The Pumping Lemma

Infiniteness Test

The Pumping Lemma

Nonregular Languages

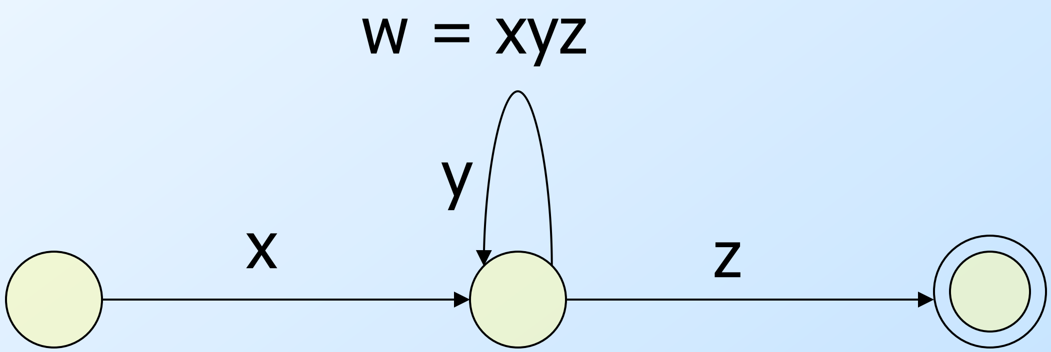
The Infiniteness Problem

- Is a given regular language infinite?
- Start with a DFA for the language.
- Key idea: if the DFA has *n* states, and the language contains any string of length *n* or more, then the language is infinite.
- Otherwise, the language is surely finite.
  - Limited to strings of length *n* or less.

Proof of Key Idea

- If an n-state DFA accepts a string w of length *n* or more, then there must be a state that appears twice on the path labeled w from the start state to a final state.
- Because there are at least n+1 states along the path.

Proof – (2)



Then xy^iz is in the language for all $i \geq 0$.

Since y is not ϵ, we see an infinite number of strings in L.

Infiniteness Test: Finding a Cycle

1. Eliminate states not reachable from the start state.
2. Eliminate states that do not reach a final state.
3. Test if the remaining transition graph has any cycles.

The Pumping Lemma

- We have, almost accidentally, proved a statement that is quite useful for showing certain languages are not regular.
- Called the *pumping lemma for regular languages*.

Statement of the Pumping Lemma

For every regular language L

There is an integer n, such that

For every string w in L of length $\geq$ n

We can write w = xyz such that:

1. $|xy| \leq n$.
2. $|y| > 0$.
3. For all $i \geq 0$, xy^iz is in L.

Number of states of DFA for L

Labels along first cycle on path labeled w

Example: Use of Pumping Lemma

- We have claimed $\{0^k1^k \mid k \geq 1\}$ is not a regular language.
- Suppose it were. Then there would be an associated n for the pumping lemma.
- Let $w = 0^n1^n$. We can write $w = xyz$, where x and y consist of 0's, and $y \neq \epsilon$.
- But then xyyz would be in L, and this string has more 0's than 1's.