Exploring UFO’s

Bobak Pezeshki¹  Radu Marinescu²  Alexander Ihler¹  Rina Dechter¹

¹University of California, Irvine
²IBM Research

CONTENTS

i. Glossary 2

ii. Abbreviations 3

iii. Notation 4

1 UFO Properties 5

2 Preliminary Tests of AOBB-UFO on UAI 2022 Competition MMAP 7

Accepted for the 39th Conference on Uncertainty in Artificial Intelligence (UAI 2023).
GLOSSARY

τ-underflowed function

\((f_\tau)\) A function such that output values less than \(\tau\) are replaced with 0.0.

\[
f_\alpha(\mathbf{a} \in D_\alpha) = \begin{cases} f_\alpha(\mathbf{a}), & f_\alpha(\mathbf{a}) \geq \tau \\ 0, & \text{otherwise.} \end{cases}
\]

graphical model

\((\mathcal{M} = \langle \mathbf{X}, \mathbf{D}, \mathbf{F} \rangle)\). Mathematical tool for modeling complex systems composed of a set of variables \(\mathbf{X}\), a set of domains \(\mathbf{D} = \{D_X | X \in \mathbf{X}\}\) for each variable \(X\), and a set of functions \(\mathbf{F}\) with each function defined over a subset of the model’s variables \(\alpha \subseteq \mathbf{X}\).

marginal maximum a-posteriori

(MMAP). The marginal likelihood associated with the configuration of a target subset of variables \(Q\) that maximizes their marginal likelihood.

In the context of discrete graphical models without evidence, with \(Q \subset \mathbf{X}\), \(S = \mathbf{X} \setminus Q\) be the variables to sum over, \(\mathbf{F}_Q = \{f_\alpha | \alpha \subseteq Q\}\) be the set of functions defined only over \(\alpha \in Q\), and \(\mathbf{F}_S = \mathbf{F} \setminus \mathbf{F}_Q\) be functions that include some \(X \in S\) in their scope,

\[
MMAP = \max_Q \sum_S \prod_{f_\alpha \in \mathbf{F}_Q} f_\alpha(q \cup s)
\]

(1)

\[
= \max_Q \prod_{f'_\alpha \in \mathbf{F}_S} f'_\alpha(q) \sum_S \prod_{f''_\alpha \in \mathbf{F}_S} f''_\alpha(q \cup s)
\]

(2)

maximum a-posteriori

(MAP). With respect to a graphical model, the likelihood value associated with the most probable explanation or MPE.

In the context of discrete graphical models without evidence, \(MAP = \max_Q \prod_{f_\alpha \in \mathbf{F}} f_\alpha(q)\).

most probable explanation

(MPE). Assignment to variables the variables of a graphical model that maximizes the conditional probability of the observed evidence.

In the context of discrete graphical models without any evidence, \(MPE = \arg \max_Q \prod_{f_\alpha \in \mathbf{F}} f_\alpha(q)\).

partial configuration

A joint assignment to a subset of the variables of a graphical model.

partition function

\((Z)\). A mathematical quantity that characterizes the distribution among a system’s possible states and serves as a normalizing constant for calculating probabilistic measures associated with these states.

In the context of discrete graphical models, \(Z = \sum_{\mathbf{x}} \prod_{f_\alpha \in \mathbf{F}} f_\alpha(\mathbf{x})\).
ABBREVIATIONS

MAP
- Maximum a-posteriori

MMAP
- Marginal maximum a-posteriori

MPE
- Most probable explanation

Z
- Partition function
NOTATION

[capital letters] (ex. X)
Representing a variable of a graphical model.

[lower-case letters] (ex. x)
Representing assignments to variable corresponding to their capitalized form. For example, x represents a particular assignment to the variable represented by X, or \( X \leftarrow x \).

[bold-faced capital letters] (ex. X)
A set of variables of a graphical model \( (X, \) in particular, often refers to the set of all variables of a graphical model).

\( X \)
The set of all variables \( X \) of a graphical model.

\( Q \)
In the context of the maximum a-posteriori or marginal maximum a-posteriori task, the subset of the variables \( Q \) that are to be maximized over (known as "query" or "MAP" variables).

\( S \)
In the context of the marginal maximum a-posteriori task, the subset of variables to be summed over. \( S = X \setminus Q \)

\( x \)
A full configuration, i.e. assignment to all variables \( X \) of a graphical model \( X \leftarrow x \in D_X \).

