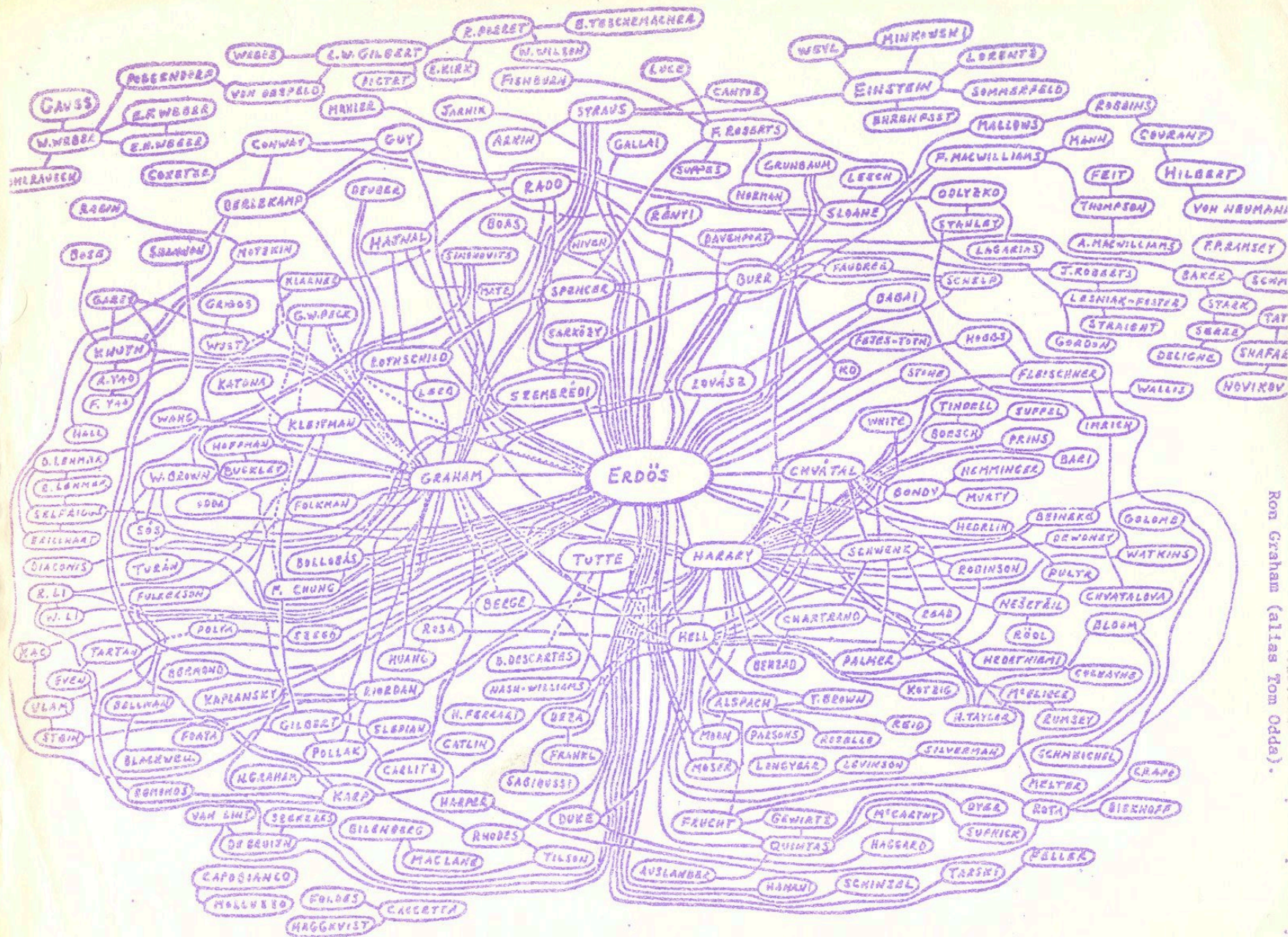


# Social Network Analysis

Some content from Lada Adamic  
and Eytan Adar

