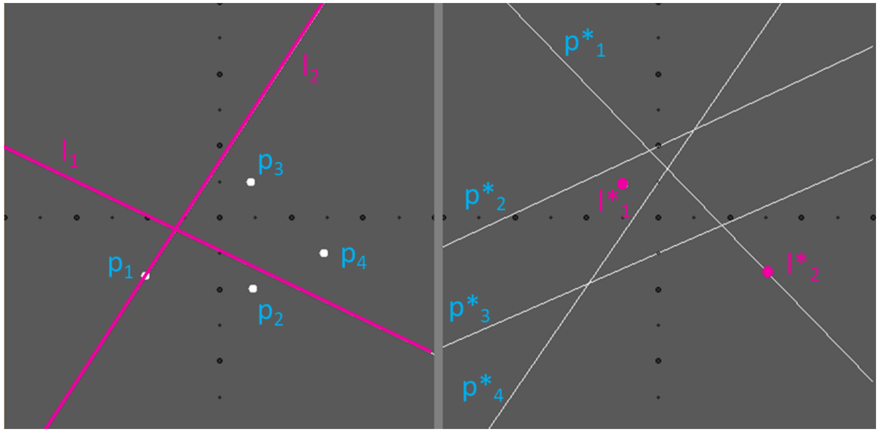
Computational Geometry



Point-Line Duality

Michael Goodrich

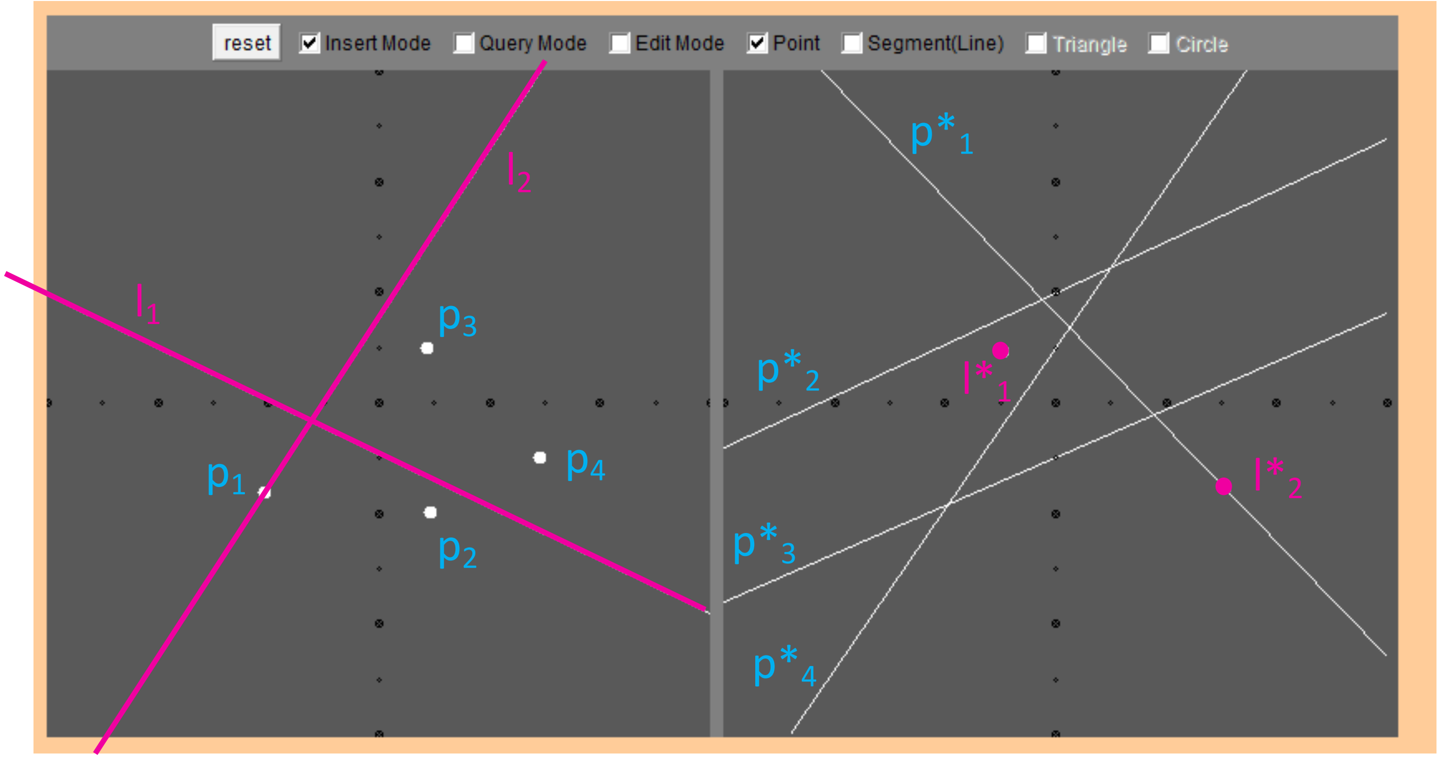
with slides from Carola Wenk

Point-Line Duality

Let $P = \{p_1, \dots, p_n\} \subseteq \mathbb{R}^2$ be a set