\( D_Y \)
The set of all possible partial configurations to the variables of the subset \( Y \subset X \). \( D_Y \) is the Cartesian product of the domains of the variables in \( Y \). \( D_Y = \bigotimes_{Y \in Y} D_Y \), where \( \bigotimes \) is the Cartesian product operator.

\( F_\tau \)
\( F_\tau = \{ f_\tau \mid f \in F \} \)

\( F_Q \)
In the context of the maximum a-posteriori or marginal maximum a-posteriori task, the subset of functions defined only on a subset of the variables \( Q \) that are to be maximized over (known as "query" or "MAP" variables). \( F_Q = \{ f_\alpha \mid \alpha \subseteq Q \} \)

\( F_S \)
In the context of the marginal maximum a-posteriori task, \( F_S = F \setminus F_Q = \{ f_\alpha \mid \exists X \in \alpha \text{} s.t. X \notin Q \} \)

\( M \)
Graphical model \( M = (X, D, F) \) with non-negative functions.

\( M_\tau \)
An altered graphical model \( M_\tau = (X, D, F_\tau) \) with non-negative functions such that each original function is replaced by its corresponding \( \tau \)-underflowed function.

\( MMAP(M, Q) \)
The marginal maximum a-posteriori of graphical model \( M \) maximizing over the subset of variables \( Q \).
1 UFO PROPERTIES

**Theorem 1.0.0.1** (Lower-bounds from \( \tau \)-underflows)
\[
\forall \text{task} \in \{Z, MAP, MMAP\}, \forall \mathcal{M}, \text{task}(\mathcal{M}_\tau) \leq \text{task}(\mathcal{M})
\]

*Proof*

The evaluated value of summation, maximization, or mixed max-sum operations over products of non-negative functions can only decrease if function values decrease (but remain non-negative). By definition, \( \tau \)-underflows either do not affect function values or decrease them to 0.0. Thus, the evaluated value of summation, maximization, or mixed max-sum operations can only decrease, and never increase, due to a \( \tau \)-underflows.

**Corollary 1.0.0.2** (Monotonicity from \( \tau \)-underflows)
\[
\forall \text{task} \in \{Z, MAP, MMAP\}, \forall \mathcal{M}, \forall \tau'<\tau'', \text{task}(\mathcal{M}_{\tau''}) \leq \text{task}(\mathcal{M}_{\tau'})
\]

*Proof*

\( \forall \mathcal{M}, \forall \tau'<\tau'', \mathcal{M}_{\tau''} = \mathcal{M}_{\tau''} \). Thus, the result follows directly from Theorem 1.0.0.1

**Corollary 1.0.0.3** (Solution persistence)
\[
\forall \text{task} \in \{Z, MAP, MMAP\}, \text{if a \( \tau \)-underflow does not affect the function values involved in the computation of the solution cost, then task(\( \mathcal{M}_{\tau} \)) = \text{task}(\mathcal{M})}.
\]

More formally, \( \forall \mathcal{M}, \forall f \in F, f_\tau = f \), then \( Z(\mathcal{M}_\tau) = Z(\mathcal{M}) \) and \( \forall \text{task} \in \{MAP, MMAP\}, \forall \mathcal{M}, \exists \mathcal{Q} \leftarrow q^* \mid \text{task}(\mathcal{M}|q^*) = \text{task}(\mathcal{M}), \forall f \in F, f_\tau(q^*) = f(q^*) \), then \( \text{task}(\mathcal{M}_\tau) = \text{task}(\mathcal{M}). \)

*Proof*

Case 1 (Z):

If \( \forall f \in F, f_\tau = f \), then \( \mathcal{M}_\tau = \mathcal{M} \) and \( \text{task}(\mathcal{M}_\tau) = \text{task}(\mathcal{M}) \) follows trivially.

Case 2 (MAP and MMAP):

If \( \exists \mathcal{Q} \leftarrow q^* \mid \text{task}(\mathcal{M}|q^*) = \text{task}(\mathcal{M}), \forall f \in F, f_\tau(q^*) = f(q^*) \), then \( \text{task}(\mathcal{M}_\tau|q^*) = \text{task}(\mathcal{M}|q^*) \). Since \( \forall q \neq q^*, \text{task}(\mathcal{M}|q) \leq \text{task}(\mathcal{M}|q^*) \) and by \( \tau \)-underflows can only decrease costs for these tasks, the optimal solution in the \( \tau \)-underflowed problem will still correspond to \( q^* \).

