2-3 Cuckoo Filters for Faster Triangle Listing and Set Intersection

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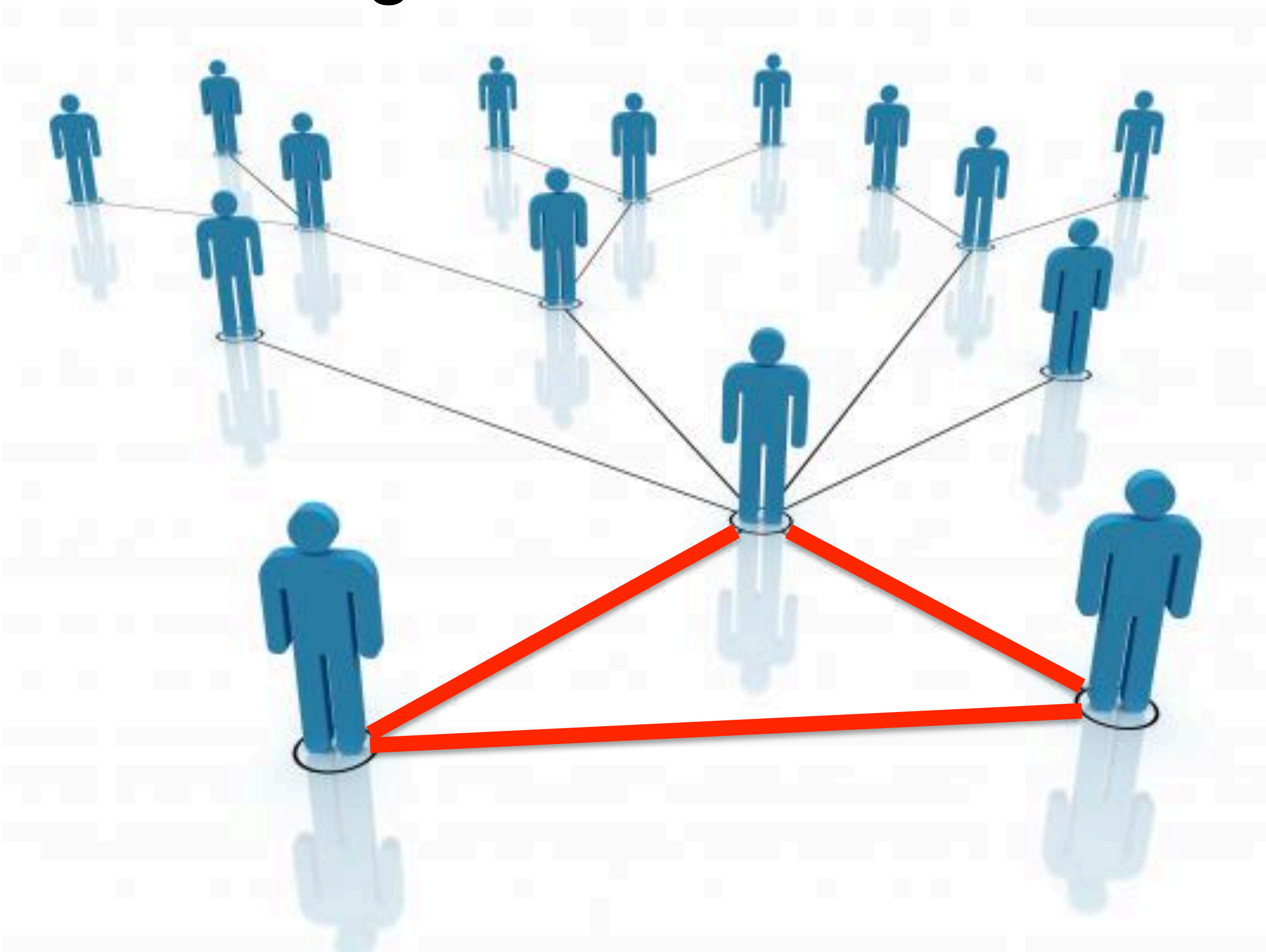
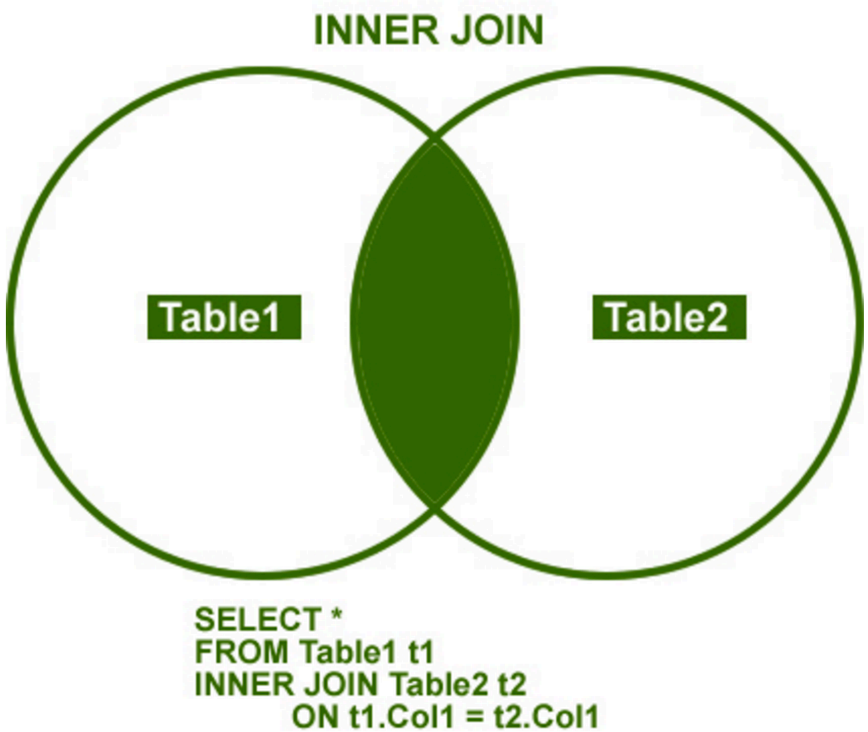
Triangle Listing Problem

