CS 261: Data Structures

Week 10: Odds and ends

Lecture 10b: Strings

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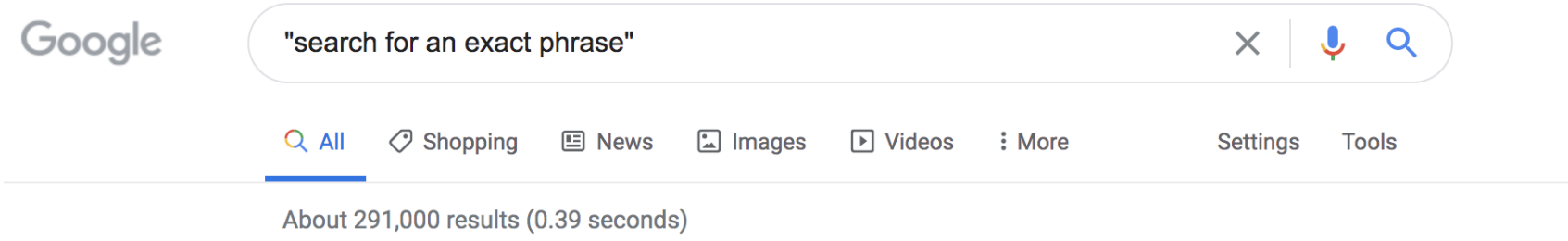
Spring Quarter, 2023

Tries

Background

If you enter a quoted string in Google, it will search for documents containing that phrase



How does it do it? Not hashing

There are too many different phrases for it to hash all of them

We need a data structure that can

- Index an enormous text document or documents
- Quickly find all copies of a string (appearing without gaps)
- Uses small space, linear in document length

Some basic definitions

Alphabet: any finite set

For text searching, might be ASCII or unicode characters

For genomics, might be the four bases adenine, thymine, cytosine, and guanine (A, T, C, G) that encode genetic information in DNA

For Google's search problem, might be the words (rather than the individual letters) appearing in its documents

Symbol: a member of the given alphabet

String: a finite sequence of symbols

Substring: a connected subsequence of the sequence (might be empty, might be the whole sequence)

A simpler warm-up problem

Given a "dictionary", a list of n strings over some alphabet, handle queries that either ask whether a query string q belongs to the dictionary, or that look up some extra information associated with the string (its definition)

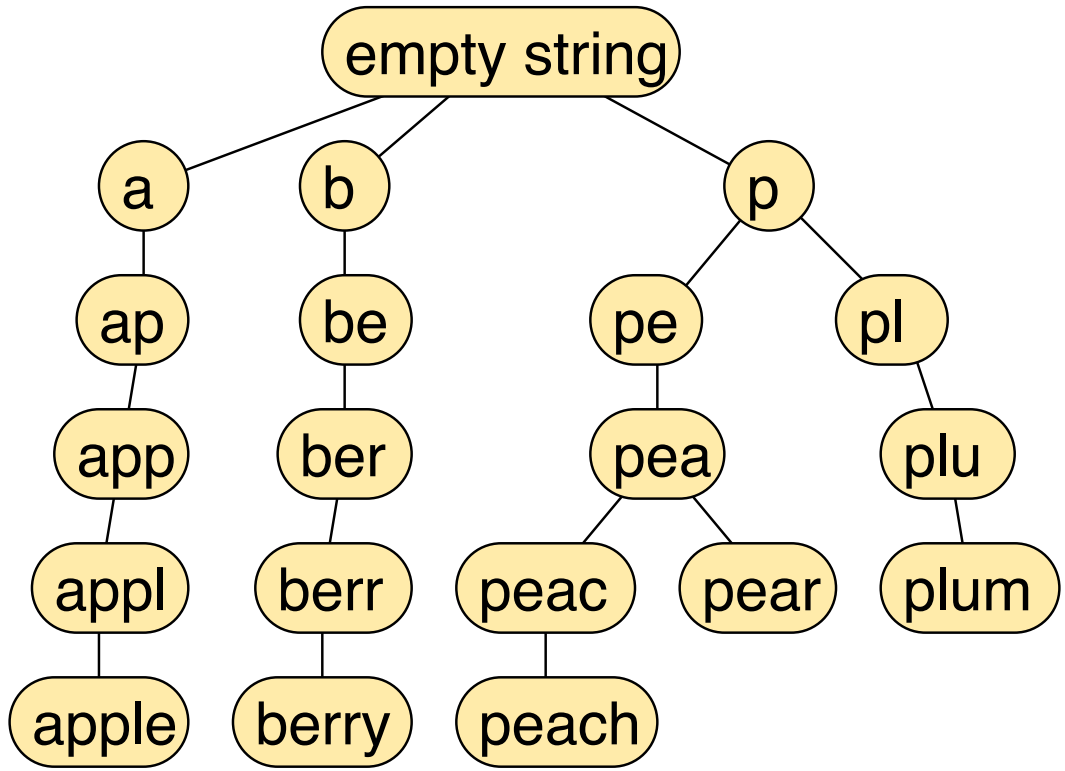
Binary search tree: query time $O(\log n) \times \text{length}(q)$