**Theorem 1.0.0.4** (Tractable upper-bound on error \( \text{MMAP}(\mathcal{M}, Q) - \text{MMAP}(\mathcal{M}_{S_\alpha}, Q) \))

Given a graphical model \( \mathcal{M} \) for computing the marginal maximum a-posteriori maximizing over query variables \( Q \subset X \) and a \( \tau \)-underflow of the model, \( \mathcal{M}_{S_\alpha} \), such that underflows are only applied to functions \( F_S \) that include summation variables in their scope \( \{f_\alpha \mid f_\alpha \in F \text{ and } 3X \in \alpha \text{ s.t. } X \in S = X \setminus Q\} \), the error \( \epsilon = \text{MMAP}(\mathcal{M}, Q) - \text{MMAP}(\mathcal{M}_{S_\alpha}, Q) \) is upper-bounded by
\[
\epsilon \leq (v_{F_Q})^{(F_Q)} \cdot |D_{X_S}| \cdot (v_{F_S})^{(F_S)-1}\cdot (\tau)
\]

*Proof*

Let \( S = X \setminus Q \) be the set of variables to sum over, \( F_Q = \{f_\alpha \mid \alpha \subseteq Q\} \) be the set of functions defined only over \( \alpha \in Q \), and \( F_S = F \setminus F_Q \) be functions that include some \( X \in S \) in their scope, the MMAP can be expressed as:
\[
\text{MMAP}(\mathcal{M}, Q) = \max_{Q} \prod_{f' \in F_Q} f'(q) \sum_{S} \prod_{f'' \in F_S} f''(q \cup s)
\]
for which the summation can be more explicitly be expressed via the notation
\[
\text{MMAP}(\mathcal{M}, Q) = \max_{Q} \prod_{f' \in F_Q} f'(q) \sum_{s \in D_S} \prod_{f'' \in F_S} f''(q \cup s)
\]
Assume the MMAP solution is due to some unique assignment \( Q \leftarrow q^* \). (Although a MMAP solution can be due to a set of possible \( q^* \)'s, it will be easy to see that the proof pertaining to a unique MMAP \( q^* \) assignment is easily extendable to cases with multiple assignments). The MMAP value can now be expressed as
\[
\text{MMAP}(\mathcal{M}, Q) = Z(\mathcal{M}, q^*) = \prod_{f' \in F_Q} f'(q^*) \sum_{s \in D_S} \prod_{f'' \in F_S} f''(q^* \cup s)
\]
Now partition $D_S$ into subsets $D_{S_{\geq \tau}}^{{q}^*}$ and $D_{S_{< \tau}}^{{q}^*}$ where
\begin{align}
D_{S_{\geq \tau}}^{{q}^*} &= \{s \mid \forall f \in F_S, f(q^* \cup s) \geq \tau\} \\
D_{S_{< \tau}}^{{q}^*} &= \{s \mid \exists f \in F_S, f(q^* \cup s) < \tau\}
\end{align}

$D_{S_{\geq \tau}}^{{q}^*}$ corresponds to terms in the summation that are not affected by a $\tau$-underflow of the problem whereas $D_{S_{< \tau}}^{{q}^*}$ corresponds to terms of the summation that end up being 0.0 after a $\tau$-underflow of the model is performed. As such, the cost of $q^*$ in the $\tau$-underflowed model is
\begin{align}
Z(M_{S_\tau}, q^*) &= \prod_{f' \in F_Q} f'(q^*) \sum_{s \in D_{S_{\geq \tau}}} \prod_{f'' \in F_{S_{\tau}}} f''(q^* \cup s) \\
&= \prod_{f' \in F_Q} f'(q^*) \cdot \left( \sum_{s \in D_{S_{\geq \tau}}} \prod_{f'' \in F_{S_{\tau}}} f''(q^* \cup s) + \sum_{s \in D_{S_{< \tau}}} \prod_{f'' \in F_{S_{\tau}}} f''(q^* \cup s) \right) \\
&= \prod_{f' \in F_Q} f'(q^*) \sum_{s \in D_{S_{\geq \tau}}} \prod_{f'' \in F_{S_{\tau}}} f''(q^* \cup s) + \prod_{f' \in F_Q} f'(q^*) \sum_{s \in D_{S_{< \tau}}} \prod_{f'' \in F_{S_{\tau}}} f''(q^* \cup s)
\end{align}

