The Pumping Lemma

Infiniteness Test
The Pumping Lemma
Nonregular Languages
The Infiniteness Problem

- Is a given regular language infinite?
- Start with a DFA for the language.
- **Key idea**: if the DFA has $n$ states, and the language contains any string of length $n$ or more, then the language is infinite.
- Otherwise, the language is surely finite.
  - Limited to strings of length $n$ or less.
Proof of Key Idea

• If an n-state DFA accepts a string $w$ of length $n$ or more, then there must be a state that appears twice on the path labeled $w$ from the start state to a final state.

• Because there are at least $n+1$ states along the path.
Proof – (2)

Then $x y^i z$ is in the language for all $i \geq 0$.

Since $y$ is not $\varepsilon$, we see an infinite number of strings in $L$. 
Infiniteness Test: Finding a Cycle

1. Eliminate states not reachable from the start state.
2. Eliminate states that do not reach a final state.
3. Test if the remaining transition graph has any cycles.
The Pumping Lemma

• We have, almost accidentally, proved a statement that is quite useful for showing certain languages are not regular.
• Called the *pumping lemma for regular languages*. 
Statement of the Pumping Lemma

For every regular language $L$
There is an integer $n$, such that
For every string $w$ in $L$ of length $\geq n$
We can write $w = xyz$ such that:

1. $|xy| \leq n$.
2. $|y| > 0$.
3. For all $i \geq 0$, $xy^i z$ is in $L$. 

Number of states of DFA for $L$

Labels along first cycle on path labeled $w$
Example: Use of Pumping Lemma

- We have claimed \( \{0^k1^k \mid k \geq 1\} \) is not a regular language.
- Suppose it were. Then there would be an associated \( n \) for the pumping lemma.
- Let \( w = 0^n1^n \). We can write \( w = xyz \), where \( x \) and \( y \) consist of 0’s, and \( y \neq \epsilon \).
- But then \( xyyz \) would be in \( L \), and this string has more 0’s than 1’s.