CS 162 — Automata Theory — Fall 2015 — Goodrich — Final Exam

Name:

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total:

1. (30 points). Short Answers.

(a) Draw an example of a DFA having just 1 state, over the alphabet $\{0, 1\}$ that accepts an infinite number of strings.

(b) Give a definition for *regular expression*, over an alphabet Σ, including the null string, ε, and to allow for the emptyset symbol, $\emptyset$.

(c) How can you tell if the language for a DFA is empty or not?

2. (30 points). More Short Answers.

(a) What is the Church-Turing thesis?

(b) What is the Halting problem?

(c) Draw a Venn diagram that shows the known relationships between the following set of languages: P, NP, PSPACE, regular languages, decidable languages.

3. (30 points). Draw a DFAs recognizing each of the following languages, over the alphabet $\Sigma = \{0, 1\}$.

(a) $\{w \mid w$ contains at least three 1's$\}$

(b) $\{w \mid w$ contains at exactly two 0's$\}$

(c) $\{w \mid w$ contains the substring 011$\}$

4. (30 points). Give a context-free grammar (CFG) for each of the following languages, over the alphabet $\Sigma = \{0, 1, 2\}$.

(a) $\{0^i 1^j 2^n \mid i, j, n \geq 1 \text{ and } i + j = n\}$

(b) $\{w \mid w = w^R, \text{ that is, } w \text{ is a palindrome}\}$

5. (30 points). Consider the following context-free grammar:

$$\begin{aligned} S &\rightarrow S\texttt{+}T \mid T \\ T &\rightarrow T\texttt{*}F \mid F \\ F &\rightarrow \texttt{(}S\texttt{)} \mid \texttt{a} \end{aligned}$$

(a) Give a derivation for the string, `(a+a)*a`.

(b) Draw a parse tree for the string, `(a+a*a)`.

6. (30 points). Pumping Lemma.

(a) State the Pumping Lemma for regular languages.

(b) Use the Pumping Lemma for regular languages to show that the following language is not regular: $\{0^n 1^n \mid n \geq 0\}$.

7. (30 points). Consider the following two languages:

$$\begin{aligned} A &= \{(M, w) \mid M \text{ is a Turing machine and } M \text{ accepts input } w\} \\ H &= \{(M, w) \mid M \text{ is a Turing machine and } M \text{ halts on input } w\}. \end{aligned}$$

Use the fact that A is undecidable to show that H is undecidable.

8. (30 points).

(a) Define the complexity class, **NP**.

(b) Define the term "**NP**-complete."

(c) Define INDEPENDENT-SET as the problem that takes a graph G and an integer k and asks if G contains an independent set of vertices of size k. That is, G contains a set W of vertices of size k such that, for any u and v in W, there is no edge (u, v) in G. Show that INDEPENDENT-SET is **NP**-complete (remember that there are two parts to this). You may assume the **NP**-completeness of either VERTEX-COVER or 3SAT for the sake of this proof.

9. (30 points). Suppose you are given a DFA, A, which recognizes a language, L, and a DFA, B, which recognizes a language, M. Describe an algorithm for using the descriptions of A and B to produce a DFA, C, that recognizes the language $L - M$, that is each string in L that is not also in M.