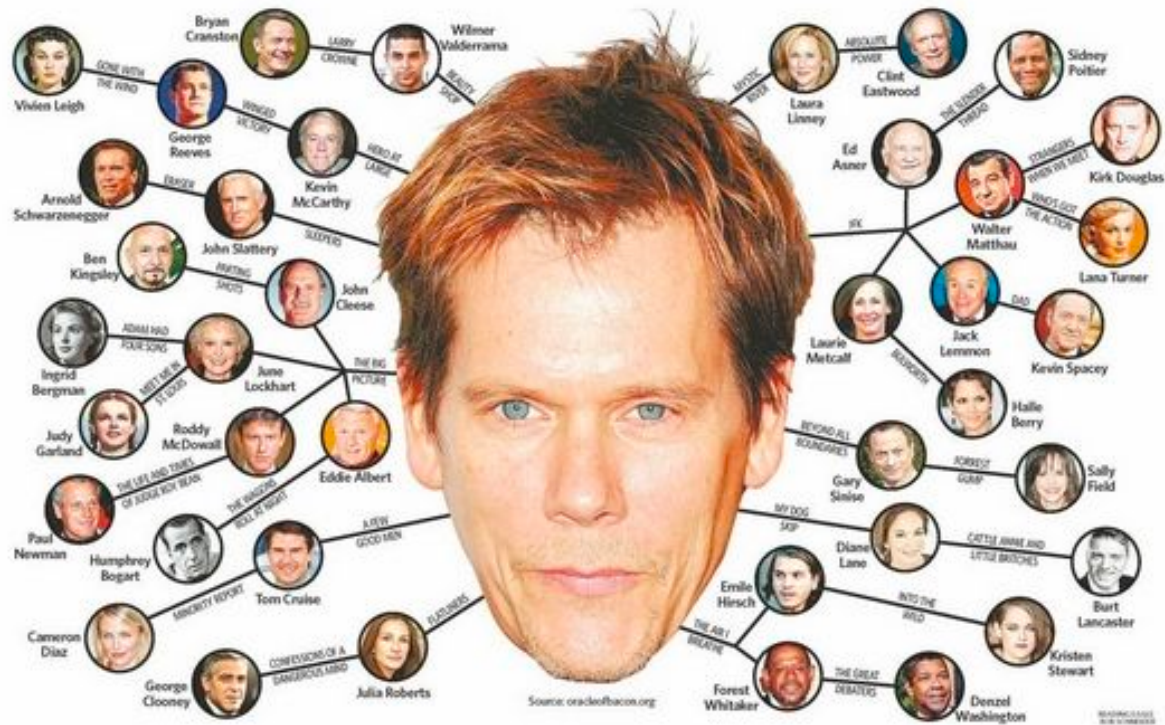


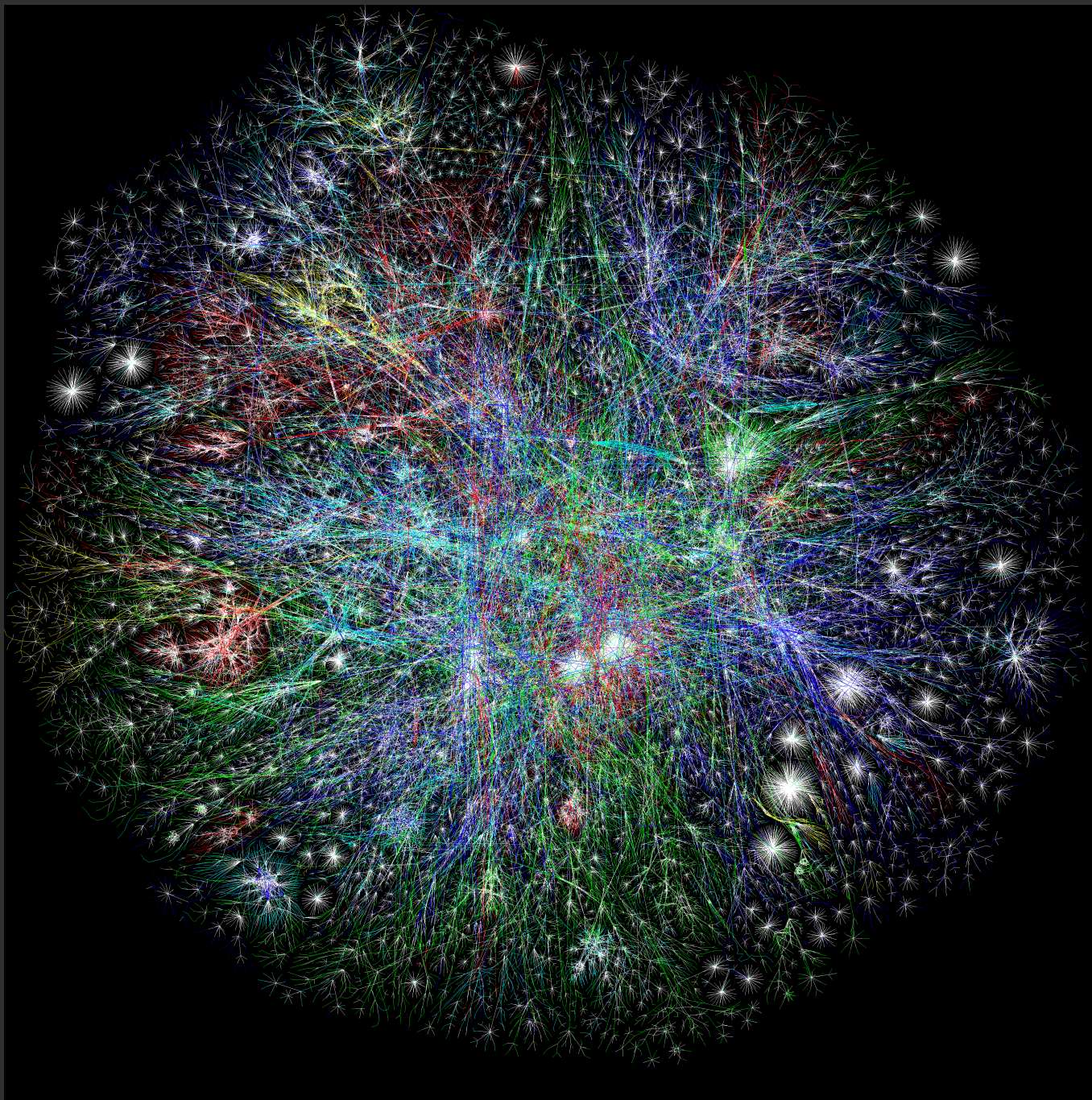
Ron Graham (alias Tom Oda).



