**Summary**

**Tree Decomposition for Influence Diagrams and Limited Memory Influence Diagrams**
- Graph separation criteria for identifying single-stage decision problems
- Valuation algebra over submodels using graph-based operations
- Submodel-Tree Clustering and Elimination Scheme

**Submodel Decomposition Bounds**
- Bounding MEU by exponentiating utility functions
- Re-use decomposition bounds in Marginal MAP inference

**Contributions**
- Generate submodel-tree with lower tree-width by removing irrelevant variables and functions in each submodel
- Scalable convex upper bounds for MEU

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**Influence Diagrams**

\[ \mathcal{M} := (X, \mathbf{D}, \mathbf{P}, U, O) \]

- **Chance variables** \( X = \{X_1, X_2, \ldots, X_n\} \)
- **Decision variables** \( \mathbf{D} = \{D_1, D_2, \ldots, D_m\} \)
- **Probability functions** \( \mathbf{P} = \{P_1, P_2, \ldots, P_n\} \)
- **Utility functions** \( U = \{U_1, U_2, \ldots, U_n\} \)
- **Policy functions** \( \Delta = \{\Delta_1, \Delta_2, \ldots, \Delta_n\} \)

**Optimal strategy** \( \Delta^* = \arg \max \mathbb{E} \left[ \sum_{U_i \in \mathbf{U}} U_i \right] \)

**Maximum expected Utility** \( \max \Delta \mathbb{E} \left[ \sum_{U_i \in \mathbf{U}} U_i \right] \)

**Graph-based Submodel Identification**
- \( \text{REL}(D', U') \) is the backdoor\(^*\) set between \( D' \) and \( U' \)
  - \( \text{Backdoor} \) (p. 2009)
  - a set \( Z \) satisfies the backdoor criterion relative to \( (X, Y) \)
  - (1) None of the nodes in \( Z \) is a descendant of \( X \)
  - (2) \( Z \) blocks every path between \( X \) and \( Y \) that contain arrow into \( X \)
- \( \text{REL}(D', U') \) is the union of all frontdoor\(^*\) set between \( pa(D') \) and \( ch(U') \)
  - \( \text{Frontdoor} \) (p. 2009)
  - a set \( Z \) satisfies the frontdoor criterion relative to \( (X, Y) \)
  - (1) \( Z \) intercept all directed paths from \( X \) to \( Y \)
  - (2) There is no backdoor path from \( X \) to \( Z \)
  - (3) All backdoor paths from \( Z \) to \( Y \) are blocked by \( X \)

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**Partial Evaluation and Local MEU**

**LMEU**

\[ \text{LMEU}_{\mathcal{M}(D', U')} := \max \Delta \mathbb{E} \left[ \sum_{U_i \in \mathbf{U}} U_i \right] \]

**Evaluate MEU over the subset of decision variables and utility functions**
- Maximize expected utility \( U_2 + U_3 \) over two decision variables \( D_2 \) and \( D_3 \)

\[ \max \Delta (D_2, D_3, C_4, C_5, C_6) \mathbb{E} [P(X, D)] \]

**Submodel in IDs**

- (Definition) Submodel \( \mathcal{M}'(D', U') \) is a relevant subset of model \( \mathcal{M} \) for computing LMEU on \( D' \subseteq D, U' \subseteq U \)

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**Graphical Models**

**Submodel-Tree Decomposition**
- Process decision nodes in reverse topological order
- Partial decision order \( \mathcal{O}_P = (D_1 < D_2 < D_3) \)

\[ \mathcal{M}(D_1, D_2, D_3, U_1, U_2, U_3) \]

**Identify 2nd Stable submodel**
- Eliminate submodel from ID

**Identify 1st Stable Submodel**
- Given an ID \( \mathcal{M} \) and the set of stable submodels \( \mathcal{M}_{O_D} \), relative to \( O_D \)
- submodel-tree decomposition is a tuple \( \mathcal{T}_{FT} := (T(C, S), \chi, \psi) \)

**Tree-clustering scheme**

**Bounding MEU of Each Submodel**
- Exponentiated Utility Bounds for MEU

\[ \max \Delta \left[ \sum_{U_i \in \mathbf{U}} U_i \right] \leq \log \max \Delta \left[ \prod_{U_i \in \mathbf{U}} e^\mathbb{E} (X_{U_i}) \right] \]

**Experiments**

**Synthetic ID Benchmarks**

| Domain       | n | m | p | q | r | s | t | u | v | w | x | y | z | n | m | p | q | r | s | t | u | v | w | x | y | z |
| ID-BN        | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| ID-DNN       | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |

- **ST-GDD**: submodel-free decomposition + GGD for MMAP
- **ST-WMD**: submodel-free decomposition + WMD for MMAP
- **ST-WMD-B**: submodel-free decomposition + WMD for MMAP
- **ST-WMD-B+B**: submodel-free decomposition + WMD for MMAP

**Synthetic ID Benchmarks**

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**SysAdmin MDP Probabilistic Planning Problem**

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