Hash table: query time $\text{length}(q)$, randomized

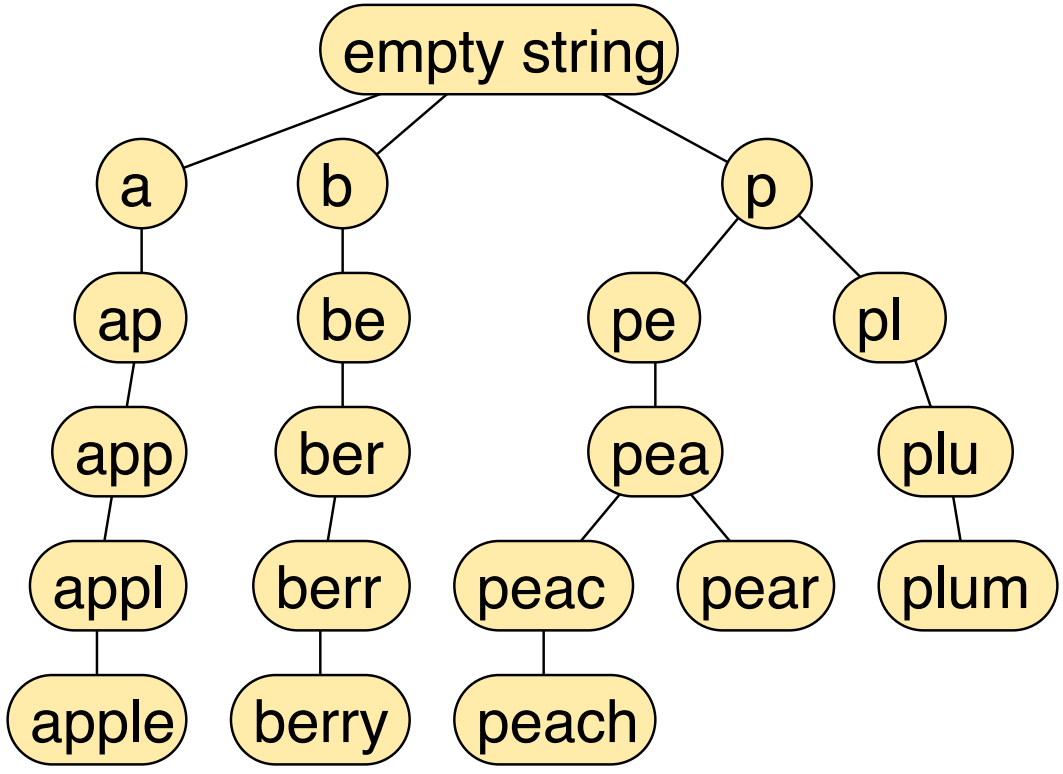
Trie (or digital search tree)

Nodes = prefixes of given strings (substrings that start at the start)

Parent of a node = prefix with one less symbol



Two complications



Leaves are words but not all words are leaves:
Is "pea" a word in this dictionary?

