Due: Friday, June 2, 11:45pm

Please answer the following questions, each of which is worth 10 points.

1. (CS 163 students only) Show that the Grötzsch graph (see link below) can be (vertex) colored using 4 colors.

https://en.wikipedia.org/wiki/Gr%C3%B6tzsch_graph

- 2. (CS 265 students only) Show that the Grötzsch graph (see link above) cannot be (vertex) colored using 3 colors.
- 3. Prove that the chromatic number of a disconnected graph is the largest chromatic number of any of its connected components.
- 4. What is the chromatic number of a graph obtained from K_n by removing one edge, where $n \geq 3$?

https://en.wikipedia.org/wiki/Complete_graph

5. The following committees need to have meetings scheduled:

```
A={Smith, Jones, Brown, Green}
B={Jones, Wagner, Chase}
C={Harris, Oliver}
D={Harris, Jones, Mason}
E={Oliver, Cummings, Larson}
```

Are three meeting times (using multiple rooms) sufficient to schedule the committees so that no member has to be at two meetings simultaneously? Justify your answer. (Hint: explain how to model this problem using a graph.)

6. The following tours of garbage trucks in Orange County are being considered by the Orange County waste management company.

```
Tour 1: The Spectrum, Diamond Jamboree, and the Great Park
Tour 2: The Bluffs, the Great Park, the Spectrum
Tour 3: Segerstrom Center, Hoag Hospital, and UCI
Tour 4: University Center and the Great Park
Tour 5: University Center, Disneyland, and the Spectrum
Tour 6: Segerstrom Center, Angels Stadium, and Hoag Hospital
Tour 7: Disneyland, Crystal Cove Beach, and Hoag Hospital
```

Assuming the sanitation workers refuse to work more than three days a week, can these tours be partitioned so that no site is visited more than once on a given day? Justify your answer. (Hint: explain how to model this problem using a graph.)