Vector Space Scoring

Introduction to Information Retrieval
CS 221
Donald J. Patterson

Content adapted from Hinrich Schütze
http://www.informationretrieval.org
Cosine Similarity Score

- Define: Euclidean Length

\[
|\vec{V}(d_1)| = \sqrt{\sum_{i=t_1}^{t_n} (\vec{V}(d_1)_i \vec{V}(d_1)_i)}
\]

| \begin{array}{cccccc}
Antony and \\
Cleopatra & Antony & Brutus & Caesar & Calpurnia & Cleopatra \\
\hline
13.1 & 11.4 & 3.0 & 2.3 & 0.0 & 17.7 \\
13.1 & 0.0 & 11.2 & 0.0 & 0.7 & 0.0 \\
13.1 & 0.0 & 0.0 & 2.3 & 0.0 & 1.2 \\
13.1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
13.1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
13.1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
13.1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
13.1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
13.1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
13.1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
\end{array} |

\[
|\vec{V}(d_1)| = \sqrt{(13.1 \times 13.1) + (3.0 \times 3.0) + (2.3 \times 2.3) + (17.7 \times 17.7) + (0.5 \times 0.5) + (1.2 \times 1.2)}
\]

\[
= 22.38
\]
Cosine Similarity Score

- Define: Euclidean Length

\[
|\vec{V}(d_1)| = \sqrt{\sum_{i=t_1}^{t_n} (\vec{V}(d_1)_i \vec{V}(d_1)_i)}
\]

<table>
<thead>
<tr>
<th></th>
<th>Antony and Cleopatra</th>
<th>Julius Caesar</th>
<th>The Tempest</th>
<th>Hamlet</th>
<th>Othello</th>
<th>Macbeth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antony</td>
<td>13.1</td>
<td>11.4</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
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<tr>
<td>Brutus</td>
<td>3.0</td>
<td>8.3</td>
<td>0.0</td>
<td>1.0</td>
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<td>Caesar</td>
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<td>0.0</td>
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<tr>
<td>Calpurnia</td>
<td>0.0</td>
<td>11.2</td>
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<td>0.0</td>
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<tr>
<td>Cleopatra</td>
<td>17.7</td>
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<tr>
<td>mercy</td>
<td>0.5</td>
<td>0.0</td>
<td>0.7</td>
<td>0.9</td>
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<td>worse</td>
<td>1.2</td>
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<td>0.6</td>
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</tr>
</tbody>
</table>

\[
|\vec{V}(d_1)| = \sqrt{(11.4 \times 11.4) + (8.3 \times 8.3) + (2.3 \times 2.3) + (11.2 \times 11.2)}
\]

= 18.15
Vector Space Scoring

Cosine Similarity Score

- Example

\[ \text{sim}(d_1, d_2) = \frac{\vec{V}(d_1) \cdot \vec{V}(d_2)}{|\vec{V}(d_1)||\vec{V}(d_2)|} \]

\[ = \frac{179.53}{22.38 \times 18.15} \]

\[ = 0.442 \]
Exercise

• Rank the following by decreasing cosine similarity.

• Assume tf-idf weighting:
  • Two docs that have only frequent words in common
    • (the, a, an, of)
  • Two docs that have no words in common
  • Two docs that have many rare words in common
    • (mocha, volatile, organic, shade-grown)
## Vector Space Scoring

### Exercise

<table>
<thead>
<tr>
<th>tf</th>
<th>df</th>
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</thead>
<tbody>
<tr>
<td>24</td>
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<tr>
<td>10</td>
<td>14</td>
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<tr>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

