Quantifying the Association Between Discrete Event Time Series

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ics.uci.edu/~galbraic/ Statistical Applications in Forensic Evidence July 31, 2018 1 / 19
Logs of User-Generated Event Data

Browser requests
Web searches
Email activity
Phone/SMS
Social media activity
GPS locations
File access
Network activity
Exercise/movement
.....
User Event Data

< ID, timestamp, action type, metadata >

Web clicks
Web searches
Emails sent
Social media posts
Files edited

Text content
Location
List of recipients

We focus on ID, timestamp, and type of actions
Problem Statement

- Consider a pair of user-generated event series $M = (A, B)$
  - Each series fully characterized by event times
  - Event types differ between series

- Quantify the likelihood that the pair was generated by the same source

\[ WLOG \text{ assume that } n_B < n_A. \]
Methodology

\[(A^*, B^*)\]
Score Function \(\Delta\)

Population-based Approach
- Sample from relevant population: \(M_i = (A_i, B_i)\) for \(i = 1, \ldots, N\)
- Estimate score-based likelihood ratio (SLR)

Resampling Approach
- Single pair: \((A^*, B^*)\)
- Estimate coincidental match probability (CMP)

Degree of Association
Need to determine suitable measures to quantify association between two event series $A$ and $B$.

- Nearest-neighbor indices (from marked point process literature)
- Distribution of inter-event times

$$\Delta(A, B) = \bar{\tau}_{BA} = \frac{1}{n_B} \sum_{i=1}^{n_B} \tau^{(i)}_{BA}$$
Population-based Approach

- Two competing propositions:
  \[ H_s : (A^*, B^*) \text{ came from the same source} \]
  \[ H_d : (A^*, B^*) \text{ came from different sources} \]

- Use sample \( M_i = (A_i, B_i) \) for \( i = 1, \ldots, N \) to estimate the score-based likelihood ratio for the observed score \( \Delta(A^*, B^*) \)

\[
SLR_\Delta = \frac{g(\Delta(A^*, B^*)|H_s)}{g(\Delta(A^*, B^*)|H_d)}
\]

- Different interpretations of denominator lead to different \( SLRs \) (Hepler et al., 2012)
Estimation of $g$

To estimate $g(\Delta(A, B) | H_d)$, repeat this process using all pairwise combinations of event series $(A_i, B_j) \ni i \neq j$. 
**Coincidental match probability**: probability that a different-source pair with observed score $\Delta(A^*, B^*)$ exhibits association by chance

$$CMP_{\Delta} = Pr(\Delta(A, B) < \Delta(A^*, B^*) | H_d)$$
Comparison of Approaches

\[ g(\Delta(A^*, B^*)| H_d) \]

\[ g(\Delta(A^*, B^*)| H_s) \]

\[ \Delta(A^*, B^*) \]

\[ \Delta(A^*, B^*) \]
Simulation Study

Simulated the equivalent of one week of data for 20k pairs of processes (10k independent & 10k associated)

Repeated for various combinations of \((\lambda_A, p, \sigma)\)
Signal-to-Noise Ratio

\[ \text{SNR} = \frac{\bar{\tau}_{AA}}{\bar{\tau}_{BA}} = \frac{\text{mean IET for process } A}{\text{mean IET from } B \text{ events to nearest } A \text{ event}} \]
Simulation Results

AUC vs SNR graph showing two lines labeled SLR and CMP. The graph indicates an increasing AUC with increasing SNR for both lines. The notation $* p = 0.20$ suggests a statistical significance level.
Case Study

- Data from a 2013-2014 study at UCI that placed logging software on 124 students’ computers that recorded all browser activity for one week (Wang et al., 2015)
- Event series created by dichotomizing browsing events to Facebook versus non-Facebook related urls
- Considered 55 students with at least 50 web browsing events of each type
### Case Study Results

<table>
<thead>
<tr>
<th>Method</th>
<th>Score Function Δ</th>
<th>TP Rate*</th>
<th>FP Rate*</th>
<th>AUC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population-based</td>
<td>Near-neighbor (mingling)</td>
<td>85.5</td>
<td>11.6</td>
<td>94.6</td>
</tr>
<tr>
<td>Population-based</td>
<td>Near-neighbor (segregation)</td>
<td>94.5</td>
<td>3.1</td>
<td>99.2</td>
</tr>
<tr>
<td>Population-based</td>
<td>Inter-event Time (mean)</td>
<td>96.4</td>
<td>2.9</td>
<td>99.6</td>
</tr>
<tr>
<td><strong>Resampling</strong></td>
<td>Inter-event Time (mean)</td>
<td><strong>98.2</strong></td>
<td><strong>0.2</strong></td>
<td><strong>99.9</strong></td>
</tr>
</tbody>
</table>

*Population-based methods use SLR with a threshold of 1*

*Sampling-based method uses CMP with threshold of 0.1%*
Conclusions

- The resampling approach shows promise in situations where no reference data is available.
- The population-based SLR is still the preferred method, given:
  - Better performance for pairs exhibiting weak association.
  - Similar performance to the CMP for strongly associated pairs.
  - Well-established approach in forensic investigation.
- R implementation available on Github: assocr.
Future Directions

- Extend methodology
  - Spatial data
  - Other types of association (e.g., exclusion and ‘causal’ patterns)
  - Incorporate more (> 2) types of events
- Develop methods for identification
- Develop theory of detectability
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Figure: Segregation

Figure: Mean IET

Figure: Mingling

Figure: Median IET
Simulation Results

![Box plots and line graph showing CMP and AUC values for different values of \( \gamma \).]
Simulation Results

Figure: $\gamma = 14.6$

Figure: $\gamma = 7.3$