ICS 161 — Algorithms — Spring 2002 — Goodrich — Second Midterm

Name:

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total:

1. (50 points). Short Answers.

(a) Define “degree of a vertex in a graph.”

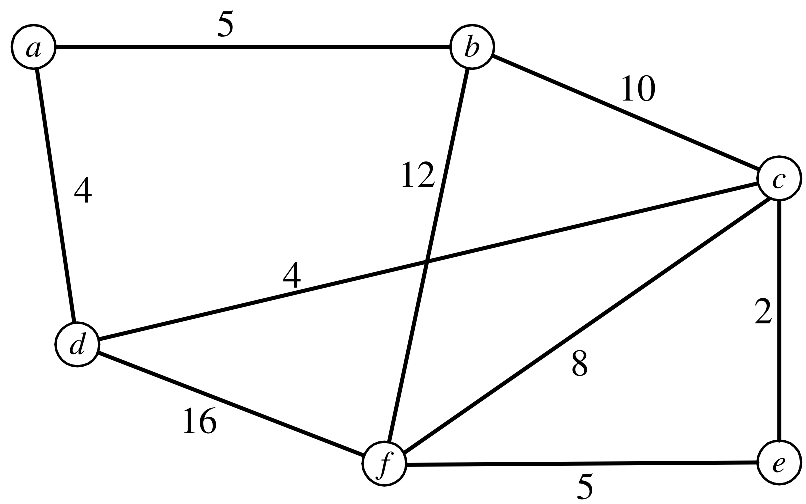
(b) Define “biconnected graph.”

(c) What is the running time of Dijkstra's algorithm for a connected graph of n vertices and m edges, assuming the graph is represented using an adjacency list and the priority queue is implemented using a heap?

(d) What is the running time of the dynamic programming algorithm discussed in class for computing the optimal parenthesization of a product of n matrices?

(e) How many edges are in the transitive closure of a strongly connected digraph G that has n vertices and m edges?

2. (50 points). Consider the following graph:



(a) What is the shortest path distance from a to each of the vertices b, c, d, and e?

(b) List all of the values of the label $d(f)$ that are assigned for the vertex f during a running of Dijkstra's algorithm on the above graph, starting from the vertex a. Note: you need to include ***all*** the different values this label takes during a running of the algorithm.

3. (50 points). Use the master theorem (or other technique) to characterize using the big-Oh notation the following divide-and-conquer recurrence relations (you may assume that $T(1) = 1$ is the base case for each):

(a) $T(n) = 4T(n/4) + n$

(b) $T(n) = 9T(n/3) + n \log n$

(c) $T(n) = 3T(n/9) + n^{1/2}$

(d) $T(n) = 2T(n/4) + n$

(e) $T(n) = T(n/4) + 1$

4. (50 points). Briefly describe an efficient algorithm for traversing a connected undirected graph $G = (V, E)$ so as to traverse each edge exactly twice (once in each direction). What is the running time of your method, in terms of $n = |V|$ and $m = |E|$?

5. (50 points). eBuy.com is selling a batch of n cameras at their online auctioning web site. Users can visit eBuy.com and submit a bid of the form (x, y), which indicates a bid of x dollars for y cameras, but users do not get to see what other users have bid. At the end of the day, eBuy.com has received m such bids and wants to send out emails to the users whose bids they will accept. Describe how we could use an algorithm discussed in class to maximize eBuy.com's dollar return for the n cameras in each of the following cases. (You do not have to give pseudo-code; just say how you would set up the algorithm.)

(a) eBuy.com is allowed to partially fulfill bids; that is, a bid (x, y) can be fulfilled using $z \leq y$ cameras, for a return of xz/y dollars. What is the running time of your algorithm in this case?

(b) eBuy.com must accept or reject bids in total; that is, it cannot partially fill orders. A bid (x, y) must be completed fulfilled or rejected. What is the running time of your algorithm in this case?