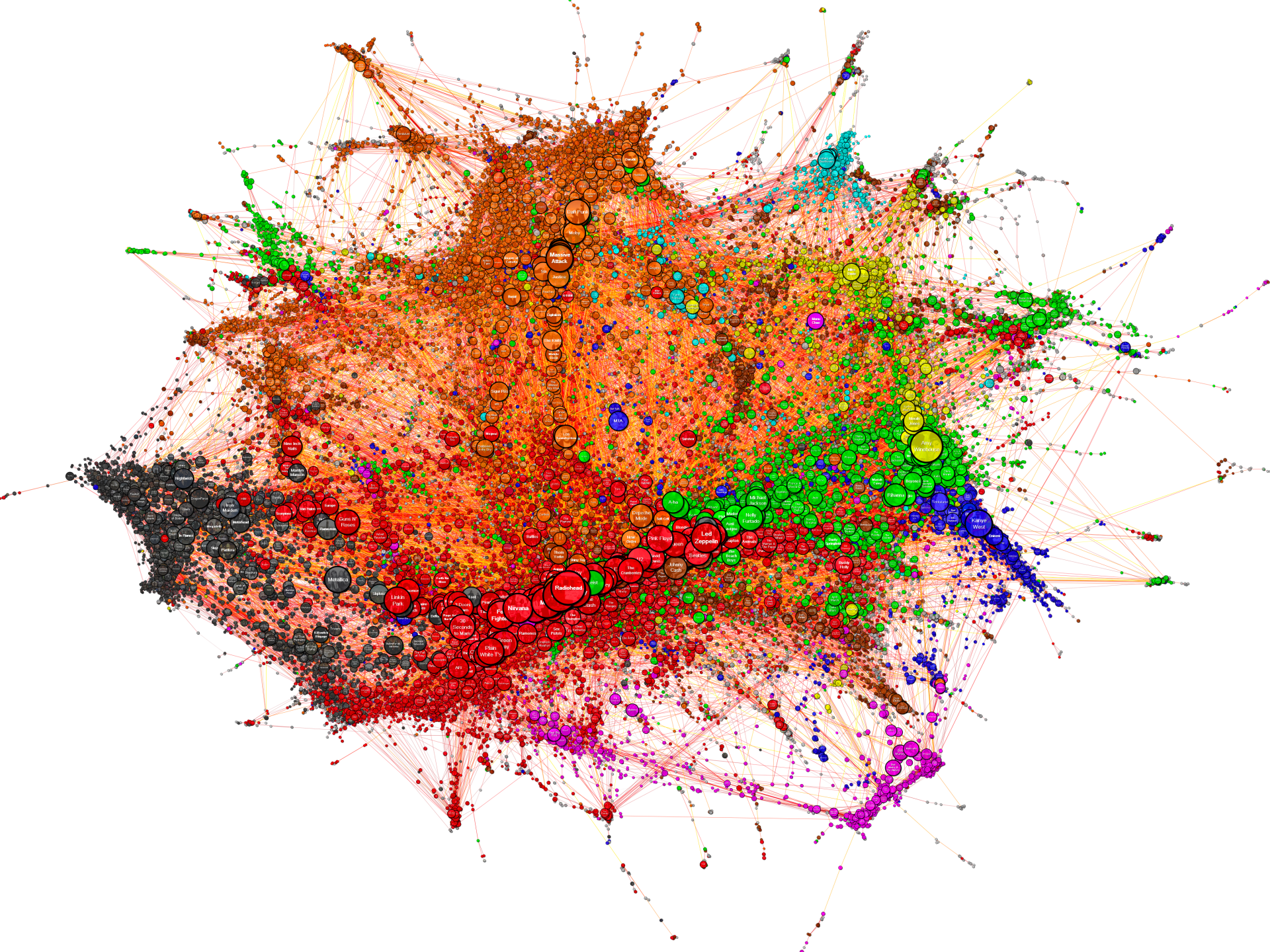
Figure 1  
To appear in Topics in Graph Theory (F. Harary, ed.), New York Academy of Sciences (1979).

