Decision Properties of Regular Languages

General Discussion of “Properties”

Membership, Emptiness, Etc.

Properties of Language Classes

- A *language class* is a set of languages.
  - We have one example: the regular languages.
  - We' ll see many more in this class.
- Language classes have two important kinds of properties:
  1. Decision properties.
  2. Closure properties.

Representation of Languages

- Representations can be formal or informal.
- Example (formal): represent a language by a RE or DFA defining it.
- Example: (informal): a logical or prose statement about its strings:
  - $\{0^n1^n \mid n \text{ is a nonnegative integer}\}$
  - "The set of strings consisting of some number of 0's followed by the same number of 1's."

Decision Properties

- A *decision property* for a class of languages is an algorithm that takes a formal description of a language (e.g., a DFA) and tells whether or not some property holds.
- Example: Is language L empty?

Subtle Point: Representation Matters

- You might imagine that the language is described informally, so if my description is "the empty language" then yes, otherwise no.
- But the representation is a DFA (or a RE that you will convert to a DFA).
- Can you tell if $L(A) = \emptyset$ for DFA A?

Closure Properties

- A *closure property* of a language class says that given languages in the class, an operator (e.g., union) produces another language in the same class.
- Example: the regular languages are obviously closed under union, concatenation, and (Kleene) closure.
  - Use the RE representation of languages.

Why Closure Properties?