Thus, the error $\epsilon = MMAP(M, Q) - MMAP(M_{S_\tau}, Q)$ can be expressed as
\begin{align}
\epsilon &= \prod_{f' \in F_Q} f'(q^*) \sum_{s \in D_{S_{\geq \tau}}} \prod_{f'' \in F_{S_{\tau}}} f''(q^* \cup s)
\end{align}

and, since for each $s \in D_{S_{\geq \tau}}$ at least one $f'' \in F_{S_S}$ has to evaluate to be zero by definition of $D_{S_{\geq \tau}}$, $\epsilon$ can be at most
\begin{align}
\epsilon \leq (v^*_{F_Q})^{\mid F_Q \mid} \cdot |D_{X_S}| \cdot (v^*_{F_S})^{\mid F_S \mid - 1} \cdot (\tau)
\end{align}

**Complexity**

Testing the condition in Corollary 1.0.0.4 can be done in linear time.
## PRELIMINARY TESTS OF AOBB-UFO ON UAI 2022 COMPETITION MMAP

Tables 1, 2 shows preliminary test results of AOBB [Marinescu et al. 2018] empowered with UFO. For these preliminary results, the UFO binary search was done over log space for two seconds and no deflation was applied to the resulting threshold. Per the competition restrictions, the algorithm was constrained to using 8G of memory.

### Table Key

For each problem, provided are:

- $|X| : |F|$: the number of variables and functions, respectively
- $w^*$: the induced width (see Extended Supplemental) due to the variable ordering used by AOBB-UFO
- $k$: the maximum domain size
- **Anytime**: the time it took AOBB-UFO to originally find its final solution
- **Time**: the time it took AOBB-UFO to terminate
- **Solution**: AOBB-UFO’s best found solution
- the best found solution for each of the competing solvers

<table>
<thead>
<tr>
<th>Problem</th>
<th>PROBLEM STATISTICS</th>
<th>AOBB-UFO</th>
<th>UAI 2022 COMPETITION COMPETING SOLVERS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$</td>
<td>X</td>
<td>:</td>
</tr>
<tr>
<td>75-17-5.Q0.5.14</td>
<td>289 : 289 : 110 : 2</td>
<td>17.19</td>
<td>18.06</td>
</tr>
<tr>
<td>75-19-5.Q0.5.12</td>
<td>361 : 361 : 133 : 2</td>
<td>15.7</td>
<td>15.66</td>
</tr>
<tr>
<td>75-22-5.Q0.5.12</td>
<td>484 : 484 : 110 : 2</td>
<td>37.6</td>
<td>37.59</td>
</tr>
<tr>
<td>75-23-5.Q0.5.13</td>
<td>529 : 529 : 177 : 2</td>
<td>17.9</td>
<td>17.95</td>
</tr>
<tr>
<td>90-22-5.Q0.5.14</td>
<td>484 : 484 : 173 : 2</td>
<td>39.4</td>
<td>39.39</td>
</tr>
<tr>
<td>90-24-5.Q0.5.12</td>
<td>576 : 576 : 131 : 2</td>
<td>27.4</td>
<td>27.37</td>
</tr>
<tr>
<td>90-26-5.Q0.5.11</td>
<td>676 : 676 : 110 : 2</td>
<td>27</td>
<td>26.96</td>
</tr>
<tr>
<td>90-30-5.Q0.5.11</td>
<td>900 : 900 : 246 : 2</td>
<td>36.1</td>
<td>36.14</td>
</tr>
<tr>
<td>90-34-5.Q0.5.12</td>
<td>1156 : 1156 : 352 : 2</td>
<td>29.6</td>
<td>29.63</td>
</tr>
<tr>
<td>90-38-5.Q0.5.14</td>
<td>1444 : 1444 : 371 : 2</td>
<td>44.9</td>
<td>44.86</td>
</tr>
<tr>
<td>90-42-5.Q0.5.14</td>
<td>1764 : 1764 : 300 : 2</td>
<td>40.6</td>
<td>40.59</td>
</tr>
<tr>
<td>90-50-5.Q0.5.13</td>
<td>2500 : 2500 : 788 : 2</td>
<td>201.2</td>
<td>201.21</td>
</tr>
<tr>
<td>bw_p24_16</td>
<td>937 : 937 : 136 : 3</td>
<td>82.8</td>
<td>3682.8</td>
</tr>
<tr>
<td>bw_p24_20</td>
<td>1169 : 1169 : 171 : 3</td>
<td>105.2</td>
<td>3705.2</td>
</tr>
<tr>
<td>bw_p34_15</td>
<td>2191 : 2191 : 294 : 3</td>
<td>59.2</td>
<td>3659.2</td>
</tr>
<tr>
<td>bw_p34_20</td>
<td>2916 : 2916 : 389 : 3</td>
<td>108.5</td>
<td>3708.5</td>
</tr>
<tr>
<td>bw_p44_15</td>
<td>4075 : 4075 : 638 : 3</td>
<td>514.1</td>
<td>4114.1</td>
</tr>
<tr>
<td>bw_p44_19</td>
<td>5155 : 5155 : 722 : 3</td>
<td>828.9</td>
<td>4428.9</td>
</tr>
<tr>
<td>bw_p54_10</td>
<td>4366 : 4366 : 777 : 3</td>
<td>911.2</td>
<td>911.25</td>
</tr>
<tr>
<td>bw_p54_16</td>
<td>6964 : 6964 : 1134 : 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>comm_p01_16</td>
<td>4477 : 4477 : 831 : 2</td>
<td>266.9</td>
<td>3866.9</td>
</tr>
<tr>
<td>comm_p01_20</td>
<td>5585 : 5585 : 1039 : 2</td>
<td>453.8</td>
<td>4053.8</td>
</tr>
</tbody>
</table>