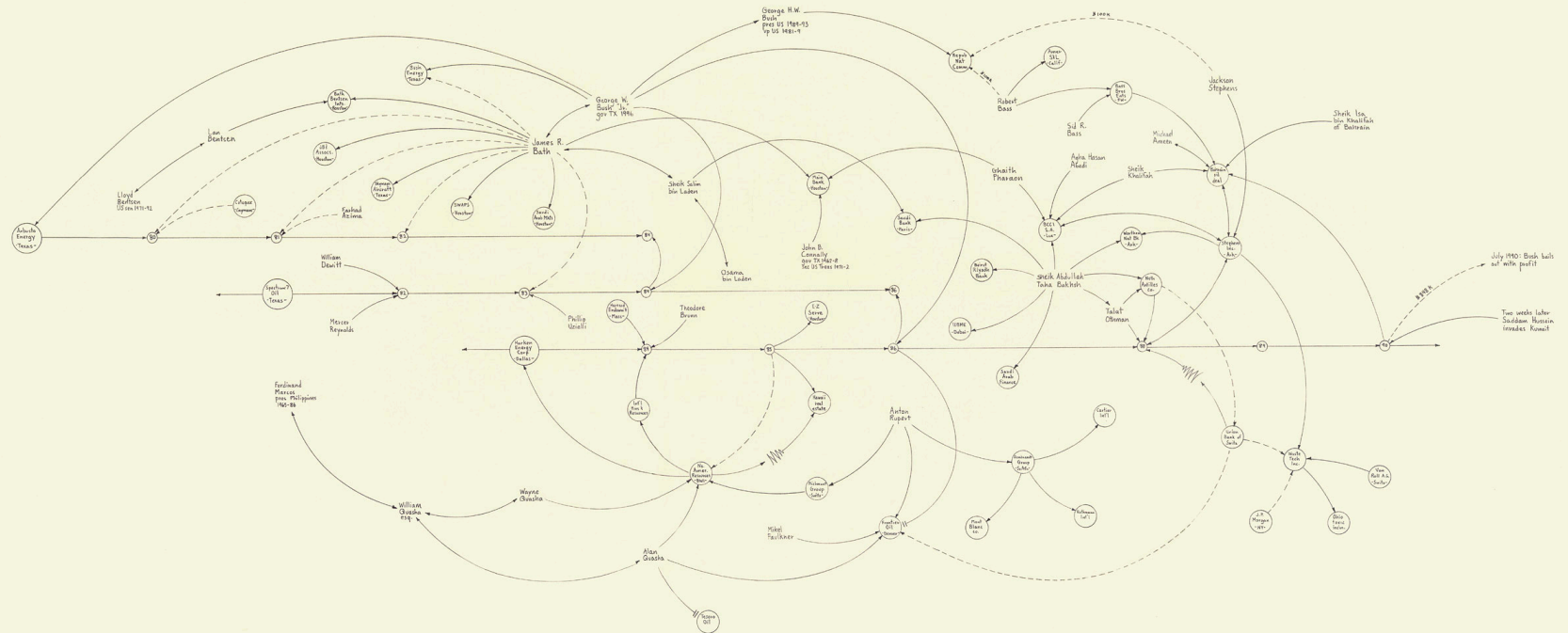




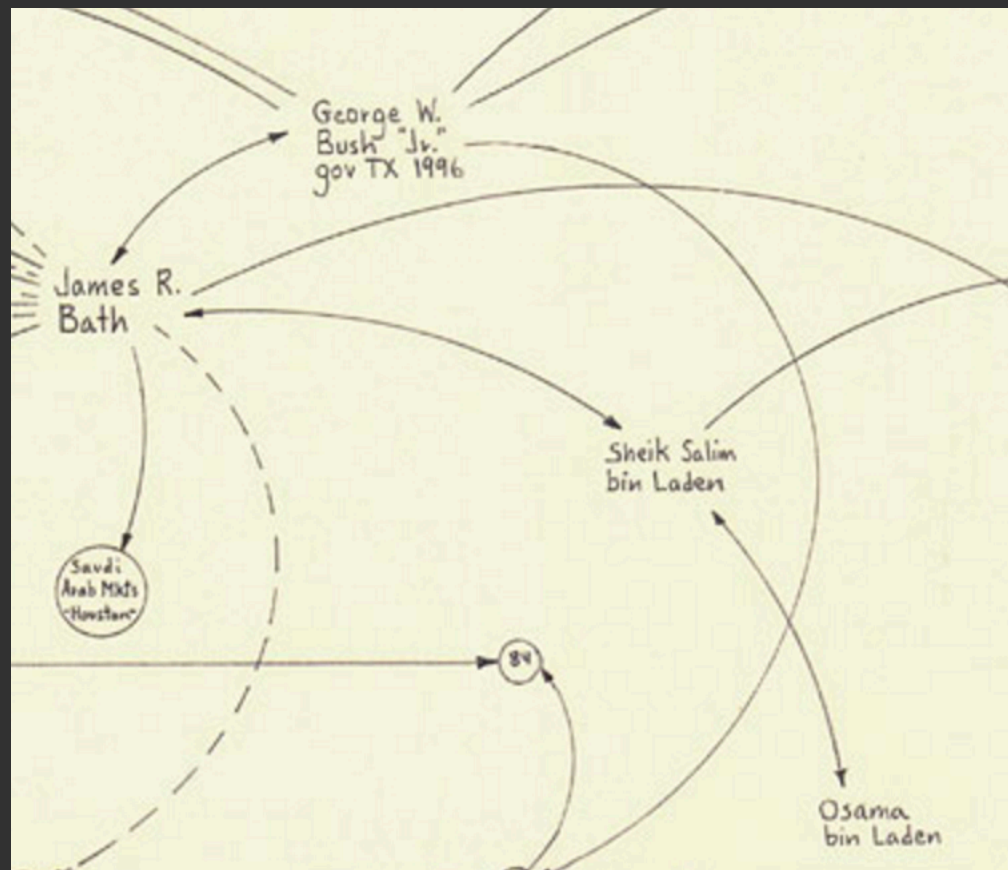






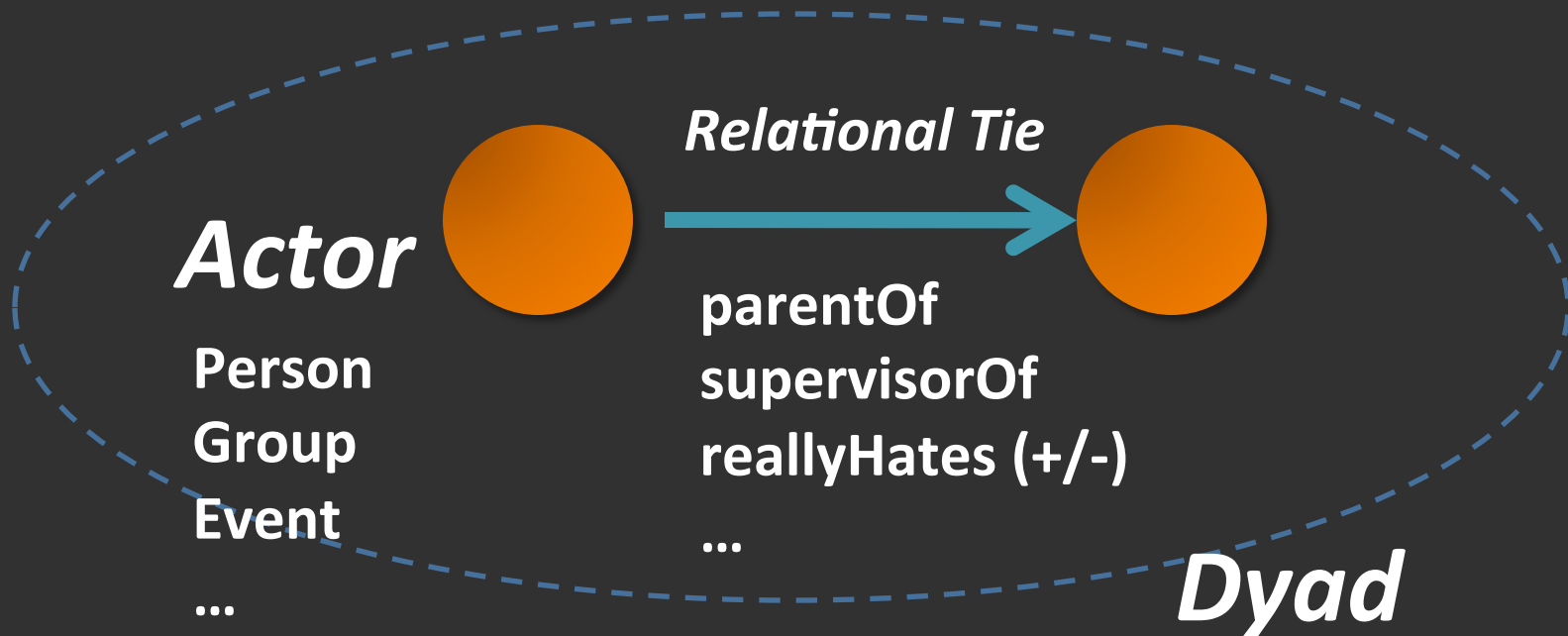






George W. Bush, Harken Energy and  
Jackson Stephens c. 1979-90  
5th version  
ML © 1999