\( tf = \) & \( df = \)
Exercise

c = 15;
tf = load('tf.txt','-ASCII');
df = load('df.txt','-ASCII');
tfidf = zeros(size(tf));

for i = 1:size(tf,1)
    for j = 1:size(tf,2)
        if tf(i,j) == 0
            tfidf(i,j) = (0) * log2(c/df(i));
        else
            tfidf(i,j) = (1+log2(tf(i,j))) * log2(c/df(i));
        end
    end
end
### Exercise

$$\text{tfidf} =$$

\[
\begin{array}{ccccccccc}
0.5559 & 0.5559 & 0 & 0.5559 & 0.5559 & 0.5559 & 0.5559 & 0.5559 & 0.5559 \\
0.4302 & 0.4302 & 0 & 0.4302 & 0.4302 & 0.4302 & 0.4302 & 0.4302 & 0.4302 \\
0.5559 & 0.5559 & 0 & 0.5559 & 0.5559 & 0.5559 & 0.5559 & 0.5559 & 0.5559 \\
0.4564 & 0.4564 & 0 & 0.4439 & 0.4439 & 0.4439 & 0.4439 & 0.4439 & 0.4439 \\
0.4302 & 0.4302 & 0 & 0.4302 & 0.4302 & 0.4302 & 0.4302 & 0.4302 & 0.4302 \\
0 & 0 & 3.9069 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 3.9069 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 3.9069 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2.9069 & 2.9069 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2.9069 & 2.9069 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2.9069 & 2.9069 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]
## Exercise

$$\text{tfidf} =
\begin{array}{cccccccc}
0.5559 & 0.5559 & 0 & 0.5559 & 0.5559 & 0.5559 & 0.5559 & 0.5559 \\
0.4302 & 0.4302 & 0 & 0.4302 & 0.4302 & 0.4302 & 0.4302 & 0.4302 \\
0.5559 & 0.5559 & 0 & 0.5559 & 0.5559 & 0.5559 & 0.5559 & 0.5559 \\
0.4564 & 0.4564 & 0 & 0.4439 & 0.4439 & 0.4439 & 0.4439 & 0.4439 \\
0.4302 & 0.4302 & 0 & 0.4302 & 0.4302 & 0.4302 & 0.4302 & 0.4302 \\
0 & 0 & 0 & 0 & 3.9069 & 0 & 0 & 0 \\
0 & 0 & 0 & 3.9069 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 3.9069 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 3.9069 & 0 & 0 \\
0 & 0 & 2.9069 & 2.9069 & 0 & 0 & 0 & 0 \\
0 & 0 & 2.9069 & 2.9069 & 0 & 0 & 0 & 0 \\
0 & 0 & 2.9069 & 2.9069 & 0 & 0 & 0 & 0
\end{array}$$
Exercise

tfidf =

```
0.5559  0.5559  0.5559  0.5559  0.5559  0.5559
0.4302  0.4302  0.4302  0.4302  0.4302  0.4302
0.5559  0.5559  0.5559  0.5559  0.5559  0.5559
0.4564  0.4564  0.4564  0.4564  0.4564  0.4564
0.4302  0.4302  0.4302  0.4302  0.4302  0.4302
     0     0     0     0     0     0
     0     0     3.9069  3.9069  3.9069  3.9069
     0     0     0     0     3.9069  3.9069
     0     0     0     0     0     0
     0     0     2.9069  2.9069  2.9069  2.9069
     0     0     0     0     0     0
     0     0     2.9069  2.9069  2.9069  2.9069
     0     0     0     0     0     0
     0     0     2.9069  2.9069  2.9069  2.9069
     0     0     0     0     0     0
```

```
>> num = tfidf(:,1)'*tfidf(:,2)
num =
     1.1964
```
Exercise

```
tfidf =
```

```
  0.5559  0.5559
  0.4302  0.4302
  0.5559  0.5559
  0.4564  0.4564
  0.4302  0.4302
    0    0
   0    0
   0    0
   0    0
   0    0
   0    0
   0    0
   0    0
  3.9069  3.9069
```

```
>> num = tfidf(:,1)'*tfidf(:,2)
num =
    1.1964
```

```
>> denom = sqrt(tfidf(:,1)'*tfidf(:,1)).*sqrt(tfidf(:,2)'*tfidf(:,2))
denom =
    1.1964
```