Table 1: AOBB-UFO on UAI 2022 Competition Final Problems (3600s)
<table>
<thead>
<tr>
<th>Problem</th>
<th>PROBLEM STATISTICS</th>
<th>AOBB-UFO</th>
<th>UAI 2022 COMPETITION COMPETING SOLVERS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>braobb</td>
<td>daoopp-lh</td>
</tr>
<tr>
<td>or_chain_11_fg.Q0.5.13</td>
<td>900; 915; 191</td>
<td>30.8</td>
<td>30.85</td>
</tr>
<tr>
<td>or_chain_16_fg.Q0.5.13</td>
<td>1675; 1700; 318</td>
<td>98.5</td>
<td>98.32</td>
</tr>
<tr>
<td>or_chain_22_fg.Q0.5.13</td>
<td>1044; 1054; 196</td>
<td>28.9</td>
<td>28.92</td>
</tr>
<tr>
<td>or_chain_24_fg.Q0.5.13</td>
<td>1155; 1171; 247</td>
<td>32.3</td>
<td>32.31</td>
</tr>
<tr>
<td>or_chain_25_fg.Q0.5.13</td>
<td>1075; 1086; 88</td>
<td>30.4</td>
<td>30.37</td>
</tr>
<tr>
<td>or_chain_32_fg.Q0.5.13</td>
<td>1466; 1478; 108</td>
<td>32</td>
<td>3632</td>
</tr>
<tr>
<td>or_chain_36_fg.Q0.5.13</td>
<td>933; 943; 91</td>
<td>30.5</td>
<td>30.46</td>
</tr>
<tr>
<td>or_chain_39_fg.Q0.5.13</td>
<td>1751; 1766; 430</td>
<td>96.3</td>
<td>96.32</td>
</tr>
<tr>
<td>or_chain_40_fg.Q0.5.13</td>
<td>988; 998; 96</td>
<td>16.3</td>
<td>16.26</td>
</tr>
<tr>
<td>or_chain_41_fg.Q0.5.13</td>
<td>1847; 1863; 203</td>
<td>118.5</td>
<td>3645</td>
</tr>
<tr>
<td>or_chain_43_fg.Q0.5.13</td>
<td>1692; 1712; 216</td>
<td>44.5</td>
<td>44.5</td>
</tr>
<tr>
<td>or_chain_6_fg.Q0.5.13</td>
<td>1849; 1876; 386</td>
<td>2351.2</td>
<td>3710.9</td>
</tr>
<tr>
<td>or_chain_60_fg.Q0.5.13</td>
<td>1997; 2023; 552</td>
<td>3691.2</td>
<td>3736.6</td>
</tr>
<tr>
<td>or_chain_63_fg.Q0.5.13</td>
<td>731; 744; 97</td>
<td>26.3</td>
<td>26.35</td>
</tr>
<tr>
<td>or_chain_70_fg.Q0.5.13</td>
<td>1195; 1203; 63</td>
<td>23.6</td>
<td>23.58</td>
</tr>
<tr>
<td>pedigree1.Q0.5.13</td>
<td>298; 334; 105</td>
<td>3319.4</td>
<td>3641.4</td>
</tr>
<tr>
<td>pedigree13.Q0.5.11</td>
<td>888; 1077; 138</td>
<td>2941.6</td>
<td>3623.5</td>
</tr>
<tr>
<td>pedigree18.Q0.5.11</td>
<td>931; 1184; 181</td>
<td>38.8</td>
<td>3638.8</td>
</tr>
<tr>
<td>pedigree19.Q0.5.14</td>
<td>693; 793; 173</td>
<td>3280</td>