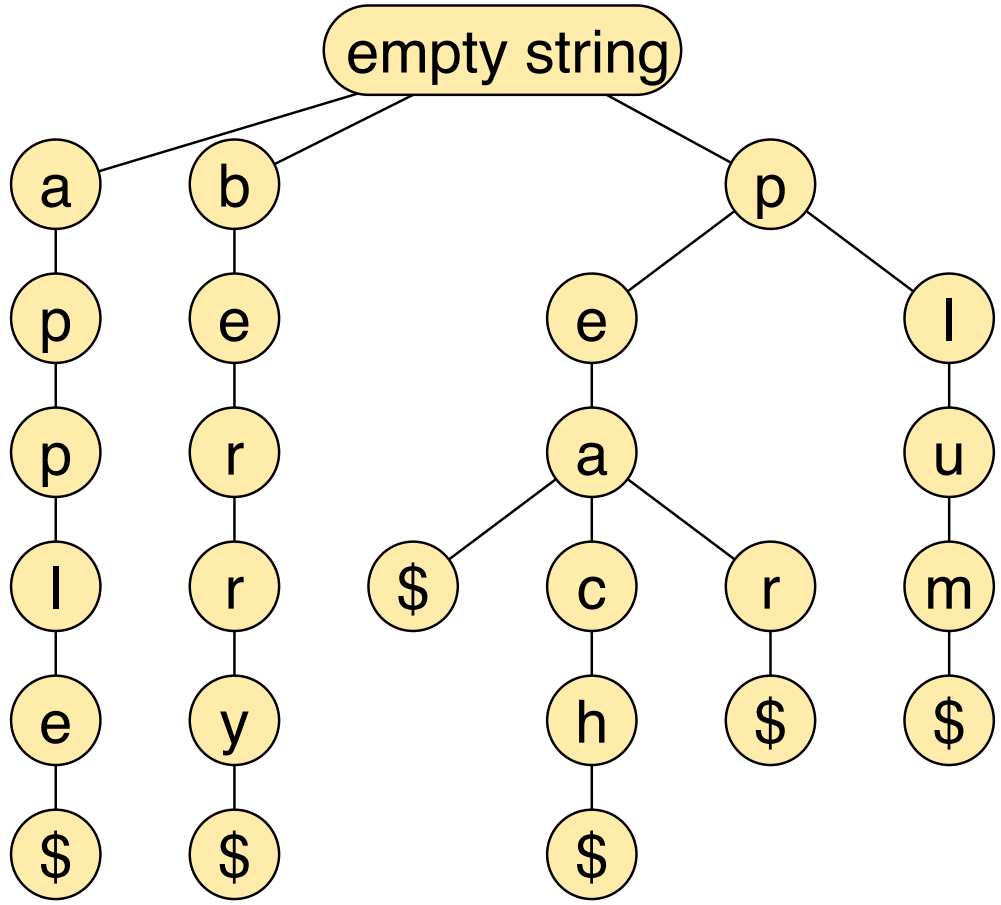
Separately storing each prefix would take too much space

Representing a trie

Add extra end-of-word symbol "$" to all words

Label vertices by single symbol added to parent's prefix (not by whole prefix)

For large alphabets, nodes can store hash table mapping alphabet symbols to corresponding children



Querying a tree

Start at root of tree (empty prefix)

For each symbol c of query $q + \$$:

    Go to the child whose label is c

    If no such child, q is not in the dictionary

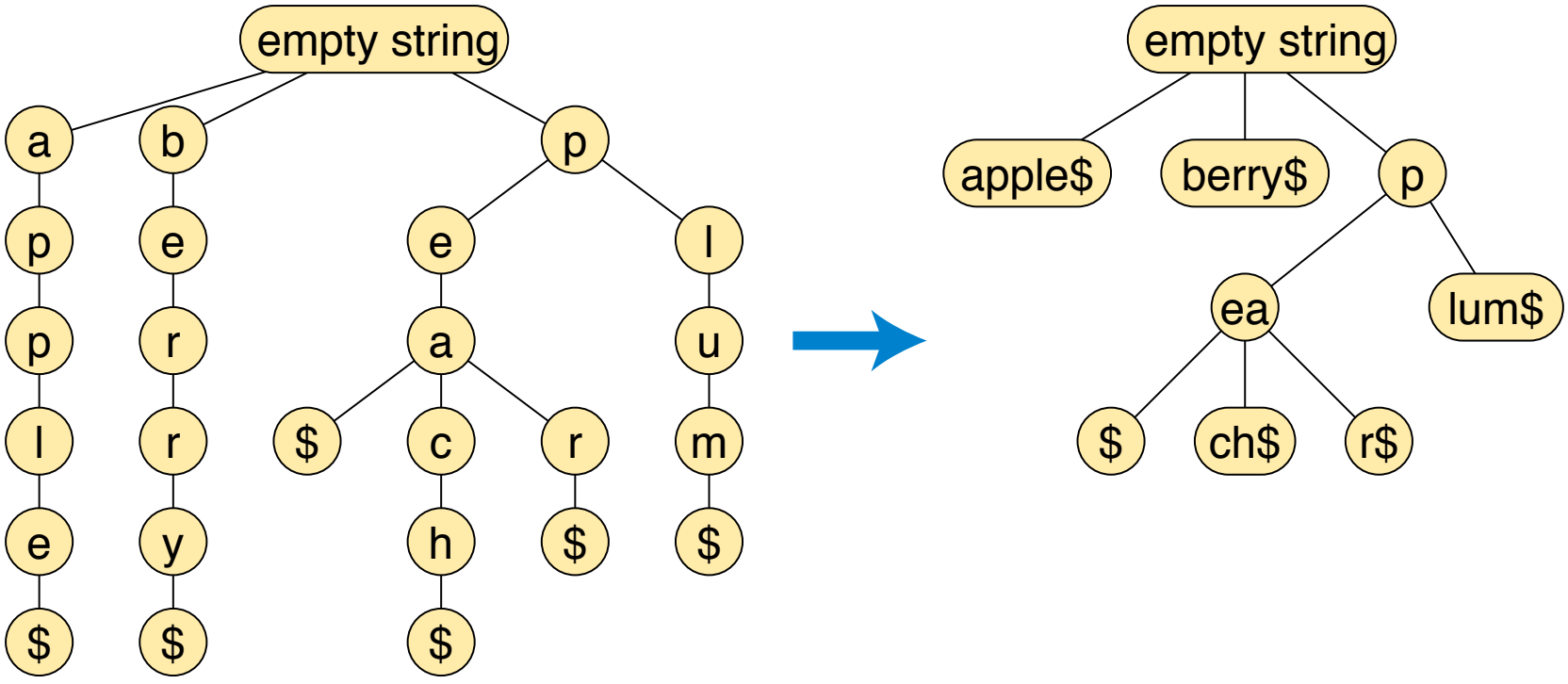
If search succeeded, q is in the dictionary, and current leaf node is its representative in the tree

Time = length(q)

Space = number of tree nodes
= total length of all dictionary words

Compressed tries

When nodes have exactly one child, merge them with their child



Tree has n leaves, no nodes with one child $\Rightarrow \leq 2n - 1$ nodes

Represent substrings by pointer to string they come from + position and length $\Rightarrow O(1)$ words of information per node

Searching a compressed trie

- Current node = root of compressed trie
- Current position in substring = 0
- For each symbol c in query $q + \$$:
  - If current position $<$ length of substring in current node, check that c matches the symbol at that position in the substring, and add 1 to current position
  - Otherwise, look for the child whose substring begins with c (by scanning all children or by using a hash table at the node mapping symbols to children), set current node to that child, and set current position = 0
- If any check fails or any child is not found, q is not in the dictionary; otherwise, it is represented in the dictionary by the current node