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Exercise

\[
\text{tfidf} =
\begin{bmatrix}
0.5559 & 0.5559 \\
0.4302 & 0.4302 \\
0.5559 & 0.5559 \\
0.4564 & 0.4564 \\
0.4302 & 0.4302
\end{bmatrix}
\]

```
>> num = tfidf(:,1)'*tfidf(:,2)
num =
    1.1964
```

```
>> denom = sqrt(tfidf(:,1)'*tfidf(:,1)).*sqrt(tfidf(:,2)'*tfidf(:,2))
denom =
    1.1964
```

```
>> score = num/denom
score =
    1.0000
```
## Vector Space Scoring

### Exercise

\[
\text{tfidf} =
\begin{pmatrix}
0.5559 & 0.5559 & 0 & 0.5559 & 0.5559 & 0.5559 & 0.5559 & 0.5559 \\
0.4302 & 0.4302 & 0 & 0.4302 & 0.4302 & 0.4302 & 0.4302 & 0.4302 \\
0.5559 & 0.5559 & 0 & 0.5559 & 0.5559 & 0.5559 & 0.5559 & 0.5559 \\
0.4564 & 0.4564 & 0 & 0.4439 & 0.4439 & 0.4439 & 0.4439 & 0.4439 \\
0.4302 & 0.4302 & 0 & 0.4302 & 0.4302 & 0.4302 & 0.4302 & 0.4302 \\
0 & 0 & 0 & 0 & 3.9069 & 0 & 0 & 0 \\
0 & 0 & 3.9069 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 3.9069 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 3.9069 & 0 & 0 \\
0 & 0 & 2.9069 & 2.9069 & 0 & 0 & 0 & 0 \\
0 & 0 & 2.9069 & 2.9069 & 0 & 0 & 0 & 0 \\
0 & 0 & 2.9069 & 2.9069 & 0 & 0 & 0 & 0
\end{pmatrix}
\]
### Exercise

**Vector Space Scoring**

```
import numpy as np

tfidf =
```

```
0.5559 0.5559 0.5559 0.5559 0.5559
0.4302 0.4302 0.4302 0.4302 0.4302
0.5559 0.5559 0.5559 0.5559 0.5559
0.4564 0.4564 0.4564 0.4564 0.4564
0.4302 0.4302 0.4302 0.4302 0.4302
0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 3.9069 0.0000 0.0000
0.0000 0.0000 3.9069 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 2.9069 0.0000 0.0000
0.0000 0.0000 2.9069 0.0000 0.0000
0.0000 0.0000 2.9069 0.0000 0.0000

```nump = tfidf(:,2)'*tfidf(:,3)```

```
num =
```

```
2.9
2.9
2.9
```

**More of the same**
## Exercise

```
tfidf =
```

<table>
<thead>
<tr>
<th></th>
<th>0.5559</th>
<th>0.5559</th>
<th>0</th>
<th>0.5559</th>
<th>0.5559</th>
<th>0.5559</th>
<th>0.5559</th>
<th>0.5559</th>
<th>0.5559</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.4302</td>
<td>0.4302</td>
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<td>0.4302</td>
<td>0.4302</td>
<td>0.4302</td>
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<td>0.5559</td>
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<tr>
<td>0</td>
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</tr>
</tbody>
</table>

```
>> num = tfidf(:,2)'*tfidf(:,3)
num =
2.9
2.9
0
2.9069
```

```
>> denom = sqrt(tfidf(:,2)'*tfidf(:,2)).*sqrt(tfidf(:,3)'*tfidf(:,3))
denom =
8.1765
```
Exercise

