ICS 163 – Spring 2012 – Midterm

Name:

Student ID:

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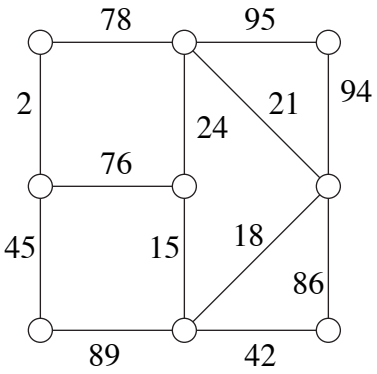
Total:

1. (18 points) Which edges in the following graph form its minimum spanning tree?



2. (6 points) Which one of Prim, Dijkstra, or Jarnik was the first to invent the Prim–Dijkstra–Jarnik algorithm?

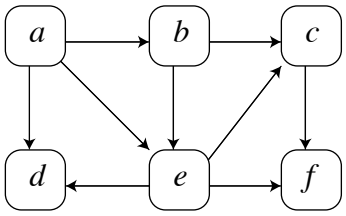
3. (18 points) Tarjan's strong connectivity algorithm involves performing a depth-first search of a graph, with a constant amount of extra work per recursive call to maintain lowlink numbers. Using an adjacency list representation, on a graph with n vertices and m edges, it takes time $O(n+m)$. How much time would it take if we instead used an adjacency matrix representation? State your answer using O-notation, as a function of n and/or m.

   (Hint: the slowest part of the algorithm in this case is listing all the neighbors of each vertex.)

4. (24 points) Suppose we are given as input an undirected graph G in which all edges have equal length, and we are also given two vertices s and t; we wish to find the shortest path in G from s to t. Which one of the following algorithms could be used to solve this problem the most efficiently? If G has n vertices and m edges, state the running time of your choice using O-notation as a function of n and/or m. (a) the DAG shortest path algorithm; (b) Dijkstra's algorithm; (c) the Bellman–Ford algorithm; (d) Johnson's algorithm; (e) breadth-first search; (f) depth-first search; (g) Hierholzer's algorithm.

5. (20 points) Give a reason why the Schulze widest-path voting method might be preferable to the instant runoff voting method.

6. (24 points) Draw a depth-first search tree for the graph below, starting from vertex a; whenever the depth-first search has a choice of more than one vertex to search next, it should choose the alphabetically smallest neighboring vertex first. After you draw the tree, list the vertices of the graph in the order that the topological ordering algorithm described in class would order them, based on the same DFS tree.



7. (10 points) For the same graph as the previous question, what is the smallest possible number of layers needed for a layered drawing of this graph?

You may use this page (or the back of the other pages) as scratch paper.