- List all triangles in a network.



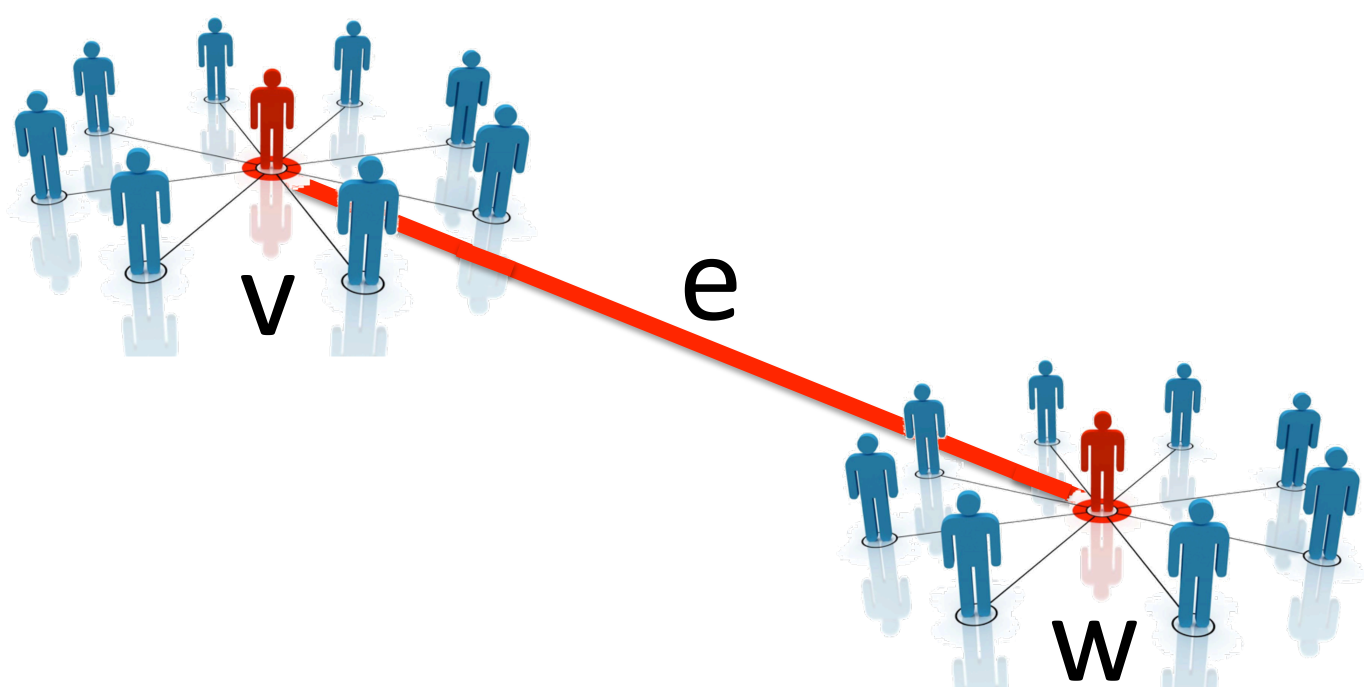
Set Intersection Problem

- Preprocess a collection of sets so as to quickly answer set-intersection queries.