Vector Space Scoring

tfidf =

\[
\begin{bmatrix}
0.5559 & 0.5559 & 0 & 0.5559 & 0.5559 & 0.5559 & 0.5559 & 0.5559 \\
0.4302 & 0.4302 & 0 & 0.4302 & 0.4302 & 0.4302 & 0.4302 & 0.4302 \\
0.5559 & 0.5559 & 0 & 0.5559 & 0.5559 & 0.5559 & 0.5559 & 0.5559 \\
0.4564 & 0.4564 & 0 & 0.4439 & 0.4439 & 0.4439 & 0.4439 & 0.4439 \\
0.4302 & 0.4302 & 0 & 0.4302 & 0.4302 & 0.4302 & 0.4302 & 0.4302 \\
0 & 0 & 3.9069 & 0 & 0 & 3.9069 & 0 & 0 \\
0 & 0 & 3.9069 & 0 & 0 & 3.9069 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2.9069 & 2.9 & 2.9 & 2.9 & 2.9 & 2.9 \\
0 & 0 & 2.9069 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2.9069 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2.9069 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\gg \text{num} = \text{tfidf(:,2)}' \times \text{tfidf(:,3)} \\
\text{num} = \\
\begin{bmatrix}
2.9 \\
2.9 \\
2.9 \\
2.9 \\
\end{bmatrix}
\]

\[
\gg \text{denom} = \sqrt{\text{tfidf(:,2)}' \times \text{tfidf(:,2)}} \times \sqrt{\text{tfidf(:,3)}' \times \text{tfidf(:,3)}} \\
\text{denom} = 8.1765
\]

\[
\gg \text{score} = \text{num}/\text{denom} \\
\text{score} = 0
\]
### Exercise

**tfidf =**

<p>| | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</tbody>
</table>

More of the same
### Exercise

**tfidf** =

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5559</td>
<td>0.5559</td>
<td>0</td>
<td>0.5559</td>
<td>0.5559</td>
</tr>
<tr>
<td>0.4302</td>
<td>0.4302</td>
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<td>0.4302</td>
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<tr>
<td>0.5559</td>
<td>0.5559</td>
<td>0</td>
<td>0.5559</td>
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</tr>
<tr>
<td>0.4564</td>
<td>0.4564</td>
<td>0</td>
<td>0.4439</td>
<td>0.4439</td>
</tr>
<tr>
<td>0.4302</td>
<td>0.4302</td>
<td>0</td>
<td>0.4302</td>
<td>0.4302</td>
</tr>
</tbody>
</table>

More of the same

```matlab
>> num = tfidf(:,3)'*tfidf(:,4)
```

```
num =

25.3500
```

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## Exercise

$$\text{tfidf} =$$

<p>| | | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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$$\text{>> num} = \text{tfidf}(:,3)'*\text{tfidf}(:,4)$$

\[
\text{num} = \\
\begin{array}{cccc}
3.9069 & 0 & 0 & 0 \\
3.9069 & 0 & 0 & 0 \\
3.9069 & 0 & 0 & 0 \\
2.9069 & 2.9069 & 0 & 0 \\
2.9069 & 2.9069 & 0 & 0 \\
2.9069 & 2.9069 & 0 & 0 \\
\end{array}
\]

\[
25.3500 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\]

$$\text{>> denom} = \sqrt{\text{tfidf}(:,3)'*\text{tfidf}(:,3)}.*\sqrt{\text{tfidf}(:,4)'*\text{tfidf}(:,4)}$$

\[
\text{denom} = \\
38.5062 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\]
## Vector Space Scoring

### Exercise

```matlab
tfidf =

0.5559 0.5559
0.4302 0.4302
0.5559 0.5559
0.4564 0.4564
0.4302 0.4302
0 0
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3.9069
3.9069
3.9069
3.9069
0 0
0 0
0 0
0 0
0 0
0 0
0 0
2.9069 2.9069
2.9069 2.9069
2.9069 2.9069
```

```matlab
>> num = tfidf(:,3)'*tfidf(:,4)
num =
25.3500
```

```matlab
>> denom = sqrt(tfidf(:,3)'*tfidf(:,3)).*sqrt(tfidf(:,4)'*tfidf(:,4))
denom =
38.5062
```

```matlab
>> score = num/denom
score =
0.6583
```
Vector Space Scoring

Exercise

• Rank the following by decreasing cosine similarity.