of n points. Now define a set $P^* = \{p_1^*, \dots, p_n^*\}$ of n lines as follows:

Primal plane
Point: $p = (p_x, p_y)$
Line: $l: y = mx + b$

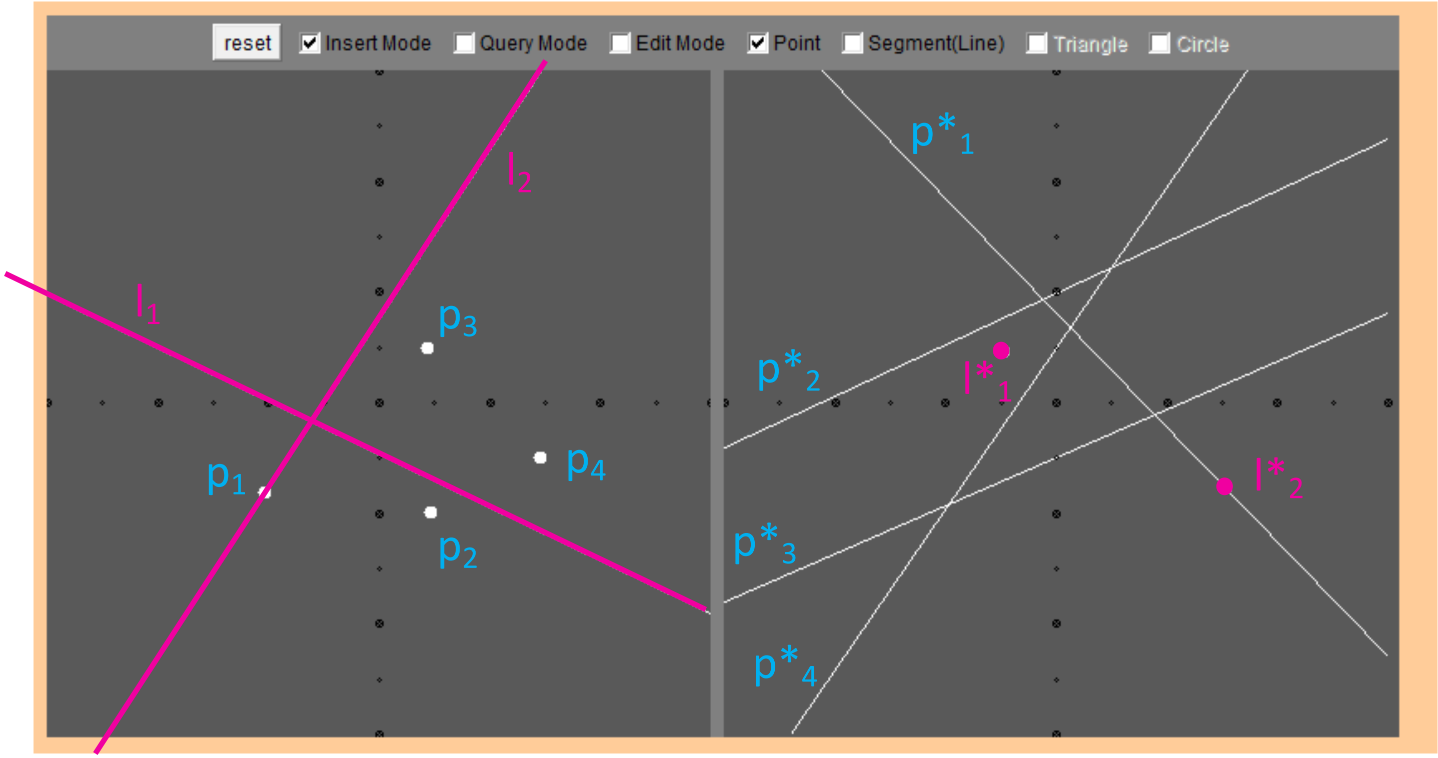
Dual plane
Line: $p^*: y = p_x x \ - p_y$
Point: $l^* = (m, -b)$