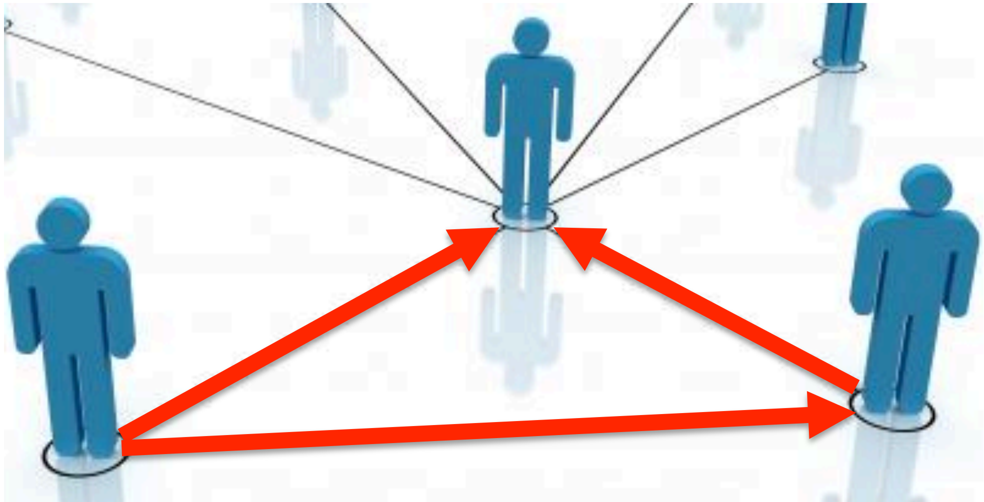
Listing Triangles Using Set Intersection Queries

- For each edge, e=(v,w):
  - Intersect the adjacency lists for v and w.



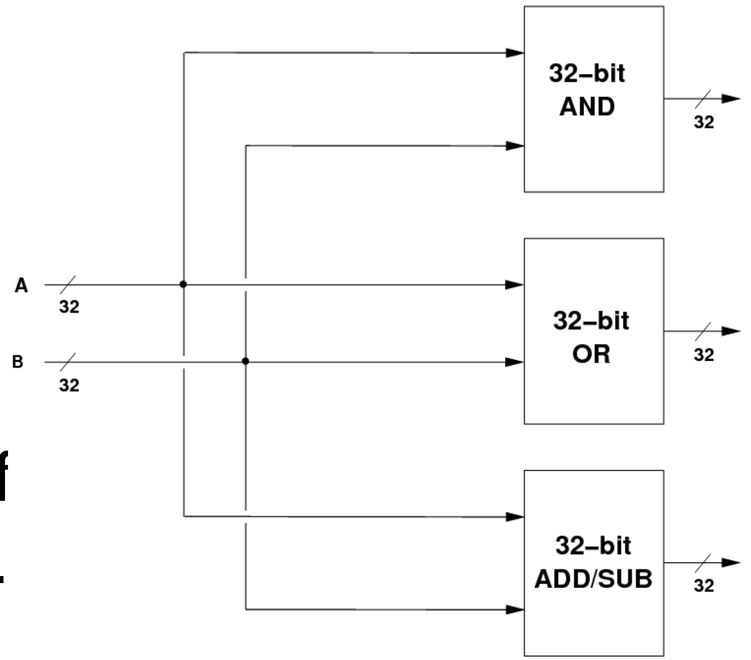
Improved Algorithm

- Order G's vertices by a **k-degeneracy order**:
  - Each vertex has out degree at most k.
  - Can be done by a greedy algorithm.
  - k is proportional to the arboricity, $\mathrm{A}(G)$, of the graph.
- Improved Algorithm: for each edge e=(v,w):
  - Intersect the out-going adjacencies for v and w.
  - Runs in $O(m\,\mathrm{A}(G))$ time
    - [e.g., see Chiba-Nishizeki '85, Ortmann-Brandes '14]



