Vector Space Scoring

Introduction to Information Retrieval
Informatics 141 / CS 121
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Content adapted from Hinrich Schütze
http://www.informationretrieval.org

Corpus-wide statistics

- Collection Frequency, cf
  - Define: The total number of occurences of the term in the entire corpus
- Document Frequency, df
  - Define: The total number of documents which contain the term in the corpus

Corpus-wide statistics

Word	*Collection Frequency*	*Document Frequency*
insurance	10440	3997
try	10422	8760

- This suggests that df is better at discriminating between documents
- How do we use df?

Corpus-wide statistics

- Term-Frequency, Inverse Document Frequency Weights
  - "tf-idf"
  - tf = term frequency
    - some measure of term density in a document
  - idf = inverse document frequency
    - a measure of the informativeness of a term
    - it's rarity across the corpus
    - could be just a count of documents with the term
    - more commonly it is:

$$idf_t = log\left(\frac{|corpus|}{df_t}\right)$$

TF-IDF Examples

$$idf_t = log\left(\frac{|corpus|}{df_t}\right) \qquad idf_t = log_{10}\left(\frac{1,000,000}{df_t}\right)$$

$term$	df_t	idf_t
$calpurnia$	1	6
$animal$	10	4
$sunday$	1000	3
fly	$10,000$	2
$under$	$100,000$	1
the	$1,000,000$	0

TF-IDF Summary

- Assign tf-idf weight for each term t in a document d:

$$tfidf(t,d) \quad = \quad (1 + log(tf_{t,d})) * log\left(\frac{|corpus|}{df_{t,d}}\right)$$

  - Increases with number of occurrences of term in a doc.
  - Increases with rarity of term across entire corpus
  - Three different metrics
    - term frequency
    - document frequency
    - collection/corpus frequency

Now, real-valued term-document matrices

- Bag of words model
- Each element of matrix is tf-idf value

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	13.1	11.4	0.0	0.0	0.0	0.0
Brutus	3.0	8.3	0.0	1.0	0.0	0.0
Caesar	2.3	2.3	0.0	0.5	0.3	0.3
Calpurnia	0.0	11.2	0.0	0.0	0.0	0.0
Cleopatra	17.7	0.0	0.0	0.0	0.0	0.0
mercy	0.5	0.0	0.7	0.9	0.9	0.3
worser	1.2	0.0	0.6	0.6	0.6	0.0

Vector Space Scoring

- That is a nice matrix, but
  - How does it relate to scoring?
  - Next, vector space scoring

Vector Space Model

- Define: Vector Space Model
  - Representing a set of documents as vectors in a common vector space.
  - It is fundamental to many operations
    - (query,document) pair scoring
    - document classification
    - document clustering
  - Queries are represented as a document
    - A short one, but mathematically equivalent

Vector Space Model

- Define: Vector Space Model
  - A document, d, is defined as a vector: $\vec{V}(d)$
    - One component for each term in the dictionary
    - Assume the term is the tf-idf score

$$\vec{V}(d)_t \quad = \quad (1 + log(tf_{t,d})) * log\left(\frac{|corpus|}{df_{t,d}}\right)$$

    - A corpus is many vectors together.
    - A document can be thought of as a point in a multi-dimensional space, with axes related to terms.

Vector Space Model

- Recall our Shakespeare Example:

	$\vec{V}(d_1)$	$\vec{V}(d_2)$				$\vec{V}(d_6)$
	Antony and Cleopatra	*Julius Caesar*	*The Tempest*	*Hamlet*	*Othello*	*Macbeth*
Antony	13.1	11.4	0.0	0.0	0.0	0.0
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$\vec{V}(d_6)_7$

Vector Space Model

- Recall our Shakespeare Example:

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Antony	13.1	11.4	0.0	0.0	0.0	0.0
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Calpurnia	0.0	11.2	0.0	0.0	0.0	0.0
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Vector Space Model