Primal plane	Dual plane
Point: $p = (p_x, p_y)$	Line: $p^*: y = p_x x - p_y$
Line: $l: y = mx + b$	Point: $l^* = (m, -b)$

Properties

Primal Dual



- $(p^*)^* = p$
- $p \in l \iff l^* \in p^*$ *incidence-preserving*
- p lies above $l \iff l^*$ lies above p^*
- $p_1 \in l_2 \iff l^*_2 \in p^*_1$
- p_3 is above $l_1 \iff l^*_1$ is above p^*_3

Primal plane	Dual plane
Point: $p = (p_x, p_y)$	Line: $p^*: y = p_x x - p_y$
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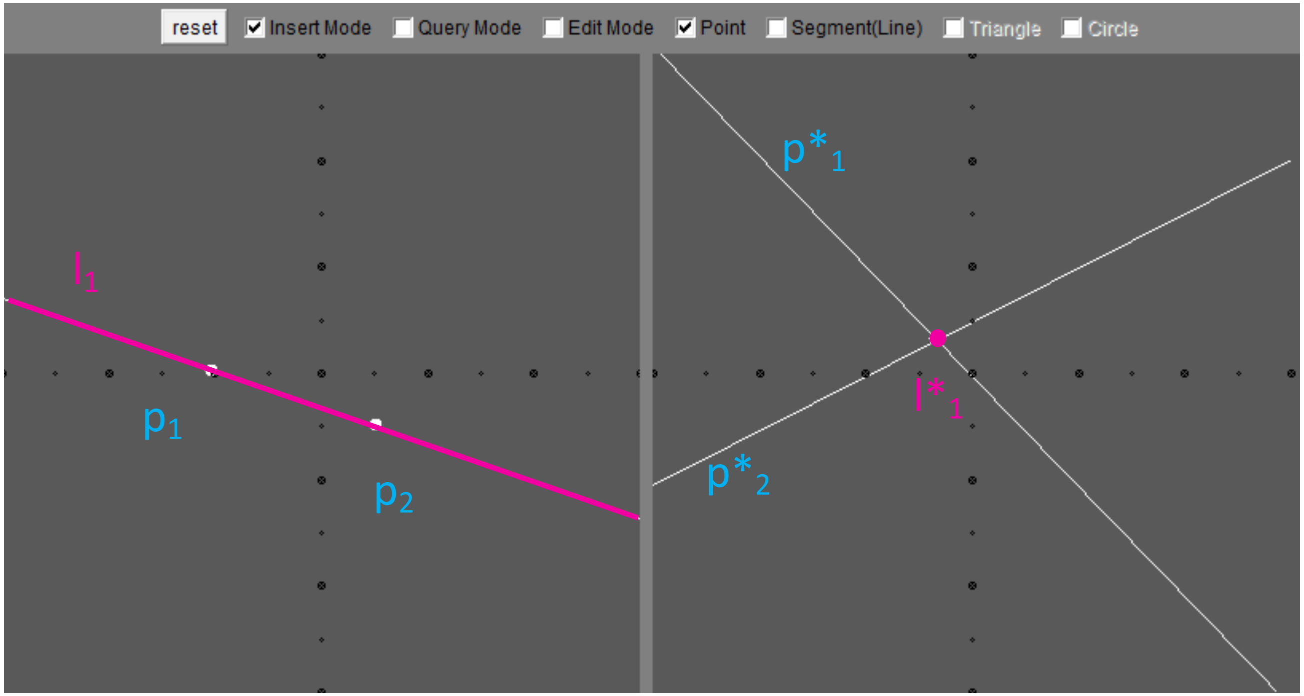
Properties