Further Improvements for Real-World Computational Models

- Take advantage of bit-level operations
- This model of computation is known as the **word-RAM** or **practical-RAM** model.
  - E.g., use built-in operations of C, C++, Java, Python, T-SQL.
- Related to external-memory model

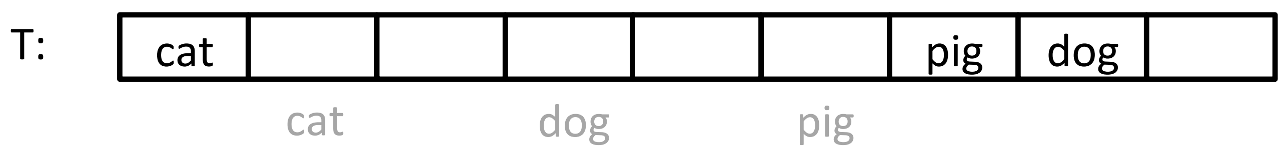


Previous Results / Our Results

- Kopelowitz et al. ‘15 introduce a set intersection data structure and use it to list the triangles in a graph G in expected time **$O(m(A(G) \log^2 w)/w + \log w + k)$.**
  - w is the word size (in bits), k is output size.
- We introduce a new set intersection data structure for listing the triangles in a graph in **$O(m(A(G) \log w)/w + k)$** expected time.
- We also give an external-memory version.

Review: Cuckoo Hash Tables

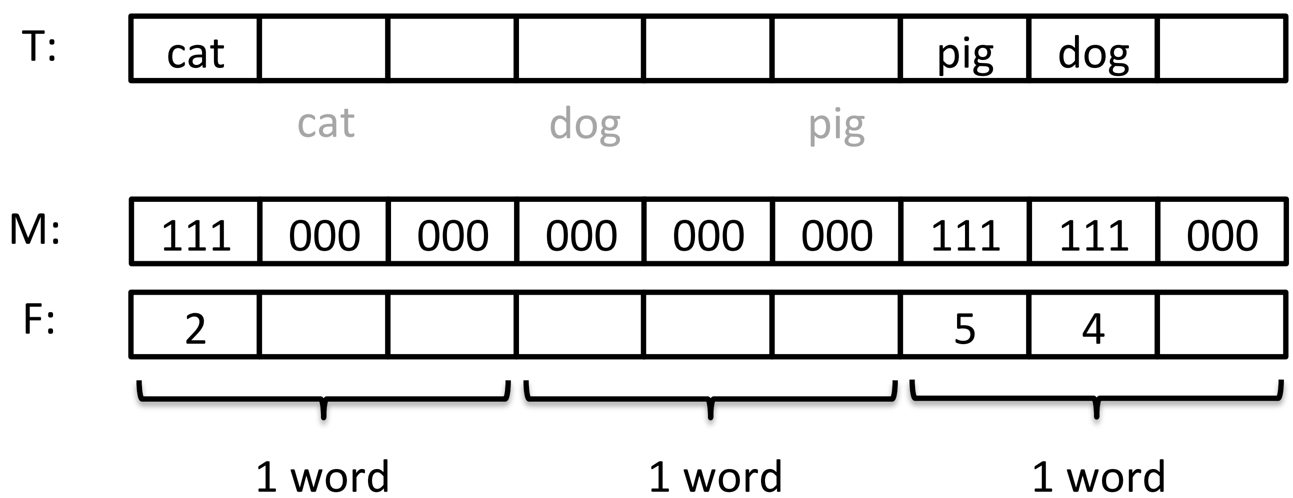
- Each element is mapped to **1-out-of-2** possible locations by a pair of random hash functions.