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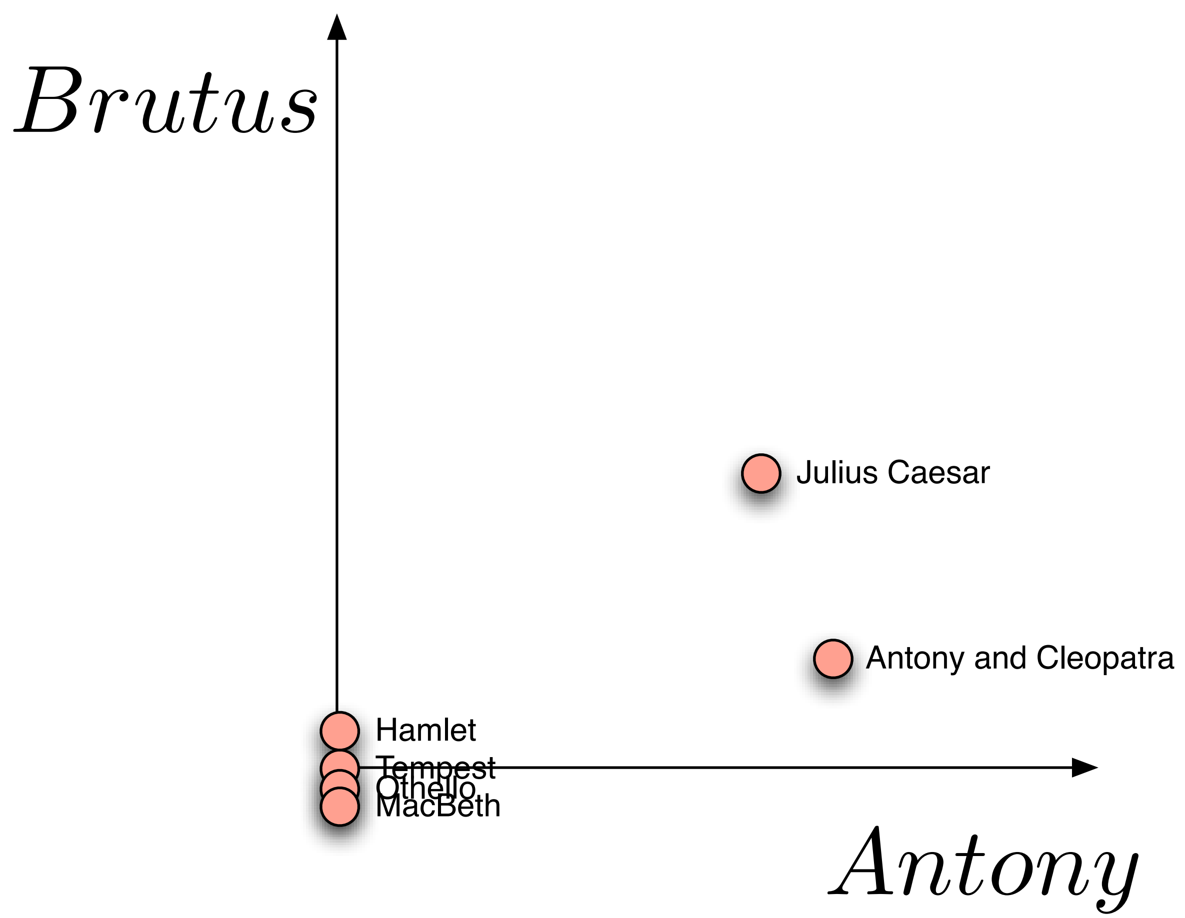

Vector Space Model

- Recall our Shakespeare Example:

	$\vec{V}(d_1)$ *Antony and Cleopatra*	$\vec{V}(d_2)$ *Julius Caesar*	*The Tempest*	*Hamlet*	*Othello*	$\vec{V}(d_6)$ *Macbeth*
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Vector Space Model