1. Helps construct representations.
2. Helps show (informally described) languages not to be in the class.

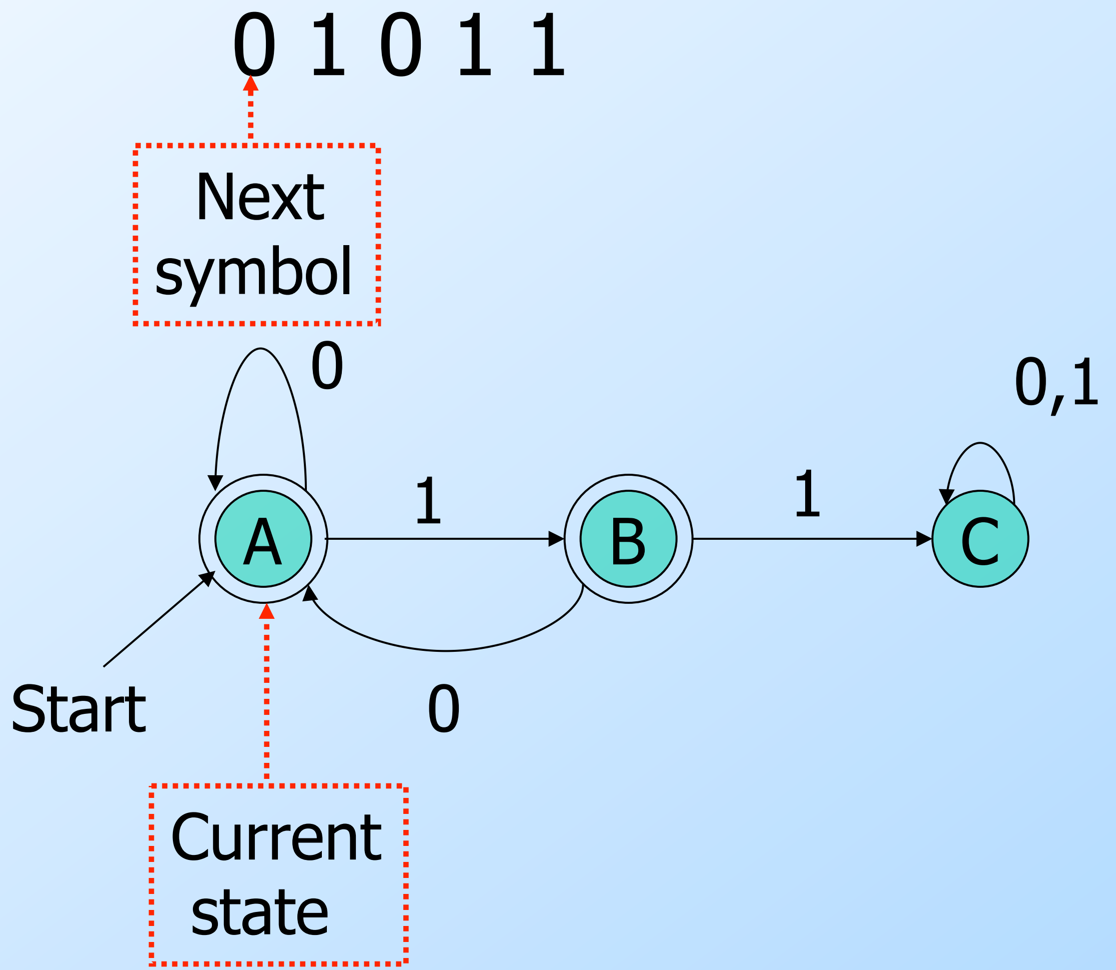
Example: Use of Closure Property

- We can easily prove $L_1 = \{0^n1^n \mid n \geq 0\}$ is not a regular language.
- L_2 = the set of strings with an = number of 0's and 1's isn't either, but that fact is trickier to prove.
- Regular languages are closed under $\cap$.
- If L_2 were regular, then $L_2 \cap L(0^*1^*) = L_1$ would be, but it isn't.

