ICS 261 – Winter 2006 – Final

Name:

Student ID:

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Total:

1. (15 points) Define the balance factor of a tree to be the ratio between the lengths of the longest and shortest root-leaf paths in the tree. How large can the balance factor be in an n-node red-black tree?

2. (25 points) An internet packet filter needs to handle a set of rules, each specifying a range of internet addresses (that is, an interval of integers), a priority, and an action to perform on packets received from addresses in its range. Describe an efficient data structure that allows the following operations: creation of a new rule, deletion of a rule, and querying the maximum-priority rule that applies to a packet whose address is given as part of the query. What is the running time of your data structure on each of these operations, assuming that there are n rules in the ruleset when the operation is performed? You may also assume that no two rules have equal priorities.

3. (20 points) Name or describe an efficient data structure for the following problem: you are given as input a set S of n points in the x-y plane, and must handle queries that specify query values a and b and return any point (x, y) in S with $y \leq ax + b$, if such a point exists. What is the space usage, preprocessing time, and query time of your data structure?

4. (20 points) Which of the persistence techniques described in class would be most appropriate for constructing a partially persistent hash table? Explain your answer.

5. (15 points) Draw a Cartesian tree for the sequence $12, 9, 11, 10, 7, 8, 13, 6, 15$.

6. (25 points) Describe a data structure for the following problem. We are given as input an n-character string, and wish to answer queries of the following form: given indices i and j, what is the minimum $k \geq 0$ such that the characters in positions $i + k$ and $j + k$ differ? For instance, if the input string $s =$ "quaquaversal", $i = 1$, and $j = 4$, then the answer to the query should be 2, because $s[i] = s[j] =$ 'u', $s[i+1] = s[j+1] =$ 'a', but $s[i+2] =$ 'q' $\neq s[j+2] =$ 'v'.

Hint: combine suffix trees and least common ancestors.

You may use this page as scratch space or as additional space for your answers.