# Vocabulary Lesson



**Relation:** collection of ties of a specific type  
(every parentOf tie)

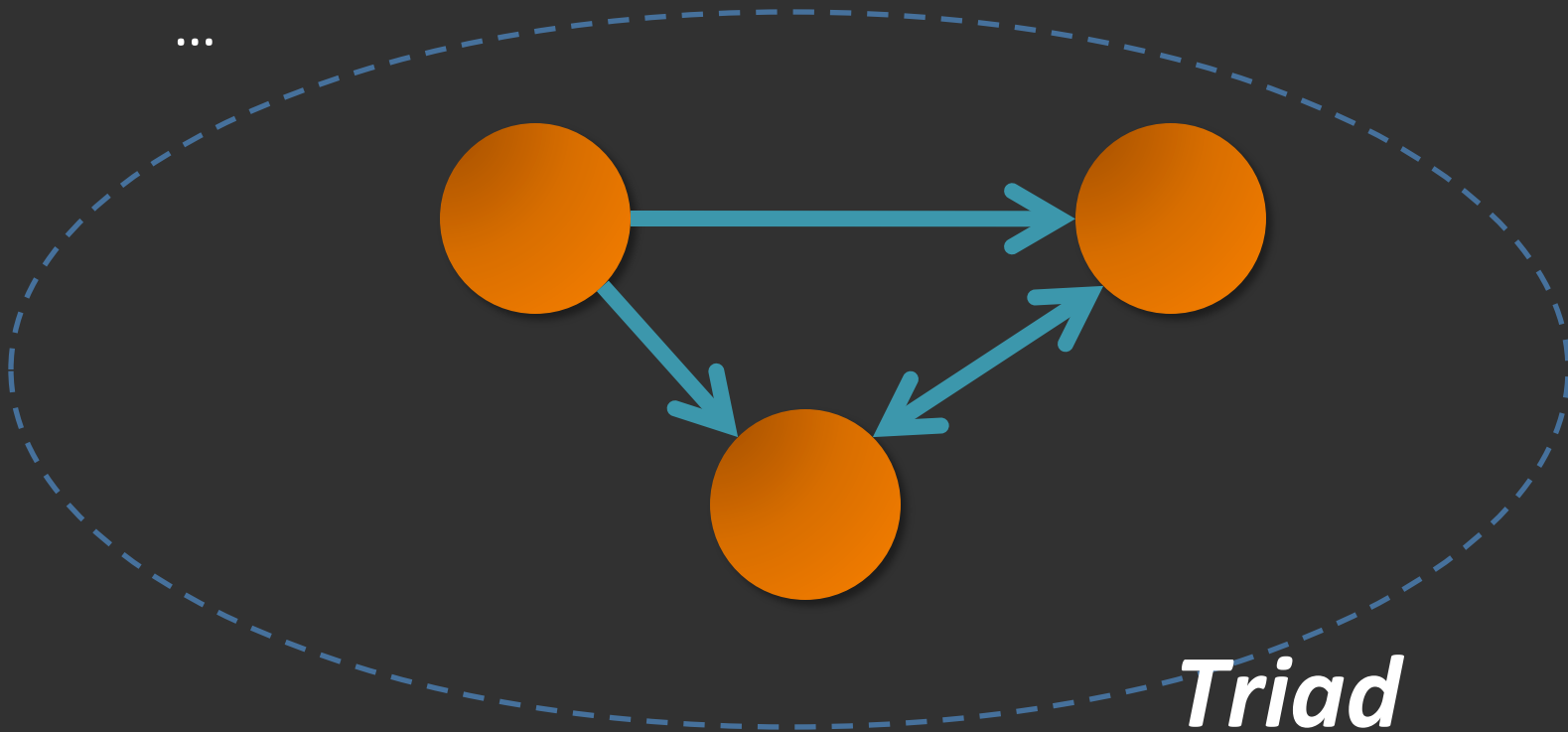


# Vocabulary Lesson

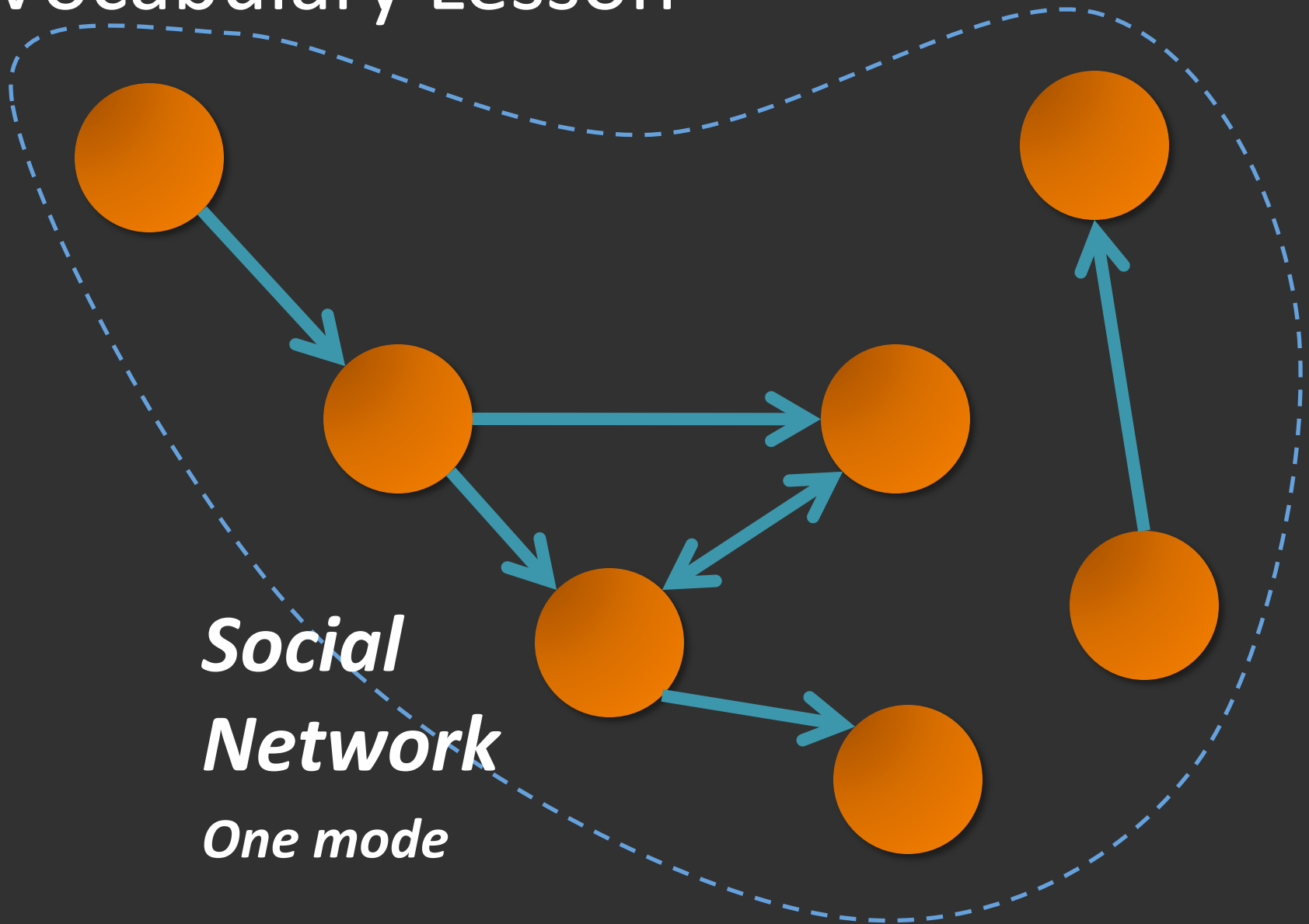
If A likes B and B likes C then A likes C (transitivity)

If A likes B and C likes B then A likes C

...