- Insertions "bounce" elements to their alternative location as needed. [Pagh, Radler '04]
- We can add a small **stash** cache of size s to reduce the probability of failure to be $1/n^s$.
  – Analysis involves characterizing the "**cuckoo graph**" defined by pairs of locations defined by each element's 2 locations [Kirsch et. al '10]

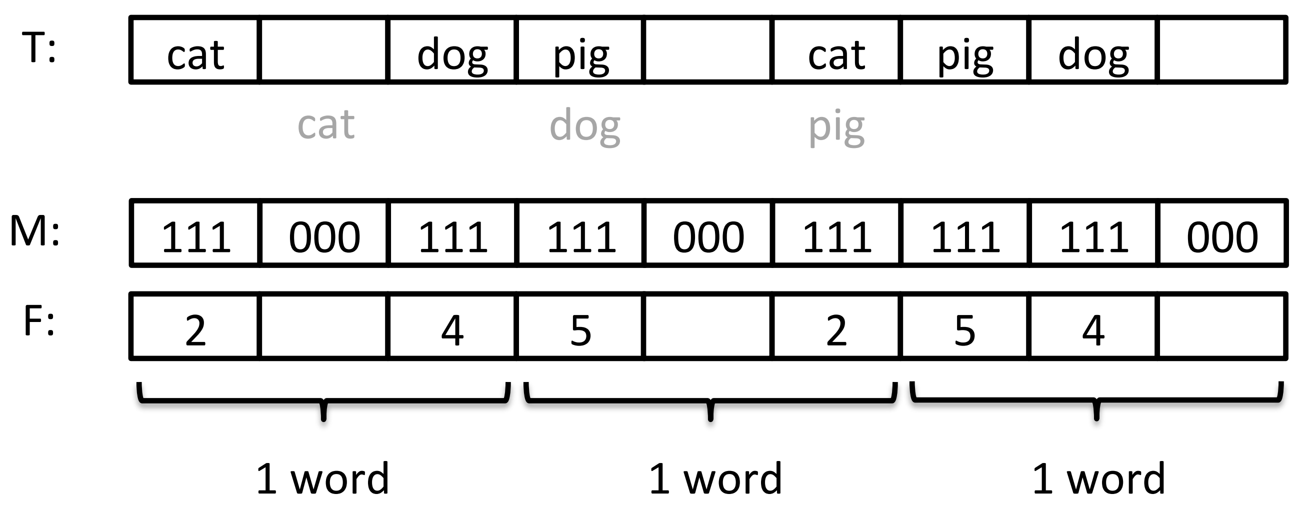
Review: Cuckoo Filters

- A cuckoo table and parallel **cuckoo filter**.
  - See Fan et al. ‘14, Eppstein ‘16.
- Provides improved functionality over Bloom filters.



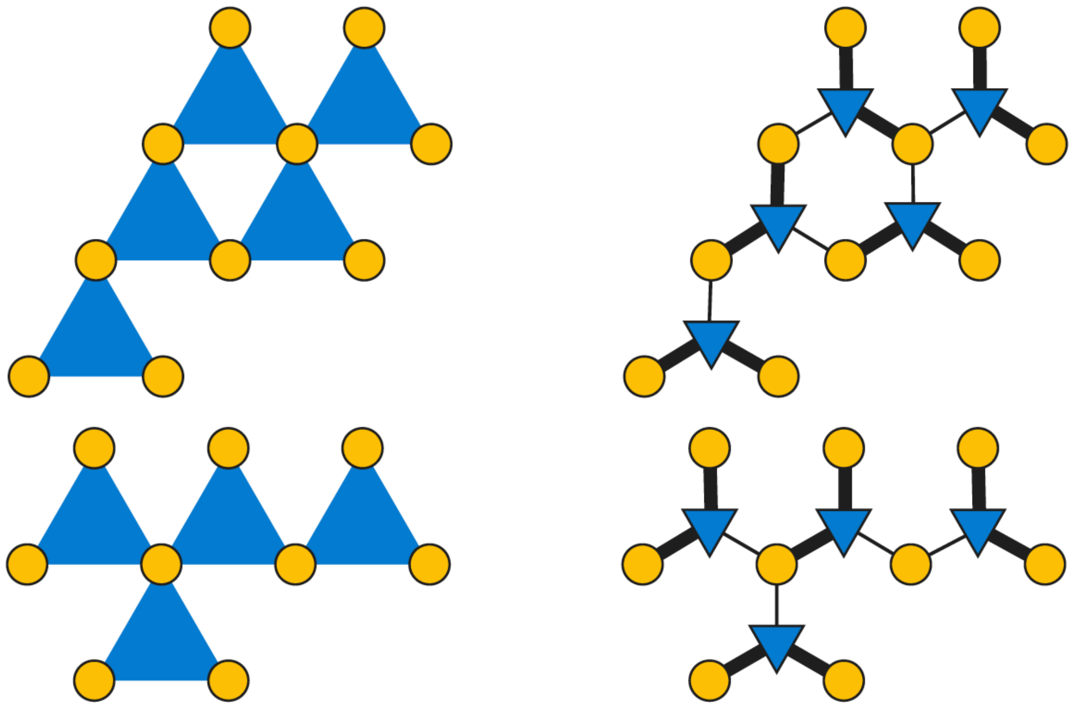
New: 2-3 Cuckoo Filters

- A cuckoo table and parallel cuckoo filter, where each element is stored in **2-out-of-3** possible locations.