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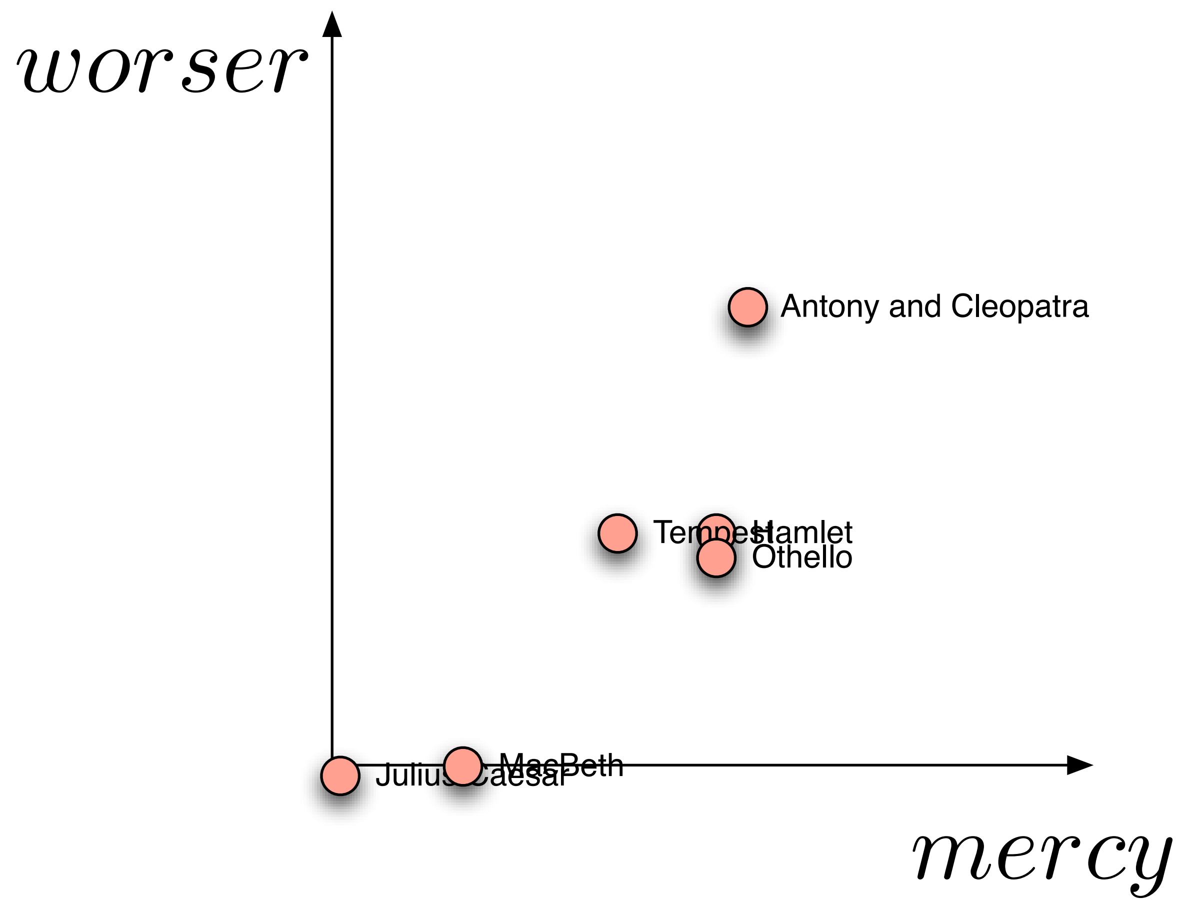
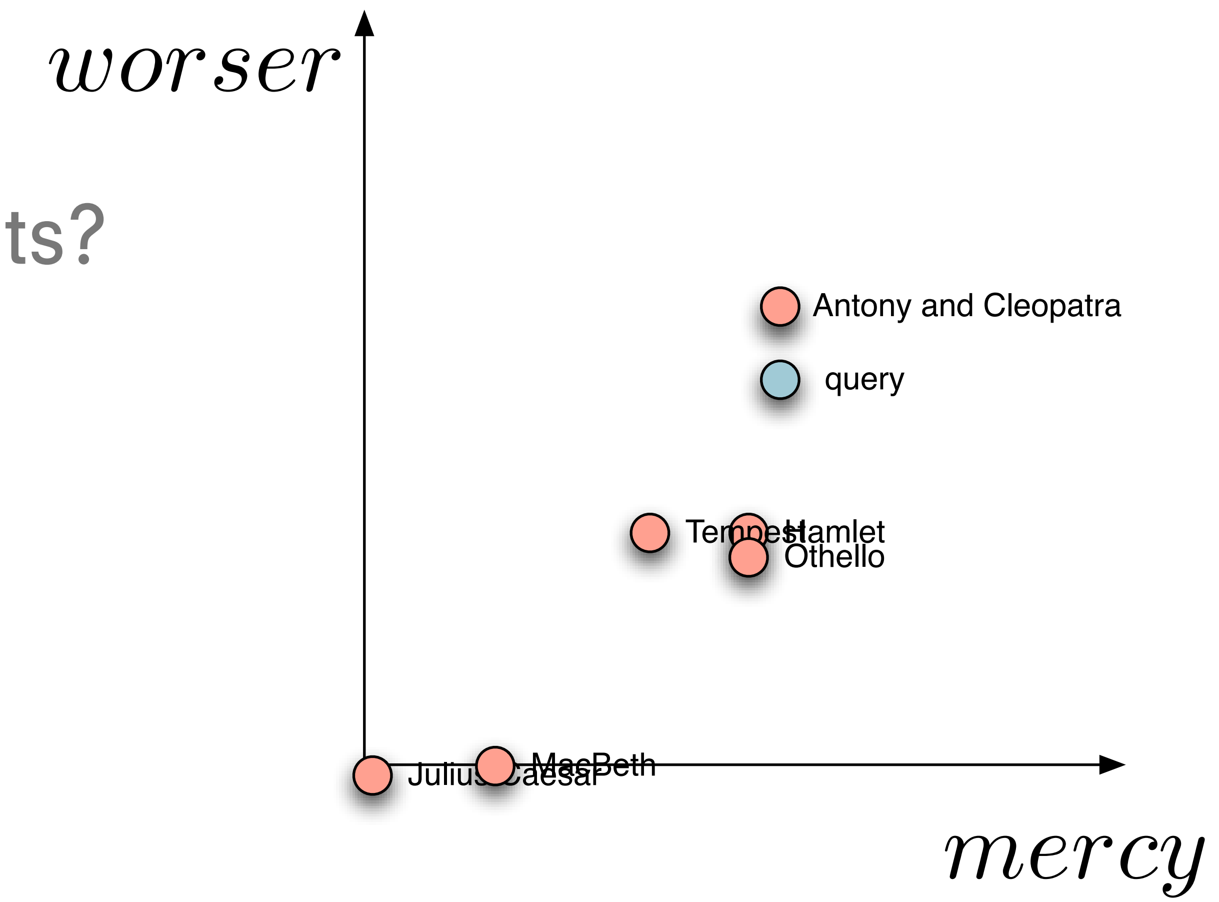