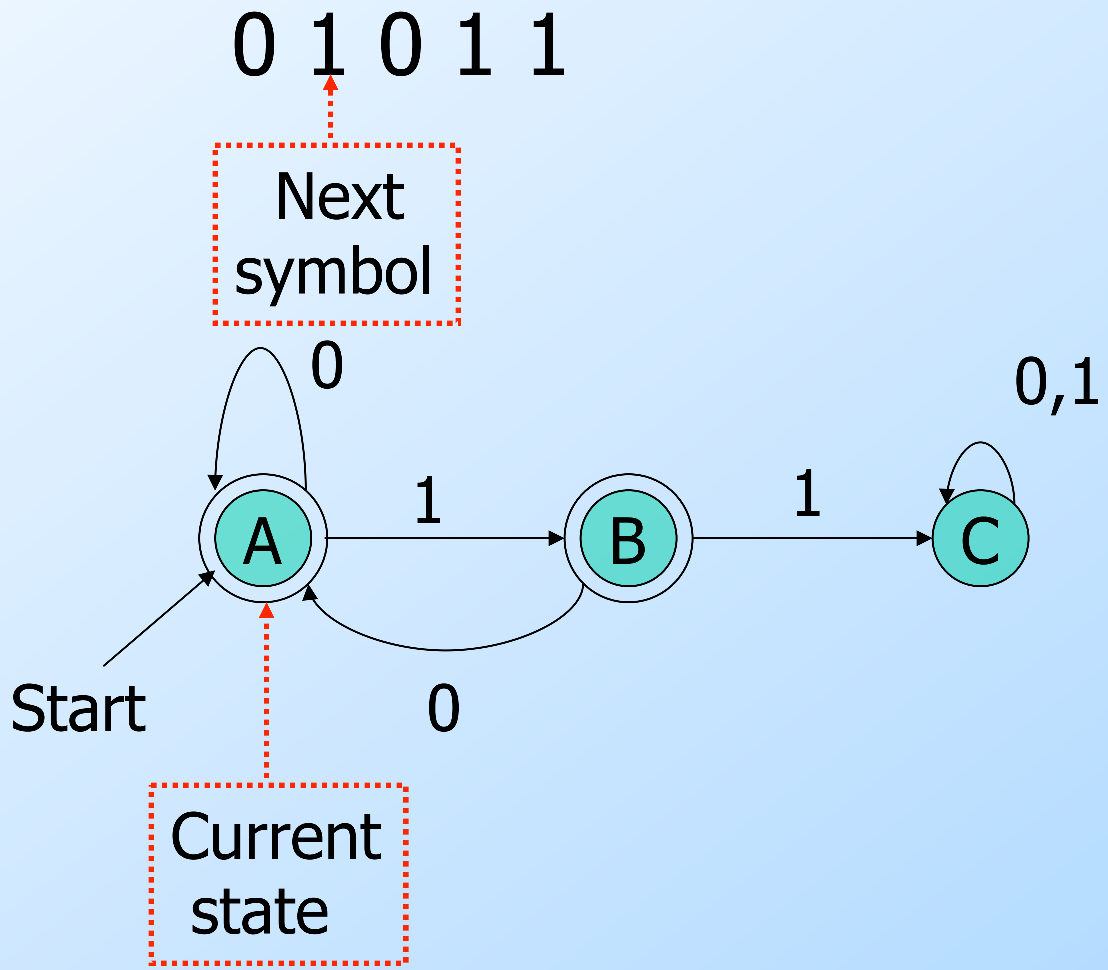
The Membership Question

- Our first decision property is the question: "is string w in regular language L?"
- Assume L is represented by a DFA A.
- Simulate the action of A on the sequence of input symbols forming w.