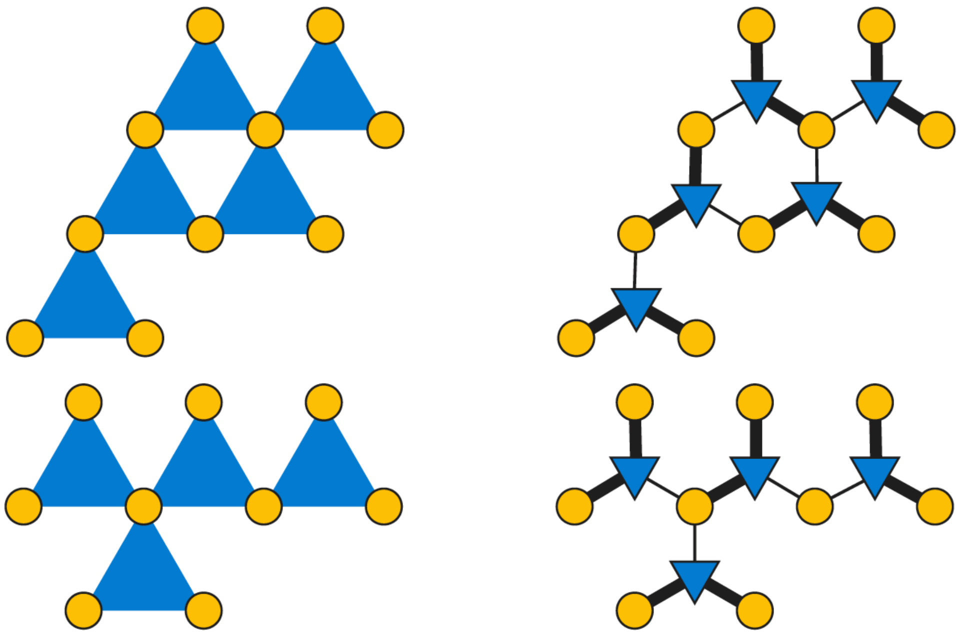
Cuckoo Hypergraph

- Instead of analysis w/ a cuckoo graph, we use a **cuckoo hypergraph** (which is 3-uniform).
- Our 2-out-of-3 paradigm corresponds to a two-assignment.



Correctness

- **Correctness Lemma**: Any 3-uniform hypergraph has a 2-assignment if and only if each of its connected components is acyclic or unicyclic.



Small Components

- Let C_v be the component containing v in the randomly chosen hypergraph, and let E_v represent the set of edges in C_v.

LEMMA 7. *There exists a constant* $\beta \in (0,1)$ *such that for any fixed vertex* v *and integer* $k > 0$,

$$\Pr(|E_v| \geq k) \leq \beta^k.$$

  - Implies that components are small with high probability.

Cyclomatic Numbers

- Let the **cyclomatic number** α(H) be the smallest number of triangles which should be removed from a 3-uniform hypergraph H in order to make H become acyclic.

LEMMA 8. *For every vertex v and $t, k \geq 1$, $k \leq m^{1/3}$,*

$$\Pr(\alpha(C_v) \geq a \mid |E_v| \leq k) \leq 2\left(\frac{126e^5k^3}{m}\right)^a,$$

2-3 Cuckoo Hashing with a Stash

- Skipping two pages of equations…

THEOREM 2. *For any constant integer $s \geq 1$, for a sufficiently large constant C, the size S of the stash in a 2-3 cuckoo hash table after all items have been inserted satisfies* $\Pr(S \geq s) = \tilde{O}(n^{-s})$.

The $\tilde{O}$ notation allows for extra polylogarithmic factors.

