USER'S GUIDE of COPL_QP
Computational Optimization Program Library: Convex Quadratic Programming

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1 Introduction

This software package uses an interior-point algorithm to solve convex quadratic programs:

$$
\text{(QP)} \qquad \begin{array}{ll} \text{Min or Max} & c^T x + (1/2) x^T Q x \\ \text{Subject to} & a_i x \ (\leq, =, \geq) \ b_i, \ i = 1, \ldots, m \\ & l_j \leq x_j \leq u_j, \ j = 1, \ldots, n \end{array}
$$

where

- $c \in R^n$: the objective function vector,
- $Q \in R^{n \times n}$: the (symmetric) objective function matrix,
- $A \in R^{m \times n}$: the constraint matrix, and its ith row is denoted by a_i,
- $b \in R^m$: the right-hand-side vector,
- $l_j, u_j \in R$: the lower and upper bounds for jth decision variable x_j.

This package contains the following files:

1. Makefile: used to compile source codes and link them into the executable code.
2. Source files for the executable code, say `opt`, to solve QP problems:

```
QPcnt.h   QPdef.h   QPfun.h
qpalloc.c  qpdata.c   qpmatrx.c  qpprepr.c  qpproc.c
qpsolv.c   qpvect.c   qpchol.c   qpmain.c   qporder.c
qpprint.c  qpshut.c   qpsymbo.c  qpxlist.c
```

3. COPLQP.spc, the default specification file.
4. Example data files:

```
afiro.qps    example.qps
```

2 Setup

The executable codes named **opt** can be created by:

1. Download all the files into a directory of your choice,
2. Compile the set of source codes and link them together to form the executable program **opt**.

3 Usage

To solve a problem using **opt**, type any of the following command lines

```
opt [-f {NAME1}] [-o {NAME2}] [-s {NAME3}]
opt [-f:{NAME1}] [-o:{NAME2}] [-s:{NAME3}]
opt [-f{NAME1}]  [-o{NAME2}]  [-s{NAME3}]
```

The contents in '`[ ]`' are optional, '`/`' means 'or', and the contents in "{}" are required if the corresponding option appears in the command line. Here

-f: Specify the input data file name. If this option is omitted, the system will prompt the user for a file name.

-o: Specify the output file name. The output information will be saved in a file named "`coplfile.out`" if this option is omitted.

-s: Specify the SPC (specification) file name. The system will look COPLQP.spc as the default specification file name. The system will use the default settings if this option is omitted and file COPLQP.spc can not be found.

4 Data Input: The QPS file

An QPS file is required for each QP problem. It is used to name variables and constraints, and to input numerical data. The QPS file has the identical form of the MPS file for linear programming, except one more section named "QUADS" to input the symmetric objective matrix Q. The QPS file has a *fixed* format, i.e. each item of data must appear in specific columns.

The QPS file is divided into the following sections:

```
NAME
ROWS
  .
COLUMNS
  .
RHS
  .
RANGES
  .
BOUNDS
  .
QUADS
  .
ENDATA
```

Here we explain the QUADS section since the other sections are identical to those in the MPS file, see "lpguide" for the MPS format in COPL_LP. Also note that all the variables must be defined in section COLUMNS as long as they appear in the objective hessian matrix, even though some of them do not appear in the constraint matrix.

Section QUADS is used to describe the symmetric matrix Q in the objective function. More specificly, in QUADS section only the nonzero elements on diagonal and upper off-diagonal positions of Q need to be specified.

In QUADS section, *Key* is blank, *Name0* is the column name, and *Name1*, *Value1* give a column name and value for some nonzero elements in that column (or row) of Q. If there is another column name and value for the same column, they may appear as *Name2*, *Value2* on the same card, or they may be on the next card.

For example, suppose we have

$$Q = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

and we have defined the column names as X01, X02, X03, and X04 in order in COLUMNS section, then the QUADS section could be defined as follows.

```
1   5      12 15      22 25          36  40      47 50          61 (fields)
+---+------+--+------+--+----------+---+------+--+----------+
QUADS
    x01        x01            2.0        x02           -1.0
    x02        x02            2.0
    x02        x03           -1.0
    x03        x03            2.0
    x03        x04           -1.0
    x04        x04            2.0
```

To write a correct QPS file, one needs to note that,

1) every column name appearing in QUADS section must be defined in COLUMNS section even though the column is null in the constraint matrix. For example, we have problem

$$\begin{array}{ll} \text{minimize} & 2x_1^2 + 4x_2^2 - 2x_1x_2 + 2 * x_3^2 - 4x_2 + x_3 \\ \text{subject to} & x_2 + x_3 = 2, \\ & x_1, x_2, x_3 \geq 0, \end{array}$$

and the columns 1, 2, and 3 are given names c01, c02, and co3, respectively, then COLUMNS section should defined like the following.

```
1   5      12 15      22 25          36  40      47 50          61 (fields)
+---+------+--+------+--+----------+---+------+--+----------+
CLOUMNS
```

```
    c01       cost          0.0
    c02       r01           1.0         cost          -4.0
    c03       r01           1.0         cost           1.0
```

In this example, `c01` is the null column in the constraint matrix.