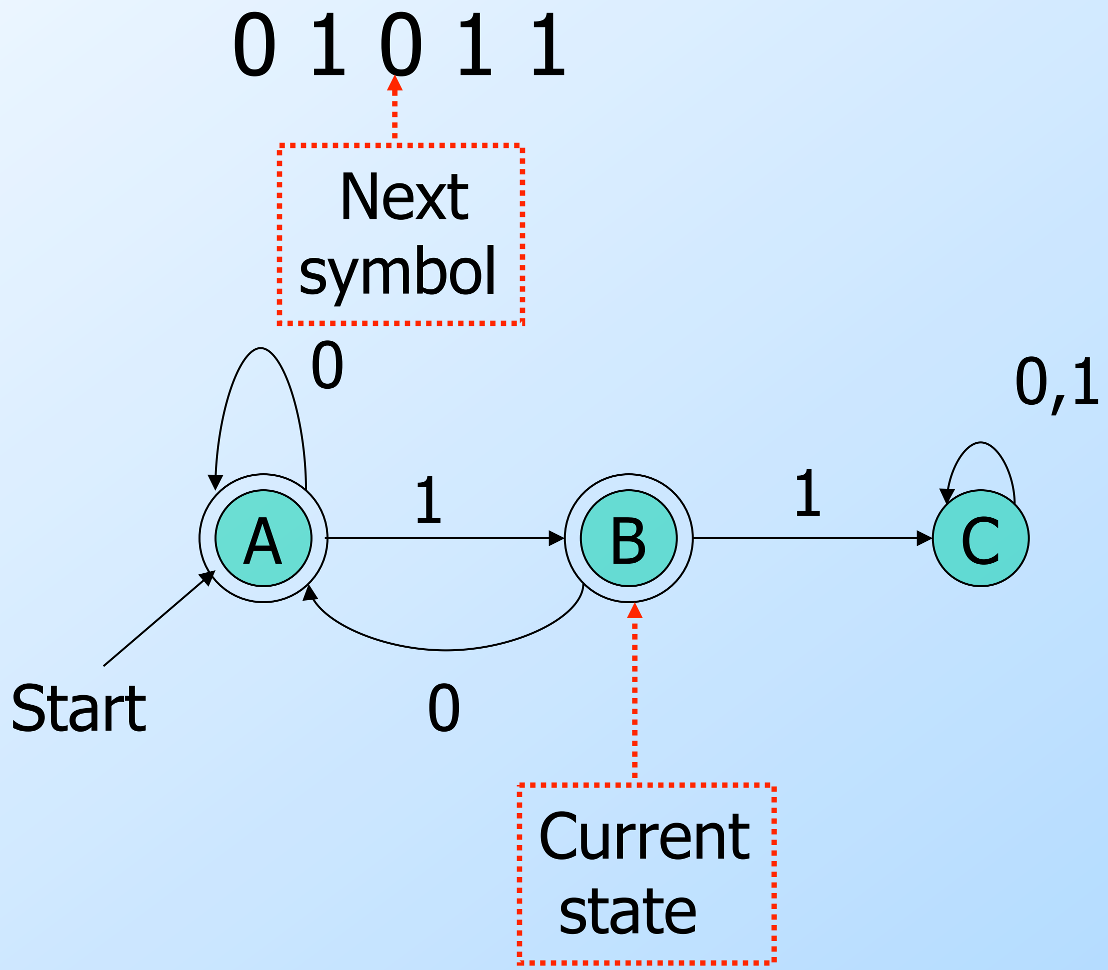
Example: Testing Membership



Example: Testing Membership

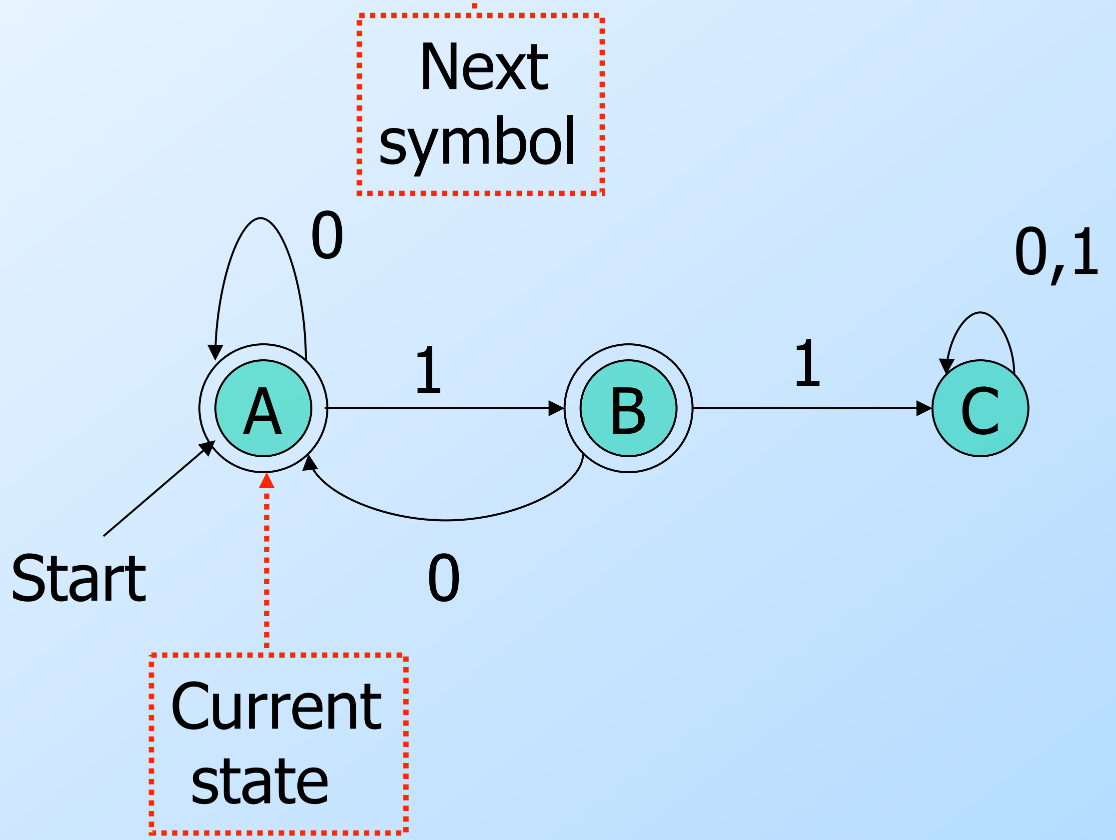


Example: Testing Membership



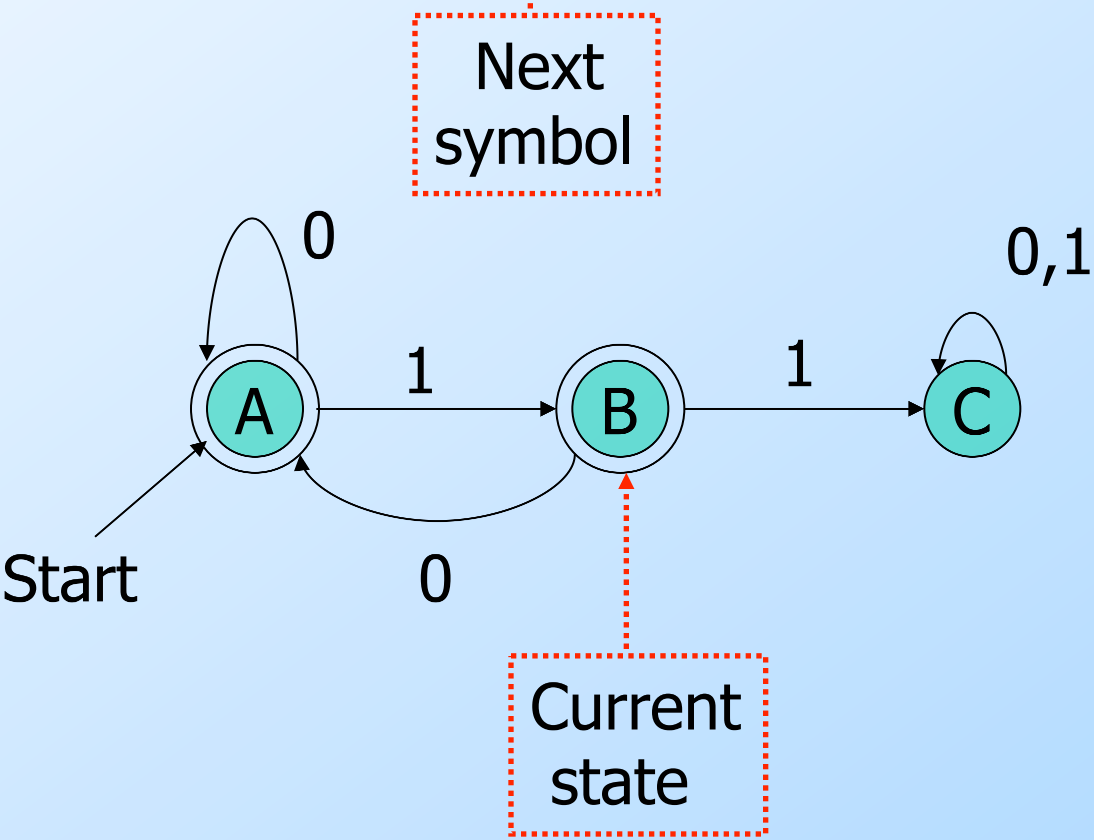
Example: Testing Membership

0 1 0 1 1