Mangrove Cuckoo

Black-billed Cuckoo

Yellow-billed Cuckoo

Intersecting Two 2-3 Cuckoo Filters

- At least one location **must** overlap:

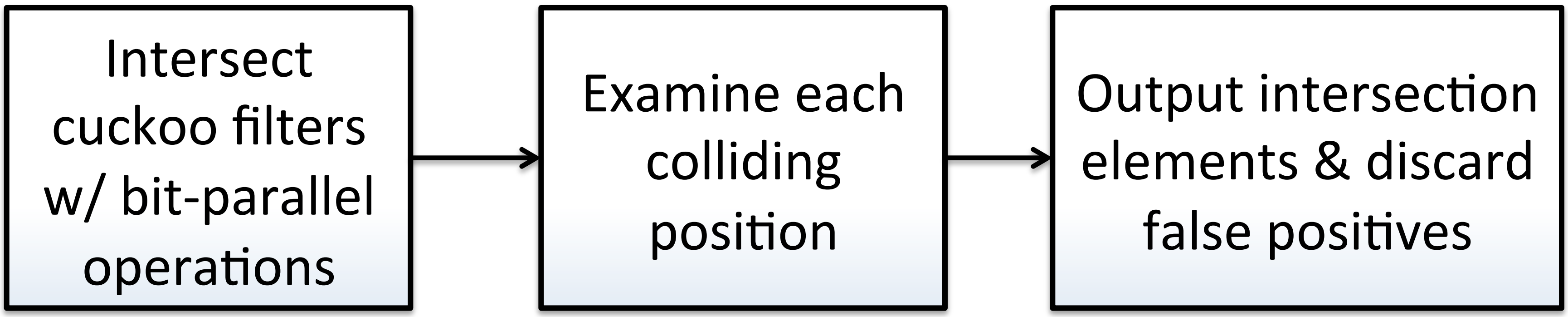
$$A = (M_i \text{ AND NOT } (F_i \text{ XOR } F_j))$$

  – We may have false-positives, though.

T_1:	cat		dog	pig		cat	pig	dog	
M_1:	111	000	111	111	000	111	111	111	000
F_1:	2		4	5		2	5	4	
	★						★	★	
T_2:	cat	cat			fox	pig	pig	fox	
M_2:	111	111	000	000	111	111	111	111	000
F_2:	2	2			4	5	5	4	
	★						★	★	

Set Intersection Analysis

- By above analysis, we can construct 2-3 cuckoo filters of size at least 2 with constant-size stashes with probability at least $1-1/w^c$.



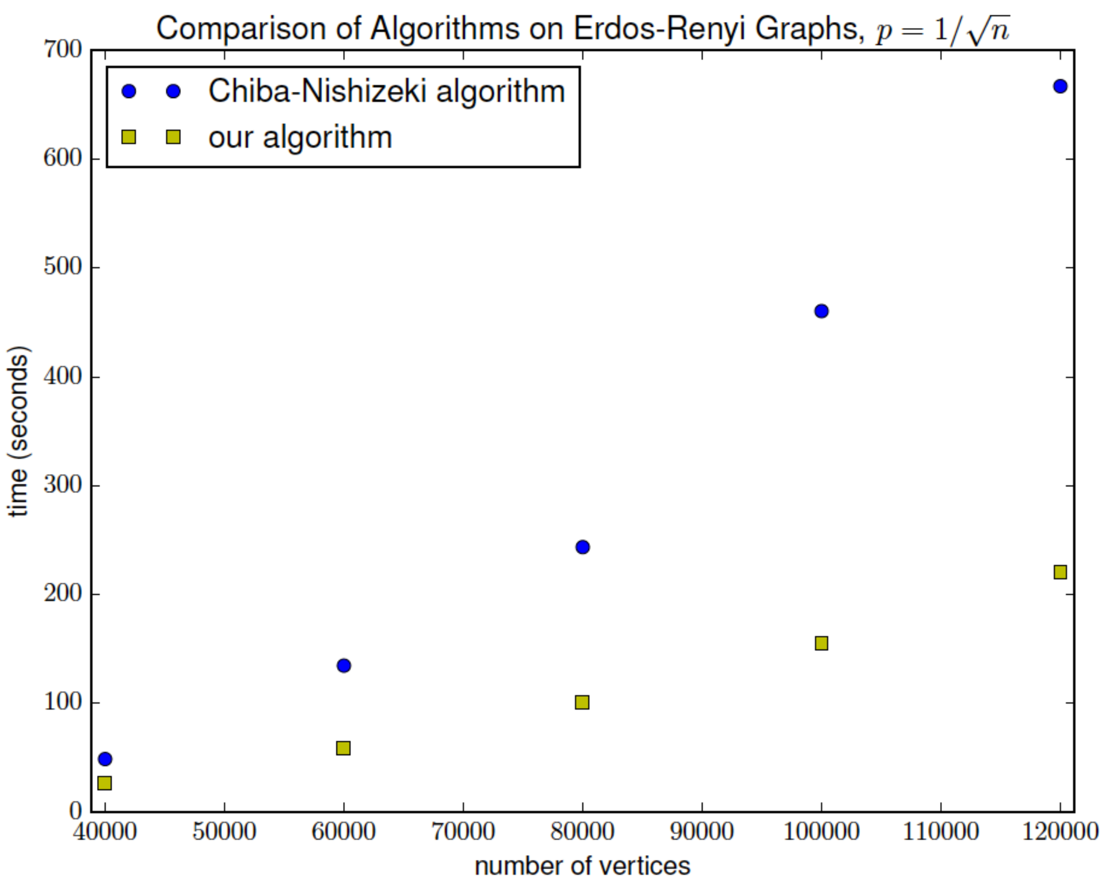
- Choosing a fingerprint size of at least log w bits implies false positives occur with probability less than 1/w.
  - Expected number of false positives is O(n/w).
- Thus, expected time for a set intersection query is **O(n(log w)/w + k)**, where k is the output size.

