elimination**

- Compute the MEU via message passing over the MCDAG, bottom-up from leaves to the root:

\[
\begin{align*}
\lambda_2 &= u_1(D_0) \\
\lambda_3 &= \sum_{C_3} P(C_3) \\
\lambda_4 &= \sum_{C_4} P(C_4) \\
\end{align*}
\]

**Cost-shifting via moment-matching**

- Use weighted elimination for variable \( X \): \( \Sigma_X f = (\Sigma_X f + 1) \mu \)
- Moment-matching between mini-buckets for SUM clusters (when eliminating variable \( X \))
  - Let \( Q = \{Q_1, \ldots, Q_R\} \) be a mini-bucket partitioning such that \( \psi_r = \prod_{f \in Q_r} f \) and \( w_r > 0 \) such that \( \sum_r w_r = 1 \)
  - Re-parameterize \( \psi_r = \psi_r(\frac{\mu}{\mu_r})^{\frac{1}{w_r}} \), \( \mu = \prod_r w_r \mu_r \), \( Y_r = var(Q_r) \)
  - Moment-matching between mini-buckets for MAX clusters (when eliminating variable \( X \))
    - If \( \psi_r = \prod_{f \in Q_r} f \) then re-parameterize \( \psi_r = \psi_r(\frac{\mu}{\mu_r})^{\frac{1}{w_r}} \), \( \mu = \max_r \mu_r \), \( Y_r = var(Q_r) \)
    - If \( \psi_r = \sum_{f \in Q_r} f \) then re-parameterize \( \psi_r = \psi_r - \mu + \frac{1}{R} \mu \), \( \mu = \max_r \mu_r \), \( Y_r = var(Q_r) \)

**Conclusion**

- Revisit MCDAGs for IDs and develop a mini-bucket approximation scheme for bounding the MEU
- MCDAGs are more sensitive to the underlying problem structure than strong join-trees (i.e., smaller induced-widths)
- Apply cost-shifting by moment-matching to tighten the bounds further
- Experiments on difficult benchmark problem instances demonstrate the effectiveness of our proposed bounding scheme compared with existing state-of-the-art approaches