Guard Placement For Wireless Localization

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Motivating example: Cyber-Café

Supply wireless internet to paying customers inside
Prevent access to non-paying customers outside
But how to tell which customers are which?
Proposed Solution

- Use multiple directional transmitters
- Customers in range of both transmitters are inside
Sculpture Garden Problem

Sculpture Garden Problem – the problem of placing angle guards to define a given polygon
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\[ F = abd + cd = (ab + c)d \]
Natural angle guards are placed at vertices of the polygon with angle of vertex = angle of guard

Non-natural guards:
Natural Guards Aren’t Enough

**Theorem.** There exists a polygon $P$ such that it is impossible to solve SGP for $P$ using a natural angle-guard vertex placement.

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**Proof.**
Theorem. Any quadrilateral can be guarded with 2 natural angle guards.
Theorem. Any pentagon can be guarded with 3 angle guards.
Theorem. Any hexagon can be guarded with 4 angle guards.
Theorem. \( n + 2(h - 1) \) guards are sufficient to define any \( n \)-vertex polygon with \( h \) holes.

Proof.

- Triangulate into \( n + 2(h - 1) \) triangles
- Partition triangulation into quadrilaterals, pentagons, and hexagons
- In each piece, \# guards = \# triangles

Definition is concise: each region is defined by \( O(1) \) guards.
Lower Bounds

Theorem. At least $\lceil \frac{n}{2} \rceil$ guards are required to solve the SGP for any polygon with no two edges lying on the same line.

Theorem. $\left\lceil \frac{n}{2} \right\rceil$ guards are always sufficient to solve SGP for any convex polygon.
Lower Bounds

**Theorem.** Any $n$-sided polygon requires $\Omega(\sqrt{n})$ guards.

**Theorem.** There exist $n$-sided simple polygons that can be guarded concisely by $O(\sqrt{n})$ guards.
Definition. *xy-monotone* polygon is an orthogonal polygon which is monotone with respect to the $x = y$ line.
**Orthogonal Polygons**

**Theorem.** $\frac{n}{2}$ natural guards are sufficient to solve SGP for any orthogonal polygons, by placing natural angle guards in every other vertex starting with a left vertex of some top edge.

**Proof.** By induction on unguarded reflex vertices.  

**Base Case**

![Diagram of orthogonal polygon with guard placement](diagram.png)
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**Inductive Hypothesis**

![Diagram](attachment:image.png)
Minimizing Number of Guards

Open Problem: How hard is it to find the minimum number of guards for a particular polygon?

Theorem (Approximation). For any polygon $P$, we can find a collection of guards for $P$, using a number of guards that is within a factor of two of optimal.

Proof.

- Place an edge guard on the line bounding each edge of $P$
- Use a Peterson-style constructive-solid-geometry formula
- In optimal solution, each line must be guarded and each guard can cover at most two lines so optimal # guards is at least half the guards used
## Summary

<table>
<thead>
<tr>
<th></th>
<th>Sometimes required</th>
<th>Always Sufficient</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Arbitrary</strong></td>
<td>$\Omega(\sqrt{n})$</td>
<td>$n + 2(h - 1)$</td>
</tr>
<tr>
<td><strong>General position</strong></td>
<td>$\lceil \frac{n}{2} \rceil$</td>
<td>$n + 2(h - 1)$</td>
</tr>
<tr>
<td><strong>Orthogonal general position</strong></td>
<td>$\frac{n}{2}$</td>
<td>$\frac{n}{2}$</td>
</tr>
<tr>
<td><strong>Convex</strong></td>
<td>$\lceil \frac{n}{2} \rceil$</td>
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Obvious open question: close factor-of-two gap for non-orthogonal general position