<td>3625.2</td>
</tr>
<tr>
<td>pedigree20.Q0.5.12</td>
<td>387; 437; 76</td>
<td>2455.1</td>
<td>3614.7</td>
</tr>
<tr>
<td>pedigree25.Q0.5.12</td>
<td>993; 1289; 178</td>
<td>3236.3</td>
<td>3702.4</td>
</tr>
<tr>
<td>pedigree30.Q0.5.12</td>
<td>1015; 1289; 109</td>
<td>30.9</td>
<td>3630.9</td>
</tr>
<tr>
<td>pedigree31.Q0.5.12</td>
<td>1006; 1183; 153</td>
<td>16</td>
<td>3616</td>
</tr>
<tr>
<td>pedigree33.Q0.5.12</td>
<td>581; 798; 118</td>
<td>28.7</td>
<td>3628.7</td>
</tr>
<tr>
<td>pedigree38.Q0.5.12</td>
<td>581; 724; 164</td>
<td>115.5</td>
<td>3621.9</td>
</tr>
<tr>
<td>pedigree41.Q0.5.12</td>
<td>885; 1062; 205</td>
<td>27.5</td>
<td>3620.7</td>
</tr>
<tr>
<td>pedigree44.Q0.5.14</td>
<td>644; 811; 186</td>
<td>3177.3</td>
<td>3628.3</td>
</tr>
<tr>
<td>pedigree50.Q0.5.11</td>
<td>478; 514; 124</td>
<td>214.5</td>
<td>3674.7</td>
</tr>
<tr>
<td>pedigree7.Q0.5.12</td>
<td>867; 1068; 89</td>
<td>3579.1</td>
<td>3628.3</td>
</tr>
<tr>
<td>pedigree9.Q0.5.13</td>
<td>935; 1118; 235</td>
<td>913.3</td>
<td>3626.6</td>
</tr>
<tr>
<td>pondp10_12_7_3_8_4</td>
<td>2673; 2701; 2599</td>
<td>32</td>
<td>1.2</td>
</tr>
<tr>
<td>pondp6_6_2_6_3</td>
<td>250; 265; 198</td>
<td>63.9</td>
<td>3663.9</td>
</tr>
<tr>
<td>pondp7_20_10_2_10_3</td>
<td>3166; 3193</td>
<td>1</td>
<td>1.2</td>
</tr>
<tr>
<td>pondp8_14_9_3_12_4</td>
<td>2145; 2189; 2057</td>
<td>48</td>
<td>1.4</td>
</tr>
<tr>
<td>pondp9_14_3_8_3_10_4</td>
<td>5277; 5313</td>
<td>1</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Table 2: AOBB-UFO on UAI 2022 Competition Final Problems (3600s)
Summary Statistics

- Total number of problems = 100
- Number of problems for which AOBB-UFO equalized or did better than all of the competing solvers = 34
- Average number of competing solvers AOBB-UFO equalized or did better than = 3.62
- Number of problems for which AOBB-UFO did strictly better than all of the competing solvers = 18
- Average number of competing solvers AOBB-UFO did strictly better than = 2.24
- Number of problems for which AOBB-UFO terminated before 120 seconds = 42
References