# ICS 261 – Winter 2011 – Midterm

Name:

Student ID:

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Total:

1. (30 points) Suppose that we wish to implement Dijkstra's algorithm on a family of graphs that have $n$ vertices and $n^{3/2}$ edges, and that we are trying to decide between using Fibonacci heaps or $d$-ary heaps for this application. Which of these two priority queue data structures would lead to a running time with a smaller $O$-notation, or would both be equal in this respect? State the worst-case running time for both algorithms using $O$-notation, in as simple a form as possible, and also state what the best choice of $d$ would be in the $d$-ary heap in this application.

2. (30 points) Suppose that, in a splay tree with $n$ items, we perform the following sequence of three operations involving two different keys $x$ and $y$ that are both among the items in the tree:

- search for key $x$
- search for key $y$
- search again for key $x$

(a) Name all the keys that could possible be stored at the root of the tree at the time that we begin the second search for $x$. (On this and the other parts of this question, you do not need to explain your answer.)

(b) What is the minimum possible number of nodes on the path from the root of the tree to $x$ at the time that we begin the second search for $x$?

(c) What is the maximum possible number of nodes on the path from the root of the tree to $x$ at the time that we begin the second search for $x$?

3. (30 points) Explain an advantage that cuckoo hashing has over hash chaining in some applications. Give an example of an application of hashing in which that advantage would be relevant, and explain why.

4. (30 points) Suppose that we have a Bloom filter that has $C$ cells in it, $n$ items stored in it, and $k$ hash functions per item. You may assume that each item may be represented as a 32-bit value, and that the $k$ hash functions for each item have random values that are completely independent of each other and independent of the hash functions for other items.

(a) Write down a formula in terms of $n$, $C$, and $k$ for the number of 32-bit memory words used by the Bloom filter.

(b) Write down a formula for the probability that a single given cell is nonzero.

(c) Write down a formula for the expected number of nonzero cells in the whole table.

You may use this page (or the back of the other pages) as scratch paper.