# CS 163 & CS 265: Graph Algorithms

## Week 1: Basics

## Lecture 1a: Course overview Web search engines and pagerank algorithm

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# Course overview

# Who is running the course?

### Instructor

David Eppstein
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### Teaching assistants

Arushi Arora, Alvin Chiu, and Ryuto Kitagawa

# Online resources

Course web site:
`https://www.ics.uci.edu/~eppstein/163/`
Online course discussion forum: Ed Discussion, via Canvas

Return graded exams: Gradescope

Confidential questions about your performance: Email us!

# Course material

Lecture notes will be linked on course web site before each lecture

## Weekly problem sets

Not graded!

We will provide answers the following week

Strongly recommended as preparation for exams

## Midterms and final exam

In person!

Three exams in total, all equally weighted (final is not comprehensive)

# But what about a textbook?

There are many graph theory textbooks but not so many on graph algorithms, and the ones I know about are at too low a level

They don't cover enough of the topics I want to cover in this course

Instead, the course web page has links to online readings, mostly from Wikipedia

Their quality is variable but they're all we have

# Expectations

What do I expect you to know already?

- Undergraduate-level algorithm design and analysis
  (as covered in CS 161): recursion, divide and conquer, dynamic programming,
  $O$-notation
- Some previous exposure to basic graph algorithms
  including depth-first search, topological sorting,
  and Dijkstra's algorithm

  (we will go over them again, but as review)

# Web search

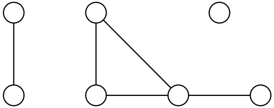
# Graphs

A graph is just a set of objects related to each other in pairs

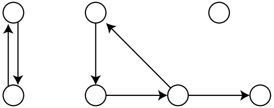
One object is called a vertex; more than one are vertices

(Do not ever call one of them a "vertice"; that is not a word.)

Pairs are called edges and their two vertices are endpoints



Undirected graph:
The endpoints are unordered



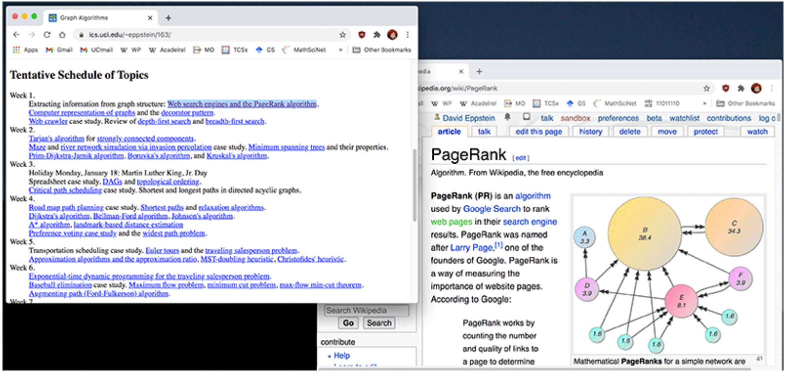
Directed graph:
Each edge goes from one endpoint to another

We will see many real-world examples and applications

# The web graph

Vertices = all web pages in the world-wide web

Directed edges = links from one web page to another



## Tentative Schedule of Topics

Week 1.
Extracting information from graph structure: Web search engines and the PageRank algorithm.
Computer representation of graphs and the decorator pattern.
Web crawler case study. Review of depth-first search and breadth-first search.
Week 2.
Tarjan's algorithm for strongly connected components.
Maze and river network simulation via invasion percolation case study. Minimum spanning trees and their properties.
Prim-Dijkstra-Jarnik algorithm, Boruvka's algorithm, and Kruskal's algorithm.
Week 3.
Holiday Monday, January 18: Martin Luther King, Jr. Day
Spreadsheet case study. DAGs and topological ordering.
Critical path scheduling case study. Shortest and longest paths in directed acyclic graphs.
Week 4.
Road map path planning case study. Shortest paths and relaxation algorithms.
Dijkstra's algorithm, Bellman-Ford algorithm, Johnson's algorithm.
A* algorithm, landmark-based distance estimation
Preference voting case study and the widest path problem.
Week 5.
Transportation scheduling case study. Euler tours and the traveling salesperson problem.
Approximation algorithms and the approximation ratio, MST-doubling heuristic, Christofides' heuristic.
Week 6.
Exponential-time dynamic programming for the traveling salesperson problem.
Baseball elimination case study. Maximum flow problem, minimum cut problem, max-flow min-cut theorem.
Augmenting path (Ford-Fulkerson) algorithm.