Triangle Listing Algorithm

- Order vertices by a k-degenerate ordering.
- Build a 2-3 cuckoo filter for each out-going adjacency list. (Each is size ($\mathcal{A}$(G)log w)/w.)
- For each edge e=(v,w):
  - Intersect the out-going adjacency lists for v and w by the above set-intersection algorithm.
  - For any adjacency lists where 2-3 cuckoo construction failed, do intersection by merging.
    - Probability of failure is at most $1/w^c$.
- Expected running time: **O(m($\mathcal{A}$(G)log w)/w + k)**.

Preliminary Experiments

- This is admittedly a theory paper, but we nevertheless did some preliminary experiments.

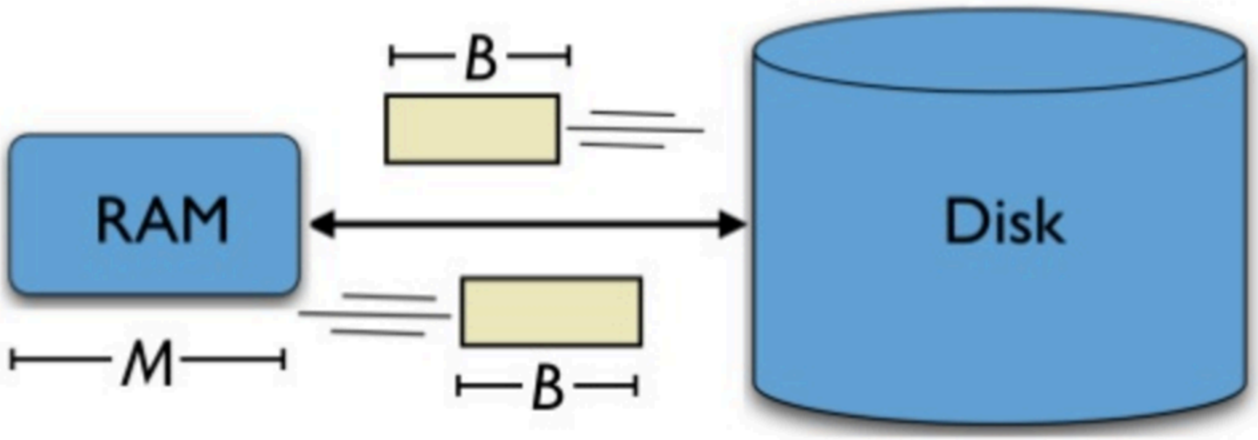


Word size = 64

Fingerprint size = 8

External-Memory Algorithm

- We also have an external-memory algorithm.



- We can list all the triangles in G using an expected number of I/Os that is:

$O(\text{sort}(n\,\mathcal{A}(G)) + \text{sort}(m(\mathcal{A}(G)\log w)/w) + \text{sort}(k))$.

Conclusion

- 2-3 cuckoo hash-filters are simple and lead to improved set-intersection queries.
- Open problem:
  - Are there other applications for 2-3 cuckoo hash-filters?