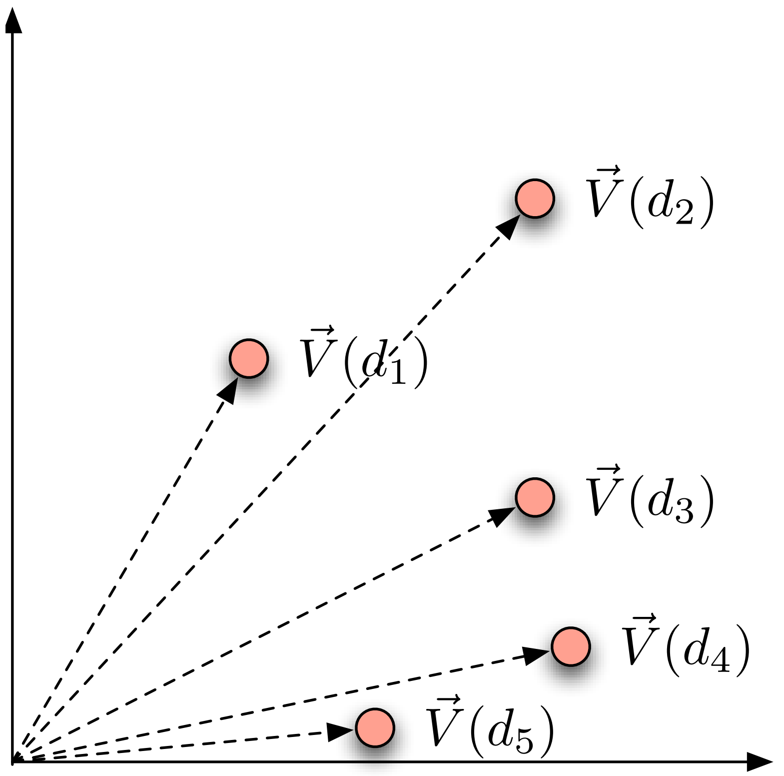
Query as a vector

- So a query can also be plotted in the same space
  - “worser mercy”
  - To score, we ask:
    - How similar are two points?
  - How to answer?



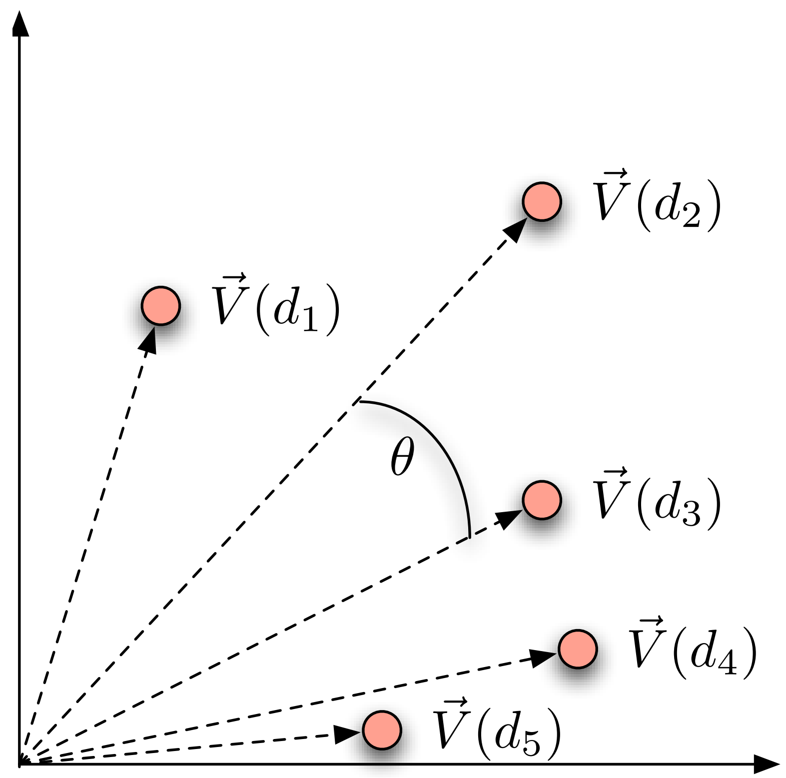
Score by magnitude

- How to answer?
    - Similarity of magnitude?
        - But, two documents, similar in content, different in length can have large differences in magnitude.