• Assume tf-idf weighting:
  • Two docs that have only frequent words in common
    • (the, a, an, of)
  • Two docs that have no words in common
  • Two docs that have many rare words in common
    • (mocha, volatile, organic, shade-grown)
Exercise

- Rank the following by decreasing cosine similarity.
- Assume tf-idf weighting:
  - Two docs that have only frequent words in
    - (the, a, an, of)
  - Two docs that have no words in common
  - Two docs that have many rare words in common
    - (mocha, volatile, organic, shade-grown)

>> score = num/denom
score = 1.0000
Exercise

- Rank the following by decreasing cosine similarity.
- Assume tf-idf weighting:
  - Two docs that have only frequent words in common
    - (the, a, an, of)
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Exercise

- Rank the following by decreasing cosine similarity.
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    - Two docs that have no words in common
    - Two docs that have many rare words in common
      - (mocha, volatile, organic, shade-grown)
Spamming indices

• This was invented before spam

• Consider:
  • Indexing a sensible passive document collection
  • vs.
  • Indexing an active document collection, where people, companies, bots are shaping documents to maximize scores

• Vector space scoring may not be as useful in this context.
Interaction: vectors and phrases

- Scoring phrases doesn’t naturally fit into the vector space world:
  - How do we get beyond the “bag of words”?
  - “dark roast” and “pot roast”
  - There is no information on “dark roast” as a phrase in our indices.
- Biword index can treat some phrases as terms
  - postings for phrases
  - document wide statistics for phrases
Vector Space Scoring

Interaction: vectors and phrases

- Theoretical problem:
  - Axes of our term space are now correlated
  - There is a lot of shared information in “light roast” and “dark roast” rows of our index

- End-user problem:
  - A user doesn’t know which phrases are indexed and can’t effectively discriminate results.
Multiple queries for phrases and vectors

- Query: “rising interest rates”
- Iterative refinement:
  - Run the phrase query vector with 3 words as a term.
  - If not enough results, run 2-phrase queries and fold into results: “rising interest” “interest rates”
  - If still not enough results run query with three words as separate terms.
Vectors and Boolean queries

- Ranked queries and Boolean queries don’t work very well together
  - In term space
    - ranked queries select based on sector containment - cosine similarity
    - boolean queries select based on rectangle unions and intersections
Vectors and wild cards
Vectors and wild cards

- How could we work with the query, "quick* print*"?
Vectors and wild cards

- How could we work with the query, “quick* print*”?
- Can we view this as a bag of words?
Vectors and wild cards

- How could we work with the query, “quick* print*”?  
- Can we view this as a bag of words?  
- What about expanding each wild-card into the matching set of dictionary terms?
Vectors and wild cards

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  • Can we view this as a bag of words? 
  • What about expanding each wild-card into the matching set of dictionary terms?
• Danger: Unlike the boolean case, we now have tfs and idfs to deal with
Vectors and wild cards

• How could we work with the query, “quick* print*”?
• Can we view this as a bag of words?
• What about expanding each wild-card into the matching set of dictionary terms?
• Danger: Unlike the boolean case, we now have tfs and idfs to deal with
• Overall, not a great idea
Vector Space Scoring

Vectors and other operators

- Vector space queries are good for no-syntact, bag-of-words queries
- Nice mathematical formalism
- Clear metaphor for similar document queries
- Doesn’t work well with Boolean, wild-card or positional query operators
- But ...
Query language vs. Scoring

• Interfaces to the rescue
  • Free text queries are often separated from operator query language
  • Default is free text query
  • Advanced query operators are available in “advanced query” section of interface
  • Or embedded in free text query with special syntax
    • aka -term -“terma termb”
Alternatives to tf-idf

- Sublinear tf scaling
- 20 occurrences of “mole” does not indicate 20 times the relevance
- This motivated the WTF score.

\[
\text{WTF}(t, d) = \\
1 \quad \text{if } t f_{t,d} = 0 \\
2 \quad \text{then return}(0) \\
3 \quad \text{else return}(1 + \log(t f_{t,d}))
\]

- There are other variants for reducing the impact of repeated terms
TF Normalization

- Normalize tf weights by maximum tf in that document

\[ ntf_{t,d} = \alpha + (1 - \alpha) \frac{tf_{t,d}}{tf_{max}(d)} \]