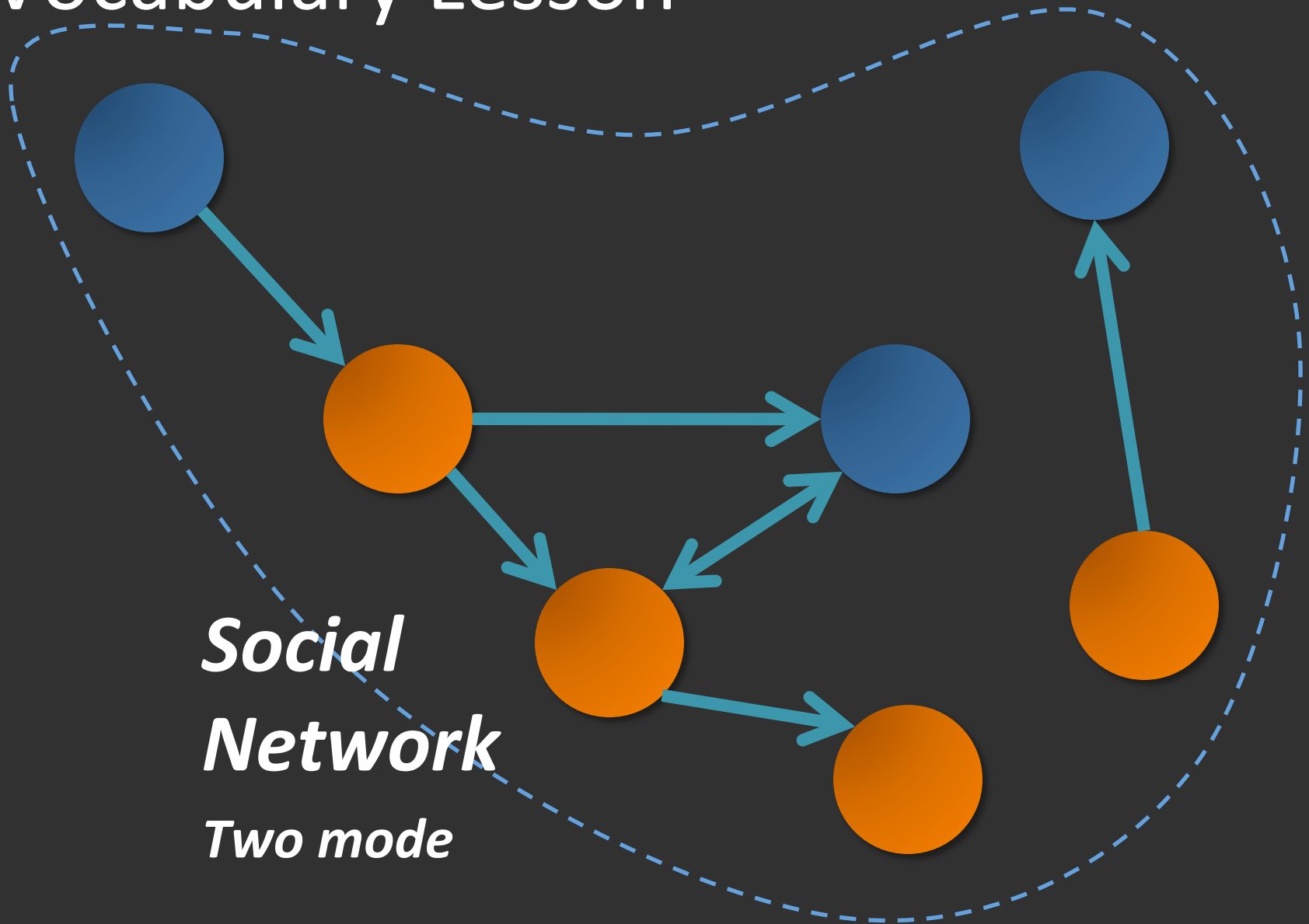


# Vocabulary Lesson



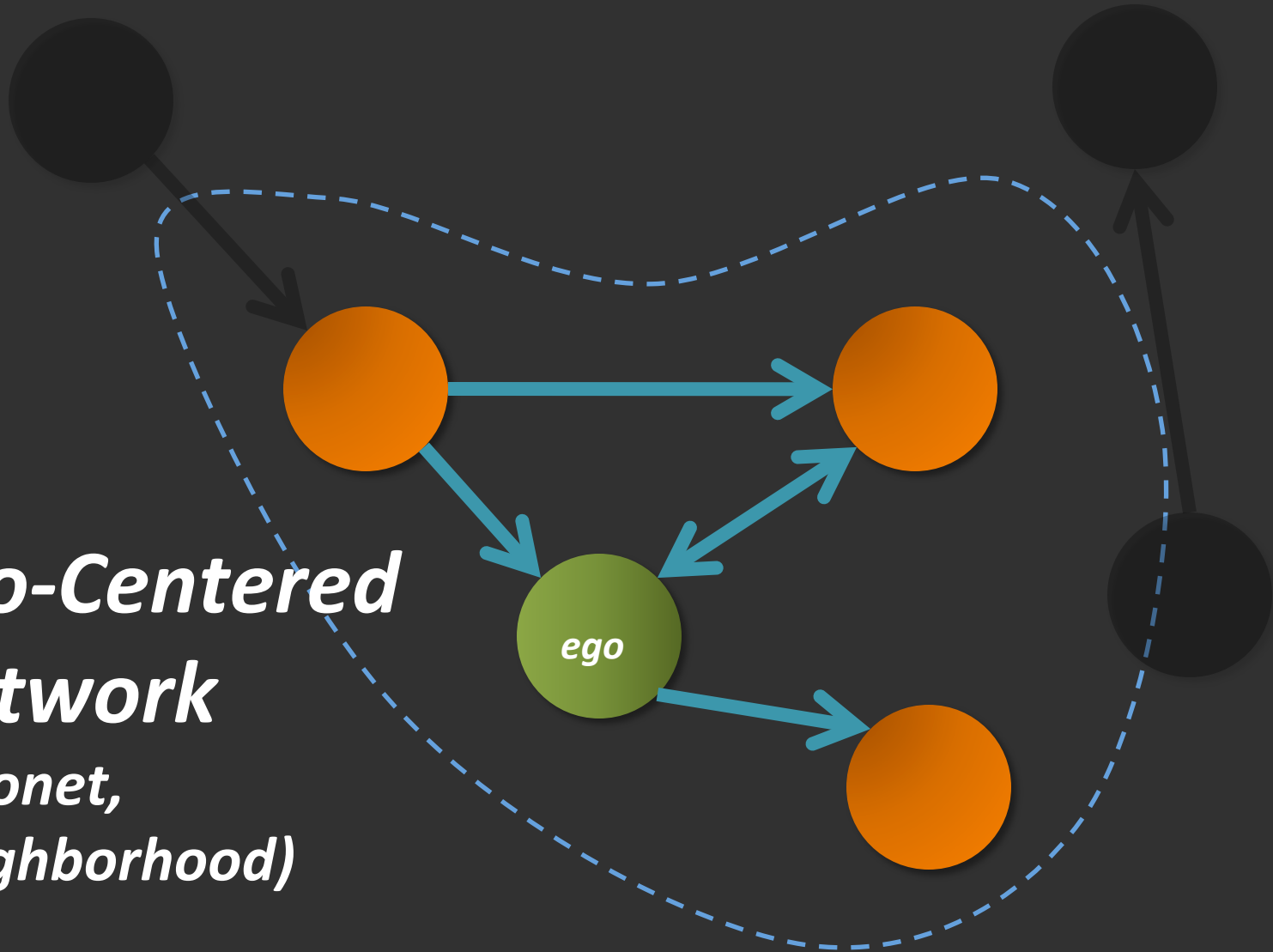


# Vocabulary Lesson



# Vocabulary Lesson

***Ego-Centered  
Network***  
(egonet,  
neighborhood)





# Describing Networks

- *Geodesic*
  - `shortest_path(n,m)`
- *Diameter*
  - $\max(\text{geodesic}(n,m))$   $n,m$  actors in graph
- *Density / Sparsity*
  - Number of existing edges / All possible edges
  - Degeneracy (number  $k$  such that every subgraph has a vertex of degree  $k$  or less)
    - Related to arboricity (number of forests that cover every edge)

# Degeneracy in the Real World

| graph               | n      | m       | d  |
|---------------------|--------|---------|----|
| zachary [48]        | 34     | 78      | 4  |
| dolphins [35]       | 62     | 159     | 4  |
| power [47]          | 4,941  | 6,594   | 5  |
| polbooks [28]       | 105    | 441     | 6  |
| adjnoun [29]        | 112    | 425     | 6  |
| football [15]       | 115    | 613     | 8  |
| lesmis [25]         | 77     | 254     | 9  |
| celegensneural [47] | 297    | 1,248   | 9  |
| netscience [39]     | 1,589  | 2,742   | 19 |
| internet [40]       | 22,963 | 48,421  | 25 |
| condmat-2005 [38]   | 40,421 | 175,693 | 29 |
| polblogs [4]        | 1,490  | 16,715  | 36 |
| astro-ph [38]       | 16,706 | 121,251 | 56 |