2) The column name appearing between column 5 and column 12 in QUADS section must be prior to the name appearing the same line between column 15 and column 22 or between column 40 and column 47 in QUADS section when they are defined in COLUMNS section. For the above example, the following QUADS section is not correct.

```
1   5      12 15     22 25         36  40     47 50          61 (fields)
+---+------+--+------+--+----------+---+------+--+----------+
QUADS
    c01       c01           2.0
    c02       c01          -1.0         c02            4.0
    c03       c03           2.0
```

3) Since the "blank space" is also a letter, the following names are treated as different names.

```
1   5      12 15     22 25         36  40     47 50          61 (fields)
+---+------+--+------+--+----------+---+------+--+----------+
COLUMNS
    c01       cost           0.0
     c01      r01            1.0         cost          -4.0
      c01     r01            1.0         cost           1.0
```

This definition is also a correct COLUMNS section for the above problem.

5 SPC file

The specification file (SPC file) is the same as that for linear programming. The following is the complete default SPC file for COPL_QP, named `COPLQP.spc`.

```
bound_set_name            : NULL
cache_memory_size         : 256
dual_feasible_tolerance   : 1.0e-6

* This item is for the initial point
* generation, which means each component
* value of the initial point should not
* be less than the setting value (1000)
initial_point_threshold   : 1000.0
```

```
iteration_limit            : 200
lower_bound_default        : 0.0
number_of_columns          : 100000
objective_set_name         : NULL
primal_dual_gap_tolerance  : 1.0e-6
primal_feasible_tolerance  : 1.0e-6
problem_type               : min
range_set_name             : NULL
rignt_hand_side_set_name   : NULL
step_factor                : 0.95
upper_bound_default        : 1.0e+40
```

6 Solution File

COPL_QP outputs the solution in two sections in the file named `coplfile.sol`:
ROWS Section

This section contains each constraint's *type* (equality or inequality), *lower* limit, *active* value at the solution, *upper* limit, and *dual value* for its (dual) Lagrangian variable.

COLUMN Section

This section contains each (primal) variable's *type* (bounded or free), *lower* limit, *active* value at the solution, *upper* limit, and *dual slack* value for its (dual) Lagrangian slack variable.

Here is an example problem which is described in the QPS file, example.qps.

$$
\begin{array}{ll}
\text{minimize} & 2x_1^2 - 2x_1x_2 + 2x_2^2 - 7x_1 - 12x_2, \\
\text{subject to} & 9x_1 + 4x_2 + x_3 = 361, \\
& 4x_1 + 5x_2 + x_4 = 202, \\
& 3x_1 + 10x_2 + x_5 = 303, \\
& -\infty < x_1, x_2 < +\infty, \\
& x_3 \geq 1, x_4 \geq 2, x_5 \geq 3, \\
& x_3, x_4 \leq 1000.
\end{array}
$$

Obviously, the lower bounded columns are $x_l = (x_3, x_4, x_5)^T$, the upper bounded columns are $x_u = (x_3, x_4)^T$, and the free variables are $x_f = (x_1, x_2)^T$. The solution file, `coplfile.sol`, is

```
COPL_QP: Solution File
======================

ROWS Section
------------
order name     type  lower         active        upper         dual value
1     LEC01     E   +3.610000E+02 +3.610000E+02 +3.610000E+02 +2.063981E-08
2     LEC02     E   +2.020000E+02 +2.020000E+02 +2.020000E+02 +4.725087E-09
3     LEC03     E   +3.030000E+02 +3.030000E+02 +3.030000E+02 +9.272339E-09

COLUMNS Section
---------------
order name     type  lower         active        upper         dual slack
1     X01       F   -infinity     +8.666667E+00 +infinity     +9.971661E-08
2     X02       F   -infinity     +1.033333E+01 +infinity     +5.733959E-08
3     X03       B   +1.000000E+00 +2.416667E+02 +1.000000E+03 -2.063981E-08
4     X04       B   +2.000000E+00 +1.156667E+02 +1.000000E+03 -4.725087E-09
5     X05       L   +3.000000E+00 +1.736667E+02 +infinity     -9.272339E-09

----------------------------------- EOF -----------------------------------
```

The QPS data file for this problem is

```
NAME          EXAMPLEa
ROWS
 N  COST
 E  LEC01
 E  LEC02
 E  LEC03
COLUMNS
    X01       LEC01             9.   LEC02             4.
    X01       LEC03             3.   COST             -7.
    X02       LEC01             4.   LEC02             5.
    X02       LEC03            10.   COST            -12.
    X03       LEC01             1.
    X04       LEC02             1.
    X05       LEC03             1.
RHS
    B         LEC01           361.   LEC02             202.
    B         LEC03           303.
BOUNDS
 FR BD        X01
 FR BD        X02
 UP BD        X03            1000.
 UP BD        X04            1000.
 LO BD        X03               1.
 LO BD        X04               2.
 LO BD        X05               3.
QUADS
    X01       X01               2.
    X01       X02              -1.
    X02       X02               2.
ENDATA
```