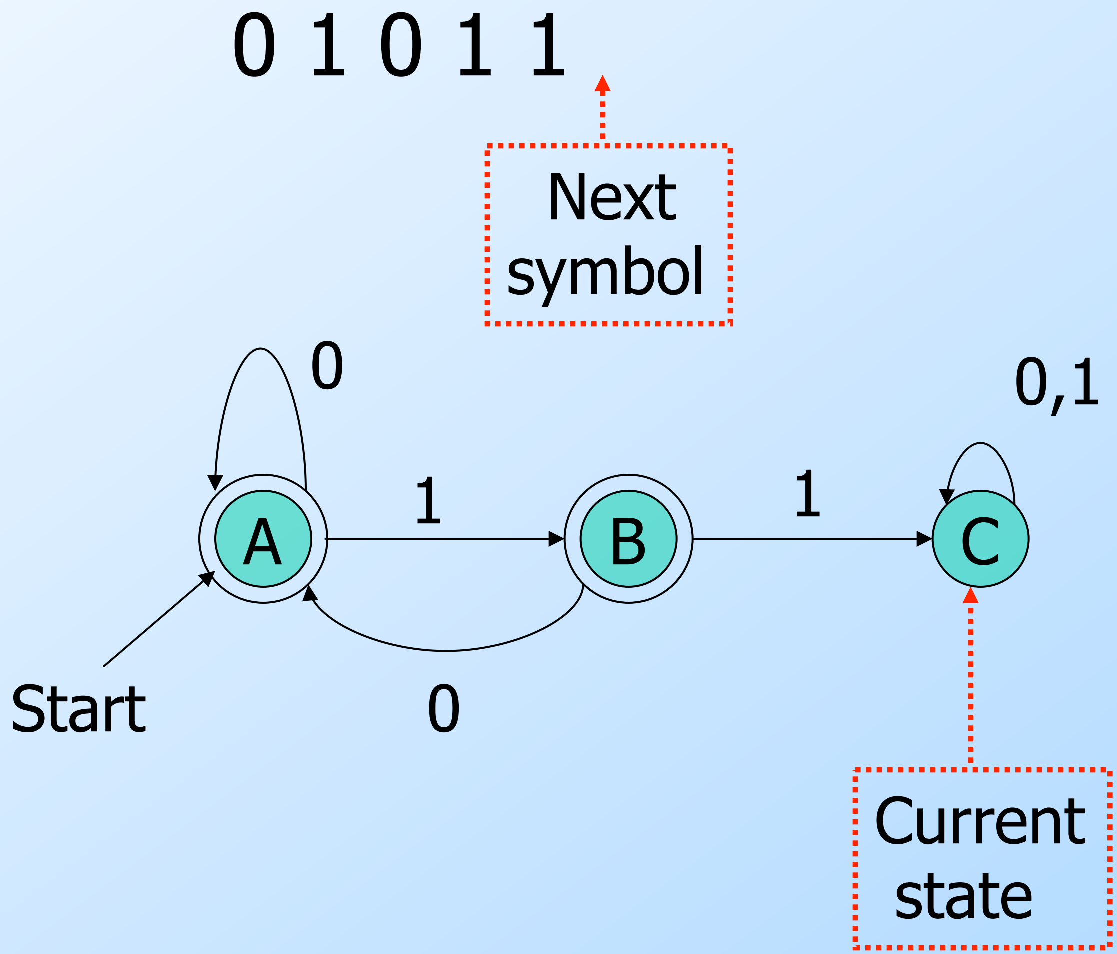


Example: Testing Membership

0 1 0 1 1

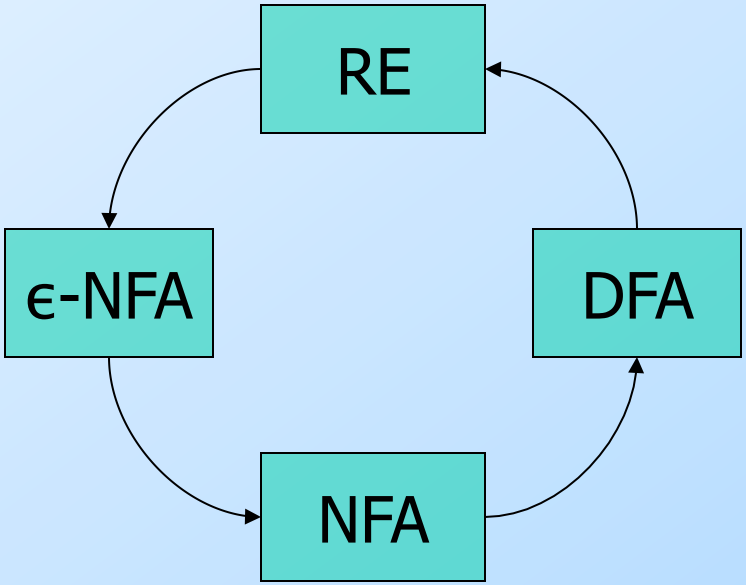


Example: Testing Membership



What if the Regular Language Is not Represented by a DFA?

- There is a circle of conversions from one form to another:



The Emptiness Problem: An Algorithm

- Given a regular language, does the language contain any string at all?
- Assume representation is DFA.
- Construct the transition graph.
- Compute the set of states reachable from the start state.
- If any final state is reachable, then yes, else no.