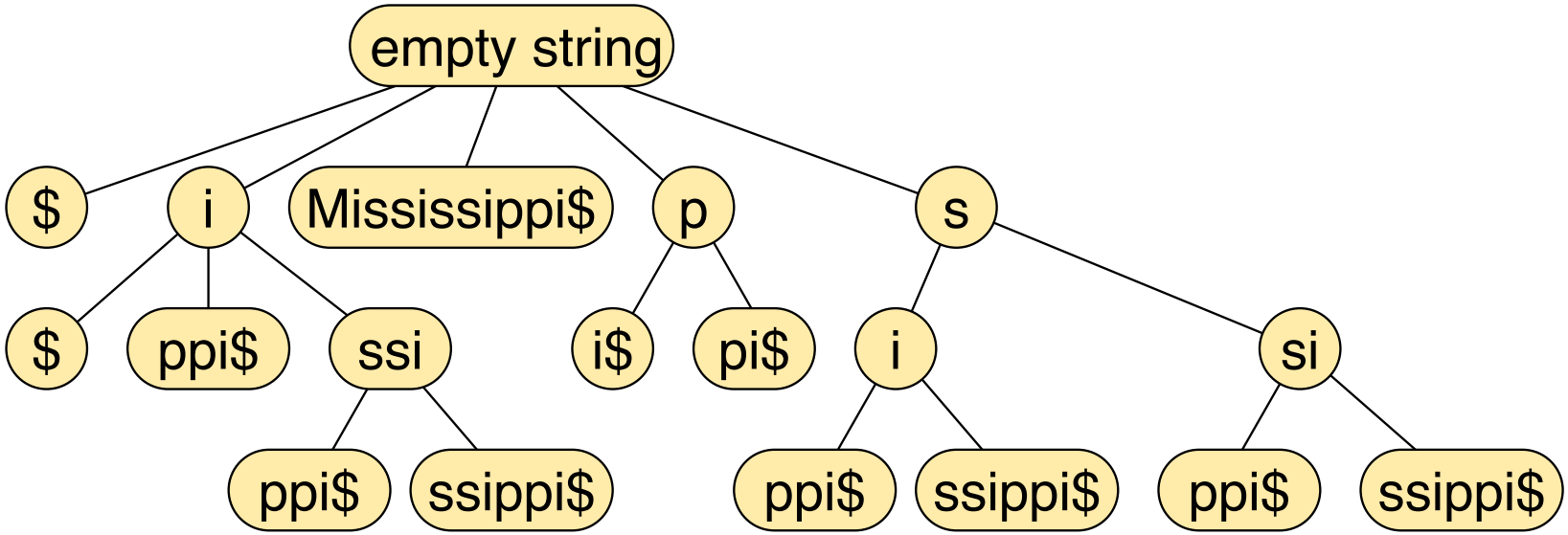
Suffix trees

Suffix trees

Suffix = substring of input string that includes the final dollar sign

Suffix tree = compressed trie of all suffixes

E.g., for input = "Mississippi":



The suffixes in order by length are \$, i\$, pi\$, ppi\$, ippi\$, sippi\$, ssippi\$, issippi\$, sissippi\$, ssissippi\$, ississippi\$, Mississippi\$

Tree leaves represent these same suffixes in alphabetical order

Searching a suffix tree

Given a query string q, search character-by-character, just like in any other compressed trie

But do not search for the \$ after the end of q

Search succeeds $\Rightarrow$ q is a substring of the input string

The positions where q appears in the input are the starting positions of the suffixes whose leaves are descendants of the node where the search finishes

Possible solution to Google's exact phrase search

Build suffix tree where input = all known web pages concatenated together, alphabet = words in each web page

Tree size: total number of words in all known web pages

Why words rather than characters: to reduce suffix tree size

Query time $\approx$ length of query phrase

Search result $\Rightarrow$ positions where q appears in concatenation $\Rightarrow$ web pages containing the given phrase

Other uses of suffix trees

Many other algorithmic problems on strings can be solved using suffix trees, for instance:

To find longest matching substrings from starting positions x and y in given string, find lowest common ancestor of leaves for suffixes starting at x and y

To sort the suffixes of an input (used in Burrows–Wheeler data compression algorithm), traverse the tree using sorted order of characters at each node

To find longest common substring of two strings s and t, build a suffix tree of $s\$t\$$ and look for a node whose leaf descendants include suffixes starting in both strings and whose distance from the root (measured by string length) is maximum

How to construct the suffix tree?

Can be done in linear time + time to sort the alphabet

Martin Farach-Colton, "Optimal suffix tree construction with large alphabets", FOCS 1997

Details are very complicated. Rough outline:

- Recursively build compressed trie T_o of suffixes starting at odd positions (almost the same as suffix tree for string of length $n/2$ whose alphabet is the set of consecutive pairs of symbols from the given string, sorted using radix sort)
- Represent suffixes at even positions as pairs (c, s) where c is a single character and s is an odd suffix, and radix sort them
- Build compressed trie T_e of even suffixes non-recursively, using sorted ordering of its leaves + tree navigation in T_o to find heights of common ancestors of consecutive leaves
- Use tree navigation to merge T_o and T_e into a single tree