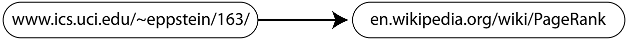
## PageRank [edit]

Algorithm. From Wikipedia, the free encyclopedia

**PageRank (PR)** is an algorithm used by Google Search to rank web pages in their search engine results. PageRank was named after Larry Page,[1] one of the founders of Google. PageRank is a way of measuring the importance of website pages. According to Google:

> PageRank works by counting the number and quality of links to a page to determine

Mathematical **PageRanks** for a simple network are



www.ics.uci.edu/~eppstein/163/

en.wikipedia.org/wiki/PageRank

# Web search engines

You type words or phrases

Database server looks up matching web pages

Shows them to you in some order

Examples: Google, Bing, DuckDuckGo, . . .

# A big change in the mid-1990s

Previous search engines:

Order search results by text (ignoring the graph structure)
Pages with many matching words shown first

Google (1998):

Order by number of incoming links (graph structure)

Intuition: if many other pages link to it, it's probably good

Refined version: Give more weight to links that come from other pages with many incoming links

This worked much better!

# How Google worked circa 1998

(How they work today: known only to Google insiders)

When you make a query on Google's servers:

- Use text data structures (beyond the scope of this class) to find a set of matching web pages (limited to 500 results)
- Show these pages to you in sorted order by pagerank

## What is pagerank?

- A number associated with each web page
- Bigger number = earlier in search results
- Computed by Google using only the link structure
  (does not depend on text of page)

# Pagerank, definition 1 (mathy version)

$$\text{pagerank}(x) = 0.05\frac{1}{\#\ \text{vertices}} + 0.95 \sum_{\text{edge } y \to x} \frac{\text{pagerank}(y)}{\#\ \text{edges out of } y}$$

Gives large system of linear equations that can be solved to compute pageranks

When $x$ has many incoming links from pages with high pageranks, its own pagerank will be high

But why this equation? What does it mean?

# Pagerank, definition 2 (intuitive version)

“Lazy web surfer”: mathematical model of how a person might view web pages

- Start at a random web page
- Repeatedly go to a new page:
    - Probability 0.95:, random outgoing link
    - Probability 0.05: get bored, restart at new random page

Define $P_i[x]$ = probability of looking at page $x$ on step $i$, with $P_0[x] = \frac{1}{\text{\# vertices}}$

After many steps, $P_i$ will get close to a “stable distribution” $P_i[x] \to P_\infty[x]$

Pagerank is just this limiting probability: $\text{pagerank}(x) = P_\infty[x]$

(0.05 and 0.95 are arbitrary and could be changed to any other numbers $p$ and $1 - p$. For $p = 0.05$, it takes $20 = 1/0.05$ steps on average to get bored.)

# How to compute pagerank? Option 1

We have a large set of linear equations

$$\text{pagerank}(x) = 0.05\frac{1}{\#\text{ vertices}} + 0.95 \sum_{\text{edge } y \to x} \frac{\text{pagerank}(y)}{\#\text{ edges out of } y}$$

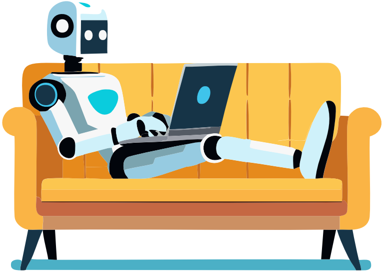
Use Gaussian elimination to solve them

But that takes time $O(n^3)$, far too slow when $n =$ billions

# How to compute pagerank? Option 2

Simulate the lazy web surfer

Estimate pagerank as the number of times the simulated surfer visits each page



But to get an accurate estimate we need to run the simulation long enough to get many visits to each page (theoretically at least proportional to $n \log n$ steps)

Still too slow

# How to compute pagerank? Option 3

Compute probability $P_i[x]$ for $i$th step of lazy surfer, using almost the same equations:

$$P_0[x] = \frac{1}{\#\text{ vertices}}$$

and, for $i > 0$,

$$P_i[x] = 0.05\frac{1}{\#\text{ vertices}} + 0.95 \sum_{\text{edge } y \to x} \frac{P_{i-1}[y]}{\#\text{ edges out of } y}$$

# Option 3 in pseudocode

```
Allocate an array $P[i, x]$
For each vertex $x$, set $P[0, x] = 1/\#$ vertices
For $i = 1, 2, \dots \#$ iterations:
    For each vertex $x$, set $P[i, x] = 0.05/\#$ vertices
    For each edge $y \to x$, add $0.95 * P[i-1, y]/(\#$ edges out of $y)$ to $P[i, x]$
```

Time: $O(\#\text{ iterations} \cdot \#\text{ edges})$

We only need approximate pageranks rather than exact solutions, and this converges quickly, so we only need very small # iterations (maybe 5) $\Rightarrow$ linear time

# The morals of the story

Throwing away other information about the vertices of a graph and using only their link structure can still provide meaningful information about the graph

By doing this, Google produced a much better search engine than previous competitors and dominated the search engine market

Getting it to work required an efficient algorithm for pagerank
(for problem sizes so big they don't fit into a single computer)