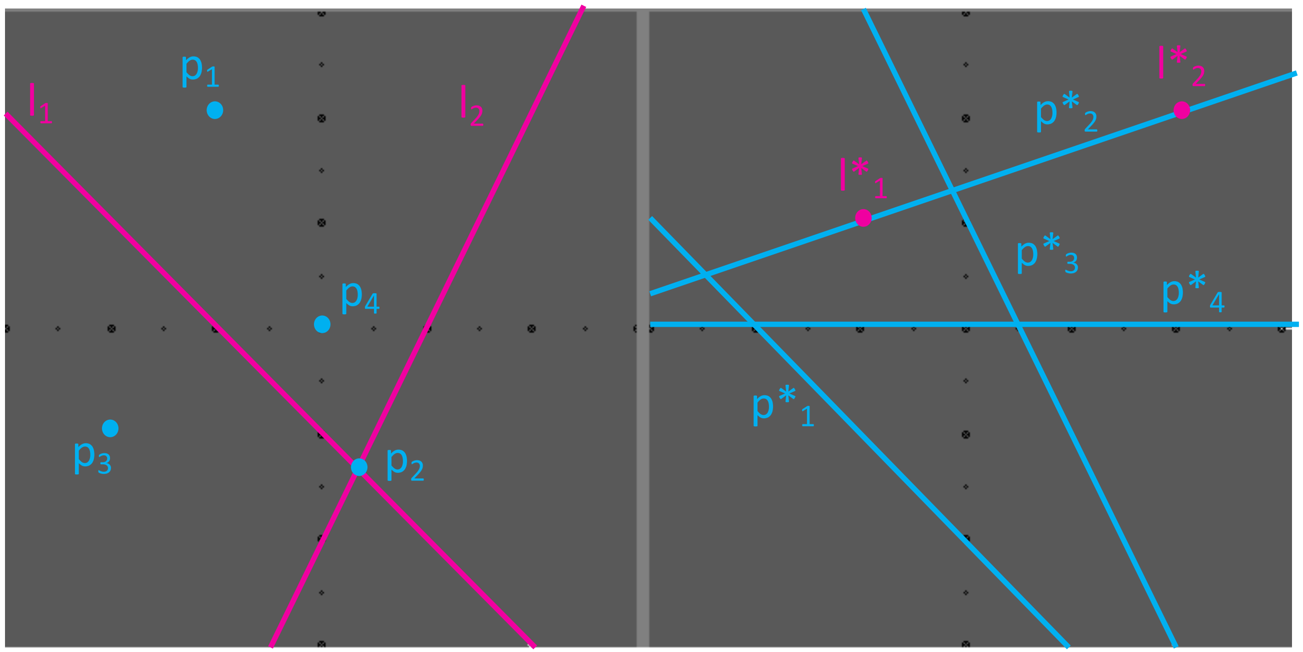
Primal

Dual



- Points p_1 and p_2 lie on line l_1
- Lines p_1^* and p_2^* contain point l_1^*

Point-Line Duality Puzzle



p_1 lies above l_1 $\Leftrightarrow$ l_1^* lies above p_1^*

p_2 lies on l_1 $\Leftrightarrow$ l_1^* lies on p_2^*

p_2 lies on l_2 $\Leftrightarrow$ l_2^* lies on p_2^*

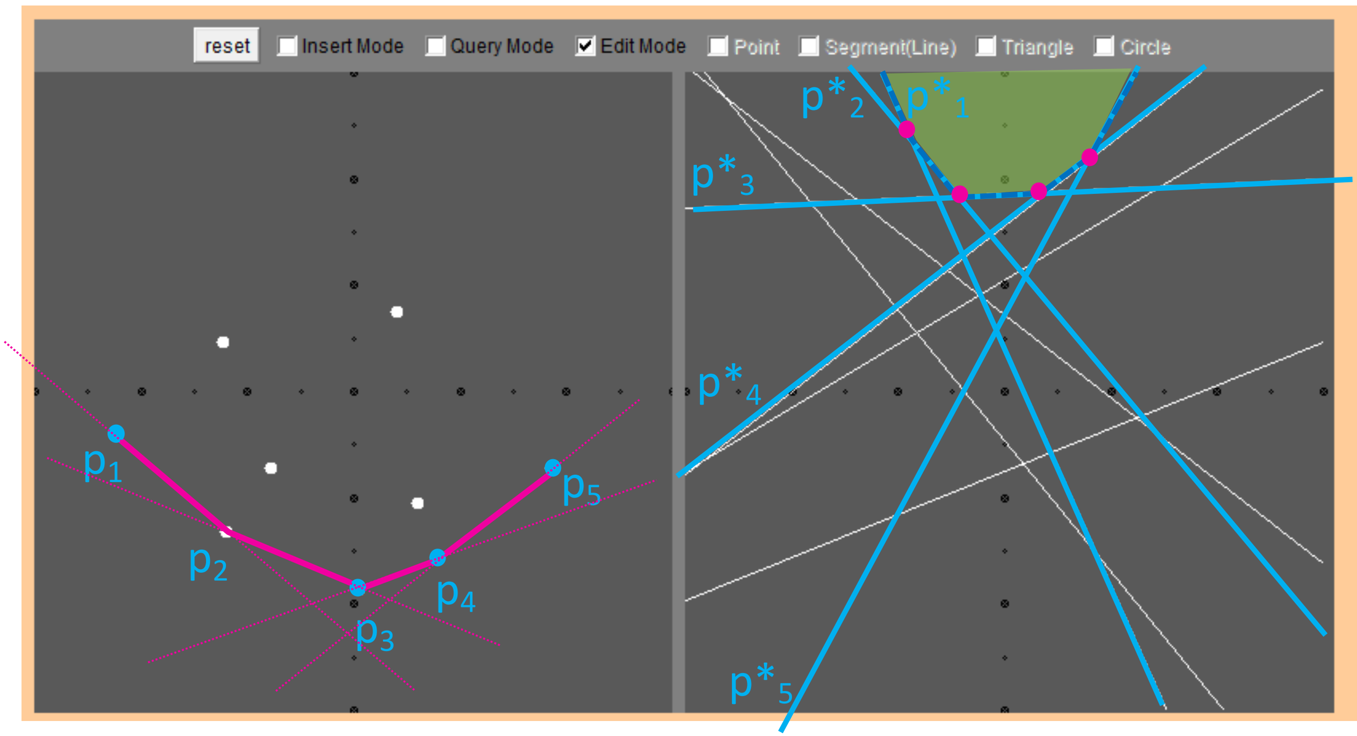
p_3 lies below l_1 $\Leftrightarrow$ l_1^* lies below p_3^*

p_4 lies above l_2 $\Leftrightarrow$ l_2^* lies above p_4^*

Primal plane	**Dual plane**
Point: $p = (p_x, p_y)$	Line: $p^*: y = p_x x - p_y$
Line: $l: y = mx + b$	Point: $l^* = (m, -b)$

LCH ≅ UE

Primal Dual



LCH lower convex hull

UE upper envelope (= pointwise maximum)
= halfplane intersection (of upper halfplanes)

- LCH = p_1, p_2, p_3, p_4, p_5
- UE = $p^*_1, p^*_2, p^*_3, p^*_4, p^*_5$