CS 163 & CS 265: Graph Algorithms

Week 0: Introduction

Lecture 1: Course overview; web search engines and pagerank algorithm

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Course overview

Who is running the course?

Instructor

David Eppstein
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Teaching assistants

Daniel Frishberg, Nitya Raju, and Evrim Ozel

Online resources

Course web site:

`https://www.ics.uci.edu/~eppstein/163/`

Online course discussion forum: Ed Discussion, via Canvas

Turn in homework and return graded homeworks: Gradescope

Confidential questions about your performance: Email us!

Course material

Lecture notes will be linked on course web site before each lecture

Weekly homeworks

Do them on your own

Midterm and final exam

In person!

But what about a textbook?

There are many graph theory textbooks but not so many on graph algorithms, and the ones I know about are at too low a level

They don't cover enough of the topics I want to cover

Instead, the course web page has links to online readings, mostly from Wikipedia

Their quality is variable but they're all we have

Expectations

What do I expect you to know already?

- Undergraduate-level algorithm design and analysis (as covered in CS 161): recursion, divide and conquer, dynamic programming, O-notation
- Some previous exposure to basic graph algorithms including depth-first search, topological sorting, and Dijkstra's algorithm

  (we will go over them again, but as review)

Web search

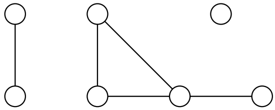
Graphs

A graph is just a set of objects related to each other in pairs

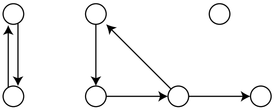
One object is called a vertex; more than one are vertices

(Do not ever call one of them a "vertice"; that is not a word.)

Pairs are called edges and their two vertices are endpoints



Undirected graph:
The endpoints are unordered



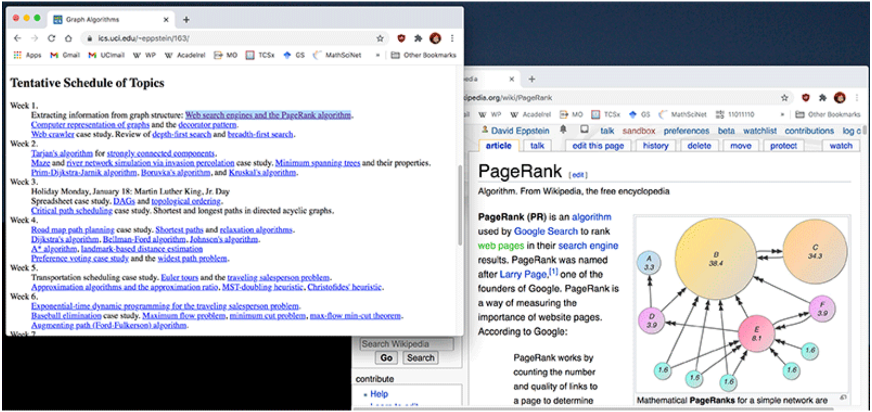
Directed graph:
Each edge goes from one endpoint to another

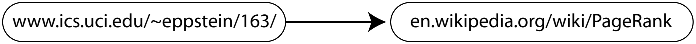
We will see many real-world examples and applications

The web graph

Vertices = all web pages in the world-wide web

Directed edges = links from one web page to another





Web search engines

You type words or phrases

Database server looks up matching web pages

Shows them to you in some order

Examples: Google, Bing, DuckDuckGo, . . .

A big change in the mid-1990s

Previous search engines:

Order search results by text (ignoring the graph structure)
Pages with many matching words shown first

Google (1998):

Order by number of incoming links (graph structure)

Intuition: if many other pages link to it, it's probably good

Refined version: Give more weight to links that come from other pages with many incoming links

This worked much better!

How Google worked circa 1998

(How they work today: known only to Google insiders)

When you make a query on Google's servers:

- Use text data structures (beyond the scope of this class) to find a set of matching web pages (limited to 500 results)
- Show these pages to you in sorted order by pagerank

What is pagerank?

- A number associated with each web page
- Bigger number = earlier in search results
- Computed by Google using only the link structure (does not depend on text of page)

Pagerank, definition 1 (mathy version)

$$\text{pagerank}(x) = 0.05 \frac{1}{\#\text{ vertices}} + 0.95 \sum_{\text{edge } y \to x} \frac{\text{pagerank}(y)}{\#\text{ edges out of } y}$$

Gives large system of linear equations that can be solved to compute pageranks

When x has many incoming links from pages with high pageranks, its own pagerank will be high

But why this equation? What does it mean?

Pagerank, definition 2 (intuitive version)

"Lazy web surfer": model of a person looking at web pages

- Start at a random web page
- Repeatedly go to a new page:
  - Probability 0.95:, choose a random outgoing link
  - Probability 0.05: get bored and start at a new random page

At each step i, each web page has some probability $P_i[x]$ of being the one the lazy web surfer is looking at

Initially, $P_0[x] = \frac{1}{\#\text{ vertices}}$

After enough steps, P_i will get very close to a "stable probability distribution" $P_i[x] \to P_\infty[x]$

Pagerank is just this limiting probability: $\text{pagerank}(x) = P_\infty[x]$

How to compute pagerank? Option 1

We have a large set of linear equations

$$\text{pagerank}(x) = 0.05\frac{1}{\#\text{ vertices}} + 0.95\sum_{\text{edge } y \to x} \frac{\text{pagerank}(y)}{\#\text{ edges out of } y}$$

Use Gaussian elimination to solve them

But that takes time $O(n^3)$, far too slow when $n =$ billions

How to compute pagerank? Option 2

Simulate the lazy web surfer

Estimate pagerank as the number of times
the simulated surfer visits each page

But to get an accurate estimate we need to run the simulation
long enough to get many visits to each page
(theoretically at least proportional to $n \log n$ steps)

Still too slow

How to compute pagerank? Option 3

Compute probability $P_i[x]$ for ith step of lazy surfer, $i = 0 \ldots 10$

Estimate pagerank $\approx P_{10}$ accurate enough for application

The equations are almost the same!

$$P_0[x] = \frac{1}{\#\ \text{vertices}}$$

and, for $i > 0$,

$$P_i[x] = 0.05 \frac{1}{\#\ \text{vertices}} + 0.95 \sum_{\text{edge } y \to x} \frac{P_{i-1}[y]}{\#\ \text{edges out of } y}$$

Linear time if we can loop over all edges $y \to x$ quickly

How do we do this? Next time...

The morals of the story

Throwing away other information about the vertices of a graph and using only their link structure can still provide meaningful information about the graph

By doing this, Google produced a much better search engine than previous competitors and dominated the search engine market

Getting it to work required an efficient algorithm for pagerank (for problem sizes so big they don't fit into a single computer)