1. Your friend owns a business with a client base in two different cities, Detroit and Seattle. In any given month, the business can be operated out of either city. Based on projections of rental costs and storage costs for the necessary equipment, your friend has estimate the cost for a given month operating out of either city for the next \( n \) months: if the business operates out of Detroit in month \( i \) the cost for the month will be \( D_i \), and if the business operates out of Seattle in month \( i \) it will be \( S_i \).

If the business operates out of one city in month \( i \) and out of the other one in month \( i + 1 \), there is an additional cost of \( T \) for the transition.

For a given sequence of \( n \) months, a schedule is a sequence of \( n \) locations, where each entry in the sequence is either Detroit or Seattle and the \( i \)th entry in the sequence is the operating location for the \( i \)th month.

Give a dynamic programming algorithm that takes as input the transition cost \( T \) and the values \( D_1, \ldots, D_n \) and \( S_1, \ldots, S_n \), and produces a schedule that minimizes the total cost.

As an example, suppose \( n = 5 \) and \( T = 10 \) with the following profit estimates:

<table>
<thead>
<tr>
<th>City</th>
<th>month 1</th>
<th>month 2</th>
<th>month 3</th>
<th>month 4</th>
<th>month 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detroit</td>
<td>5</td>
<td>7</td>
<td>30</td>
<td>30</td>
<td>15</td>
</tr>
<tr>
<td>Seattle</td>
<td>30</td>
<td>40</td>
<td>4</td>
<td>6</td>
<td>20</td>
</tr>
</tbody>
</table>

The optimum schedule would be \([\text{Detroit, Detroit, Seattle, Seattle, Seattle}]\) with a total profit of \( 5 + 7 + 4 + 4 + 20 + 10 = 50 \).

In your solution provide the following:

(a) The specification of the solution for computing the cost of the optimum schedule, as prescribed on slide 6-15: subproblem domain, function/memoization table definition, goal, initial condition(s), and recurrence equation.

(b) A verbal (English) description of additional information that can be used to derive the optimal schedule (analogous to the keep array in the weighted interval scheduling problem and the truck-loading and knapsack problems).

(c) Pseudocode to compute the memoization table and the additional information.

(d) Pseudocode to compute the optimal schedule from the memoization table and the additional information.
2. Suppose you are a consultant for a company that manufactures computing equipment and ships it to distributors. For each of the next \( n \) weeks, they have a projected supply of equipment, measured by weight, which has to be shipped via air freight. The weight of the equipment to be shipped in week \( i \) is \( w_i \) (for \( i = 1, \ldots, n \)). Each week’s supply can be carried by one of two air freight shippers, A or B. The two companies calculate their shipping charges as follows:

- Shipper A charges by weight, at a fixed cost per unit of weight. In other words, there is some parameter \( d \) such that it costs \( d \cdot w_i \) to ship the equipment for week \( i \) using Shipper A.
- Shipper B will contract to ship an unlimited amount of equipment at a fixed cost of \( F \) per week, independent of the weight. However, there is a catch: contracts with shipper B must be made for four weeks at a time.

For a given sequence of \( n \) weeks, a schedule is a choice of shipper (A or B) for each week, with the restriction that company B must be chosen for blocks of 4 contiguous weeks at a time. The cost of the schedule is the total amount paid to company A and B according to the description above.

Give a dynamic programming algorithm that takes as input the values of \( d \) and \( F \) and the sequence of weights \( w_1, \ldots, w_n \), and produces a schedule that minimizes the cost.

As an example, suppose \( d = 1 \), \( F = 10 \), \( n = 10 \), and the sequence of weights is

\[
11, 9, 9, 12, 12, 12, 12, 9, 9, 11
\]

The optimal schedule would be to choose A for three weeks, then B for a block of four consecutive weeks, and then A for the final three weeks. The cost of this schedule would be

\[
11 + 9 + 9 + 4 \cdot 10 + 9 + 9 + 11.
\]

You can make the following simplifying assumption: for every \( i \), \( w_i \leq \frac{4F}{d} \). This assumption implies, for example, that it would never be better to hire shipper B just for the first three weeks and pay for four weeks, and then shift over to shipper A in week 4.

In your solution, provide the same information you were asked to provide in your solution to the previous problem.