- \( \alpha \) is a smoothing term from \((0 - 1.0) \sim 0.4\) in practice
- This addresses a length bias.
- Take one document, repeat it, WTF goes up
TF Normalization

• Normalize tf weights by maximum tf in that document

\[ ntf_{t,d} = \alpha + (1 - \alpha) \frac{tf_{t,d}}{tf_{max}(d)} \]

• a change in the **stop word list** can change weights drastically - hard to tune

• still based on bag of words model

• one outlier word, repeated many times might throw off the algorithmic understanding of the content
## Vector Space Scoring: Alternatives to tf-idf

### Laundry List

<table>
<thead>
<tr>
<th>Term Frequency</th>
<th>Document Frequency</th>
<th>Normalization</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n)atural</td>
<td>$tf_{t,d}$</td>
<td>$(n)o$</td>
</tr>
<tr>
<td>(l)ogarithm</td>
<td>$1 + \log(tf_{t,d})$</td>
<td>$(t)idf$</td>
</tr>
<tr>
<td>(a)ugmented</td>
<td>$\alpha + (1 - \alpha) \frac{tf_{t,d}}{tf_{max}(d)}$</td>
<td>$(p)robidf$</td>
</tr>
<tr>
<td>(b)oolean</td>
<td>$t_{f,t,d} &gt; 0? 1 : 0$</td>
<td>$(u)pivoted$</td>
</tr>
<tr>
<td>(L)ogaverage</td>
<td>$\frac{1}{1+\log(average(tf_{t,d}))}$</td>
<td>$(b)yte$</td>
</tr>
</tbody>
</table>

- SMART system of describing your IR vector algorithm
- $ddd.qqq$ ($ddd =$ document weighting) ($qqq =$ query weighting)
- first is term weighting, second is document, then normalization
- Inc.ltc is what?
Efficient Cosine Ranking

- Find the k docs in the corpus “nearest” to the query
  - the k largest query-doc cosines

Efficient ranking means:

- Computing a single cosine efficiently
- Computing the k largest cosine values efficiently

Can we do this without computing all n cosines?

- n = number of documents in corpus
Efficient Cosine Ranking

- Computing a single cosine
- Use inverted index
- At query time use an array of accumulators $A_j$ to accumulate component-wise sum
- Accumulate scores as postings lists are being processed (numerator of similarity score)

$$A_j = \sum_t (w_{q,t}w_{d,t})$$
Efficient Cosine Ranking

- For the web
  - an array of accumulators in memory is infeasible
  - so only create accumulators for docs that occur in postings list
    - dynamically create accumulators
  - put the tf_d scores in the postings lists themselves
  - limit docs to non-zero cosines on rare words
    - or non-zero cosines on all words
  - reduces number of accumulators
Vector Space Scoring

Efficient Cosine Ranking

\textsc{CosineScore}(q)

1. \textbf{Initialize}(Scores[d \in D])
2. \textbf{Initialize}(Magnitude[d \in D])
3. \textbf{for each term}(t \in q)
   
   do \hspace{1em} p \leftarrow \text{FetchPostingsList}(t)

4. \hspace{1em} df_t \leftarrow \text{GetCorpusWideStats}(p)
5. \hspace{1em} \alpha_{t,q} \leftarrow \text{WeightInQuery}(t, q, df_t)
6. \hspace{1em} \textbf{for each} \{d, tf_t, d\} \in p
   
   do \hspace{1em} Scores[d] + = \alpha_{t,q} \cdot \text{WeightInDocument}(t, q, df_t)

7. \hspace{1em} \textbf{for} d \in \text{Scores}
   
   do \hspace{1em} \text{Normalize}(Scores[d], Magnitude[d])

8. \hspace{1em} \textbf{return} \hspace{1em} \text{top} K \in \text{Scores}
Vector Space Scoring

Use heap for selecting the top K Scores

- Binary tree in which each node’s value > the values of children
- Takes 2N operations to construct
  - then each of k “winners” read off in 2logn steps
- For n =1M, k=100 this is about 10% of the cost of sorting