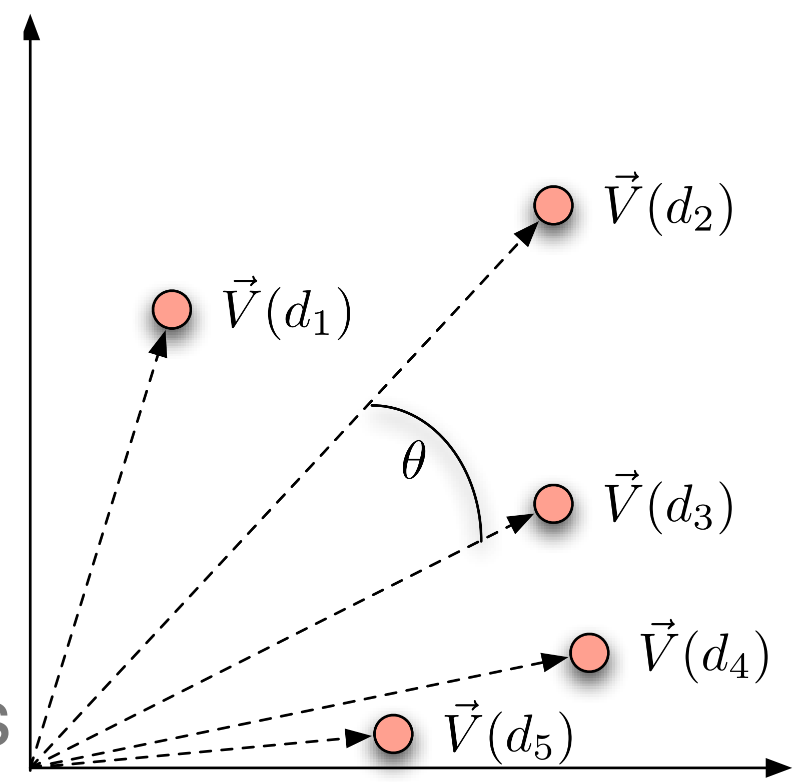
Score by angle

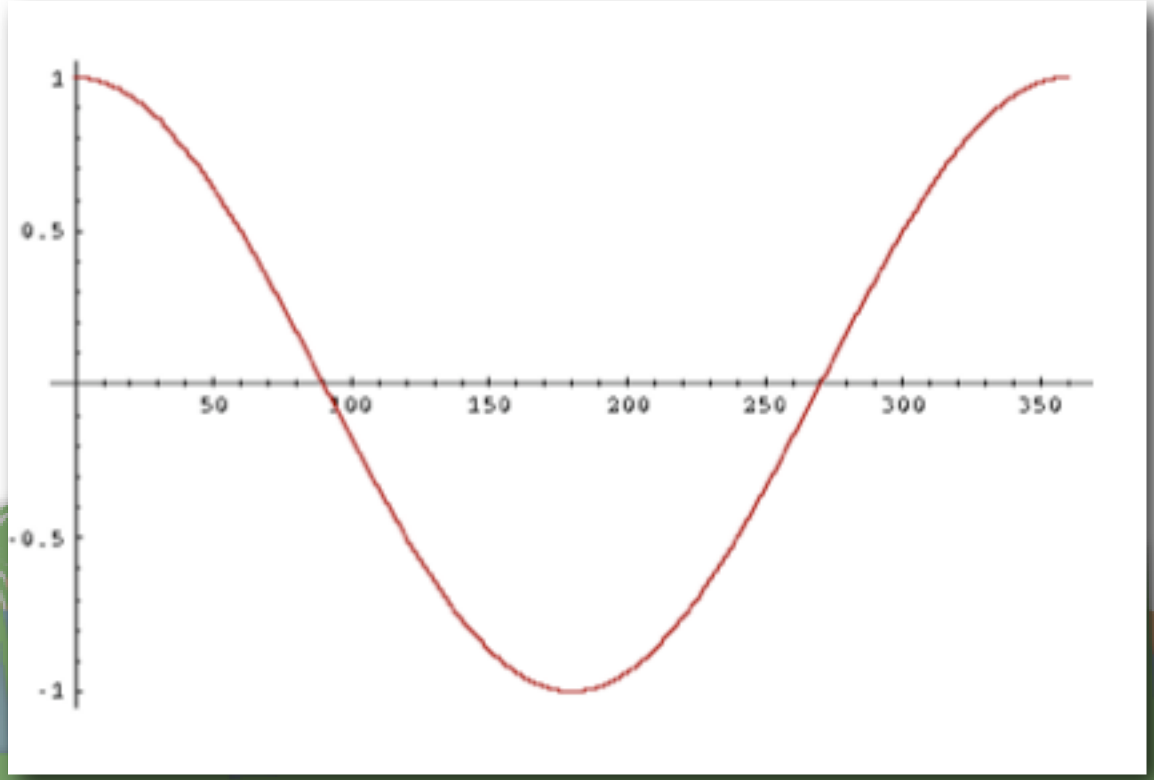
- How to answer?
  - Similarity of relative positions, or
  - difference in angle
    - Two documents are similar if the angle between them is 0.
    - As long as the ratios of the axes are the same, the documents will be scored as equal.
  - This is measured by the dot product



