ICS 261 – Winter 2006 – Midterm

Name:

Student ID:

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Total:

1. (30 points)

Consider the following problem: we must maintain a set S of integers, and handle queries that seek the smallest multiple of a given number x that belongs to S. If we describe this as a range minimization problem, what are the ranges?

2. (30 points)

Describe an efficient data structure for answering the problem defined in Question 1, where the integers in S are restricted to lie in the range from 1 to n. Your data structure should use space $O(n^{1+o(1)})$, time $O(n^{o(1)})$ to insert or delete an integer, and constant time per query.

Hint: you may use without proof the fact that any integer in this range has $O(n^{o(1)})$ distinct divisors, and that $\log n = O(n^{o(1)})$ for any epsilon.

3. (30 points)

Suppose we perform the same sequence of insertions with two different data structures, a red black tree and a splay tree.

(a) Can an individual insertion in the red black tree be faster than the same insertion in the splay tree by more than a constant factor? Why or why not?

(b) Can the total time for the whole sequence of insertions in the red black tree be faster than the total time for the splay tree by more than a constant factor? Why or why not?

4. (30 points)

Suppose we wish to maintain a set of points (x, y) in the plane, with queries that specify an interval $[x_\ell, x_r]$, and seek the greatest difference of y-coordinates for points with that interval; that is, the result of the query is

$$\max_{x_\ell \le x_i, x_j \le x_r} |y_i - y_j|.$$

With what extra information should we augment a binary search tree on the x-coordinates of the points in order to answer these queries?

You may use this page (or the back of the other pages) as scratch paper.