Score by angle

- Rather than use angle
  - use cosine of angle
  - When sorting cosine and angle are equivalent
  - Cosine is monotonically decreasing as a function of angle over (0 ... 180)





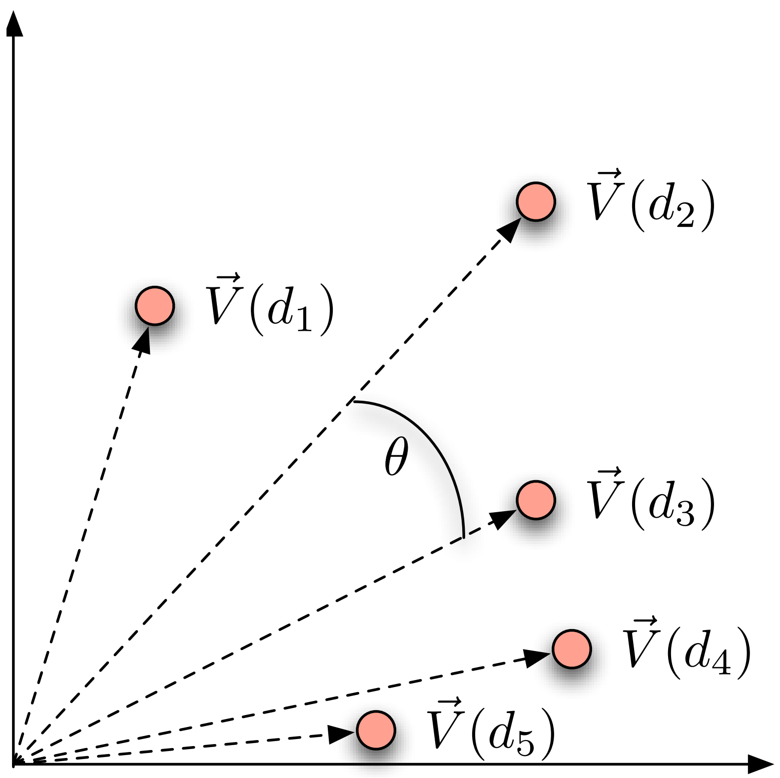
Cosine Similarity Score

- Also called cosine similarity

$$\vec{V}(d_1) \cdot \vec{V}(d_2) = \frac{|\vec{V}(d_1)||\vec{V}(d_2)|}{cos(\theta)}$$

$$cos(\theta) = \frac{\vec{V}(d_1) \cdot \vec{V}(d_2)}{|\vec{V}(d_1)||\vec{V}(d_2)|}$$

$$sim(d_1, d_2) = \frac{\vec{V}(d_1) \cdot \vec{V}(d_2)}{|\vec{V}(d_1)||\vec{V}(d_2)|}$$



Cosine Similarity Score

- Define: dot product

$$\vec{V}(d_1) \cdot \vec{V}(d_2) = \sum_{i=t_1}^{t_n} (\vec{V}(d_1)_i \vec{V}(d_2)_i)$$

	Antony and Cleopatra	*Julius Caesar*	*The Tempest*	*Hamlet*	*Othello*	*Macbeth*
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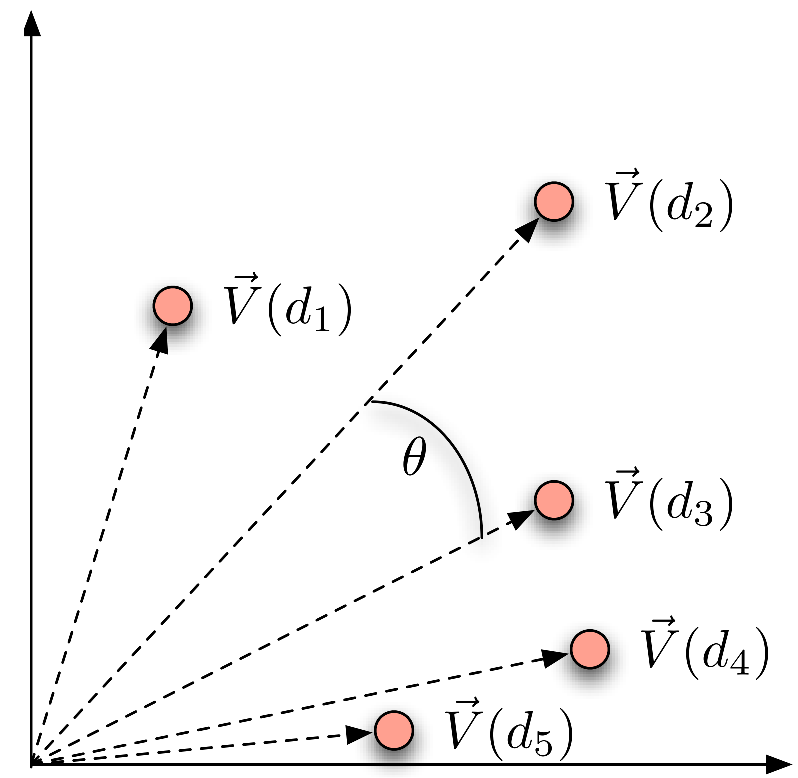

$$\begin{aligned}\vec{V}(d_1) \cdot \vec{V}(d_2) &= (13.1 * 11.4) + (3.0 * 8.3) + (2.3 * 2.3) + (0 * 11.2) + (17.7 * 0) + (0.5 * 0) + (1.2 * 0) \\ &= 179.53\end{aligned}$$

Cosine Similarity Score

- Define: Euclidean Length

$$|\vec{V}(d_1)| = \sqrt{\sum_{i=t_1}^{t_n} (\vec{V}(d_1)_i \vec{V}(d_1)_i)}$$

	Antony and Cleopatra	*Julius Caesar*	*The Tempest*	*Hamlet*	*Othello*	*Macbeth*
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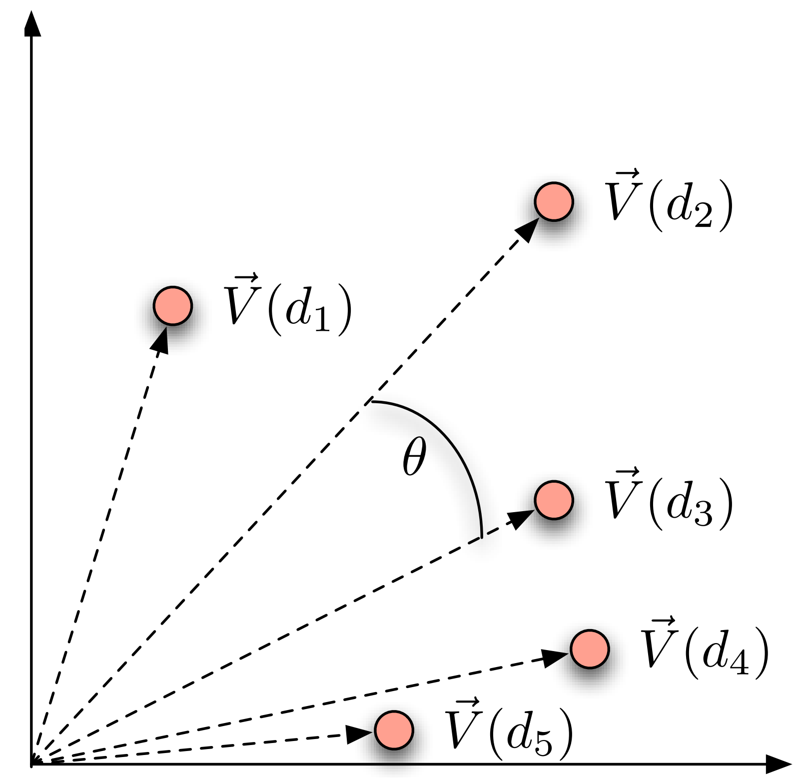

$$\begin{aligned} |\vec{V}(d_1)| &= \sqrt{(13.1 * 13.1) + (3.0 * 3.0) + (2.3 * 2.3) + (17.7 * 17.7) + (0.5 * 0.5) + (1.2 * 1.2)} \\ &= 22.38 \end{aligned}$$

Cosine Similarity Score

- Define: Euclidean Length

$$|\vec{V}(d_1)| = \sqrt{\sum_{i=t_1}^{t_n} (\vec{V}(d_1)_i \vec{V}(d_1)_i)}$$

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
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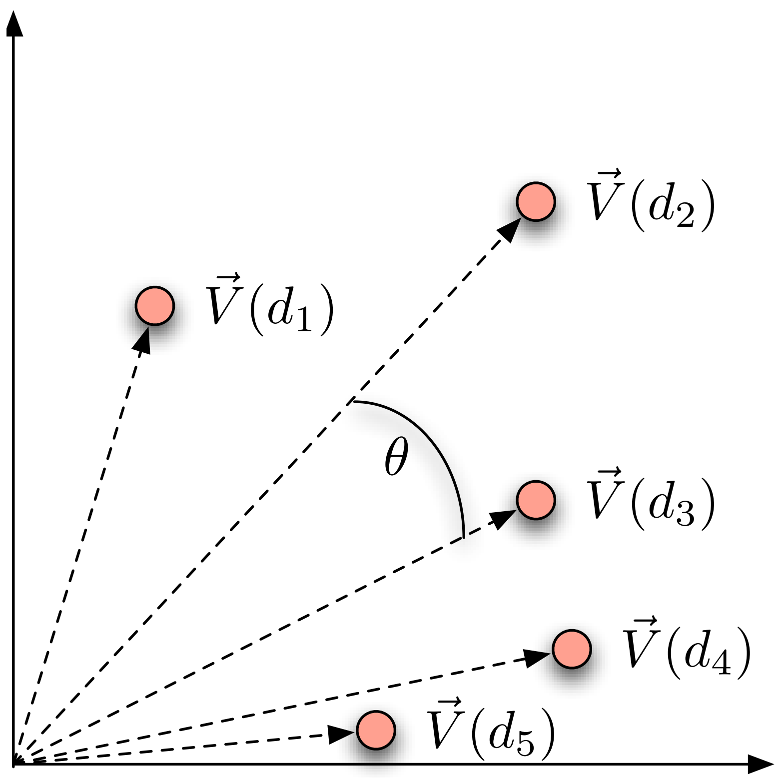
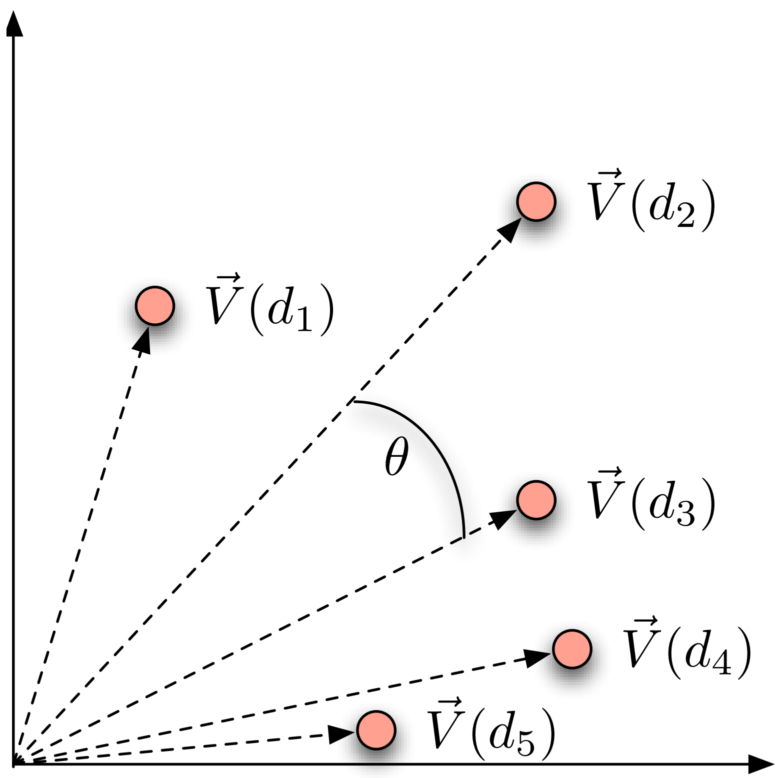


$$|\vec{V}(d_1)| = \sqrt{(11.4 * 11.4) + (8.3 * 8.3) + (2.3 * 2.3) + (11.2 * 11.2)}$$
$$= 18.15$$

Cosine Similarity Score

- Example

$$
\begin{aligned}
sim(d_1, d_2) &= \frac{\vec{V}(d_1) \cdot \vec{V}(d_2)}{|\vec{V}(d_1)||\vec{V}(d_2)|} \\
&= \frac{179.53}{22.38 * 18.15} \\
&= 0.442
\end{aligned}
$$



Big picture

- Why are we turning documents and queries into vectors
    - Getting away from Boolean retrieval
    - Developing ranked retrieval methods
    - Developing scores for ranked retrieval
    - Term weighting allows us to compute scores for document similarity
    - Vector space model is a clean mathematical model to work with

Big picture

- Cosine similarity measure
  - Gives us a symmetric score
    - if d_1 is close to d_2, d_2 is close to d_1
  - Gives us transitivity
    - if d_1 is close to d_2, and d_2 close to d_3, then
    - d_1 is also close to d_3
  - No document is closer to d_1 than itself
  - If vectors are normalized (length = 1) then
    - The similarity score is just the dot product (fast)

Exercise

- Rank the following by decreasing cosine similarity.
  - Assum tf-idf weighting:
    - Two docs that have only frequent words in common
      - (the, a , an, of)
    - Two docs that have no words in common
    - Two docs that have many rare words in common
      - (mocha, volatile, organic, shade-grown)

Queries in the vector space model

- Central idea: the query is a vector
  - We regard the query as a short document
  - We return the documents ranked by the closeness of their vectors to the query (also a vector)

$$sim(q, d_i) = \frac{\vec{V}(q) \cdot \vec{V}(d_i)}{|\vec{V}(q)||\vec{V}(d_i)|}$$

  - Note that q is very sparse!

Spamming indices

- This was invented before spam
- Consider:
  - Indexing a sensible passive document collection
  - vs.
  - Indexing an active document collection, where people, companies, bots are shaping documents to maximize scores
- Vector space scoring may not be as useful in this context.