From <https://arxiv.org/pdf/1006.5440>



# Degeneracy in the Real World

| graph         | n     | m       | d  |
|---------------|-------|---------|----|
| mouse         | 1,455 | 1,636   | 6  |
| worm          | 3,518 | 3,518   | 10 |
| plant         | 1,745 | 3,098   | 12 |
| fruitfly      | 7,282 | 24,894  | 12 |
| human         | 9,527 | 31,182  | 12 |
| fission-yeast | 2,031 | 12,637  | 34 |
| yeast         | 6,008 | 156,945 | 64 |

From <https://arxiv.org/pdf/1006.5440>

# Degeneracy in the Real World

| graph                 | n         | m          | d   |
|-----------------------|-----------|------------|-----|
| roadNet-CA [34]       | 1,965,206 | 2,766,607  | 3   |
| roadNet-PA [34]       | 1,088,092 | 1,541,898  | 3   |
| roadNet-TX [34]       | 1,379,917 | 1,921,660  | 3   |
| amazon0601 [30]       | 403,394   | 2,443,408  | 10  |
| email-EuAll [31]      | 265,214   | 364,481    | 37  |
| email-Enron [24]      | 36,692    | 183,831    | 43  |
| web-Google [2]        | 875,713   | 4,322,051  | 44  |
| soc-wiki-Vote [33]    | 7,115     | 100,762    | 53  |
| soc-slashdot0902 [34] | 82,168    | 504,230    | 55  |
| cit-Patents [18]      | 3,774,768 | 16,518,947 | 64  |
| soc-Epinions1 [42]    | 75,888    | 405,740    | 67  |
| soc-wiki-Talk [33]    | 2,394,385 | 4,659,565  | 131 |
| web-berkstan [34]     | 685,231   | 6,649,470  | 201 |

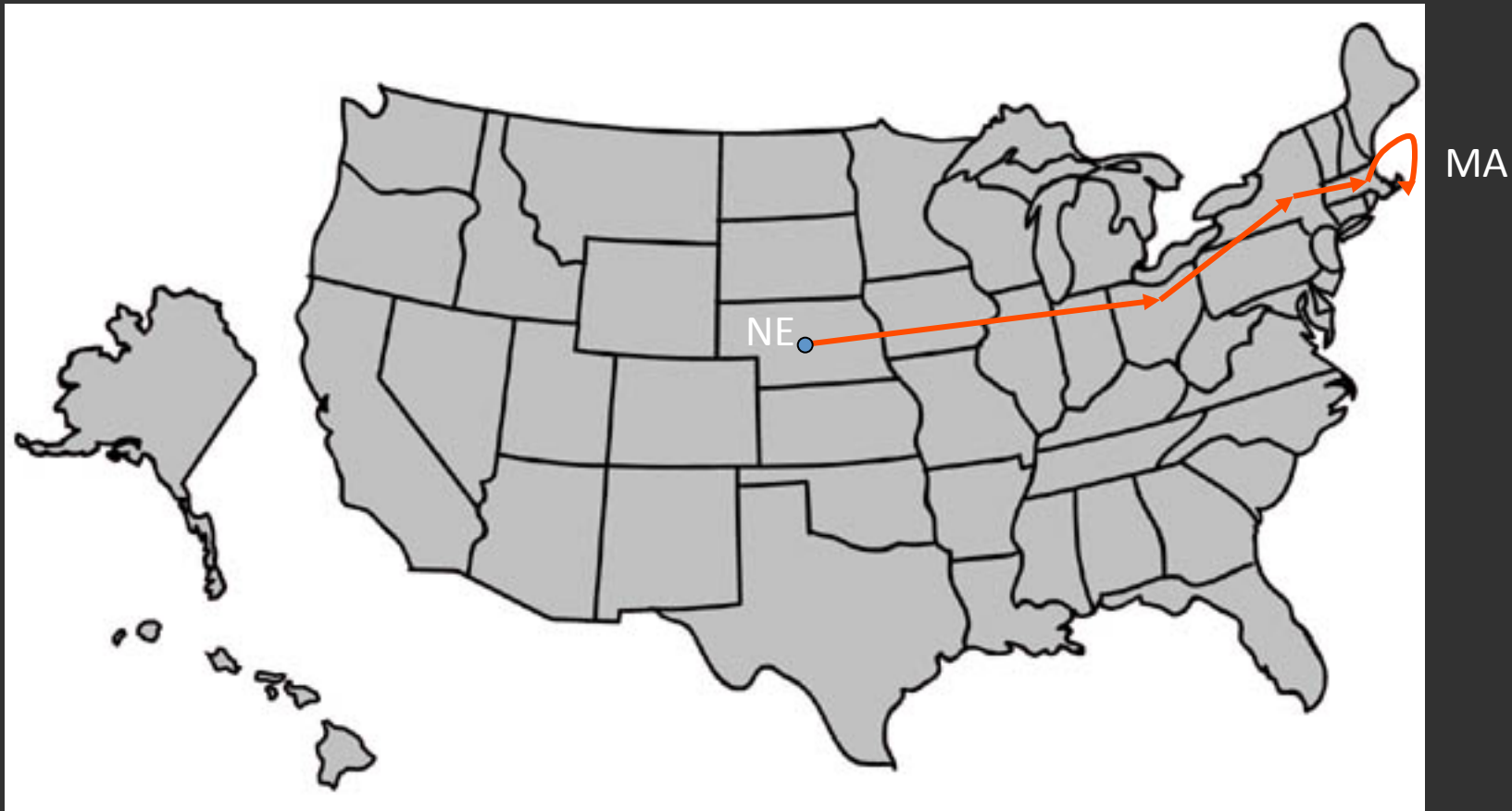
From <https://arxiv.org/pdf/1006.5440>

# Random Network Graph Models

- Two classic examples:
  - Erdős–Rényi
    - $G(n, M)$ : randomly draw  $M$  edges between  $n$  nodes
    - $G(n, p)$ : randomly draw edges between  $n$  nodes, each with probability  $p$ .
  - These models don't really model the real world, in that they don't show:
    - Small world phenomenon
    - Power laws
    - Sparsity

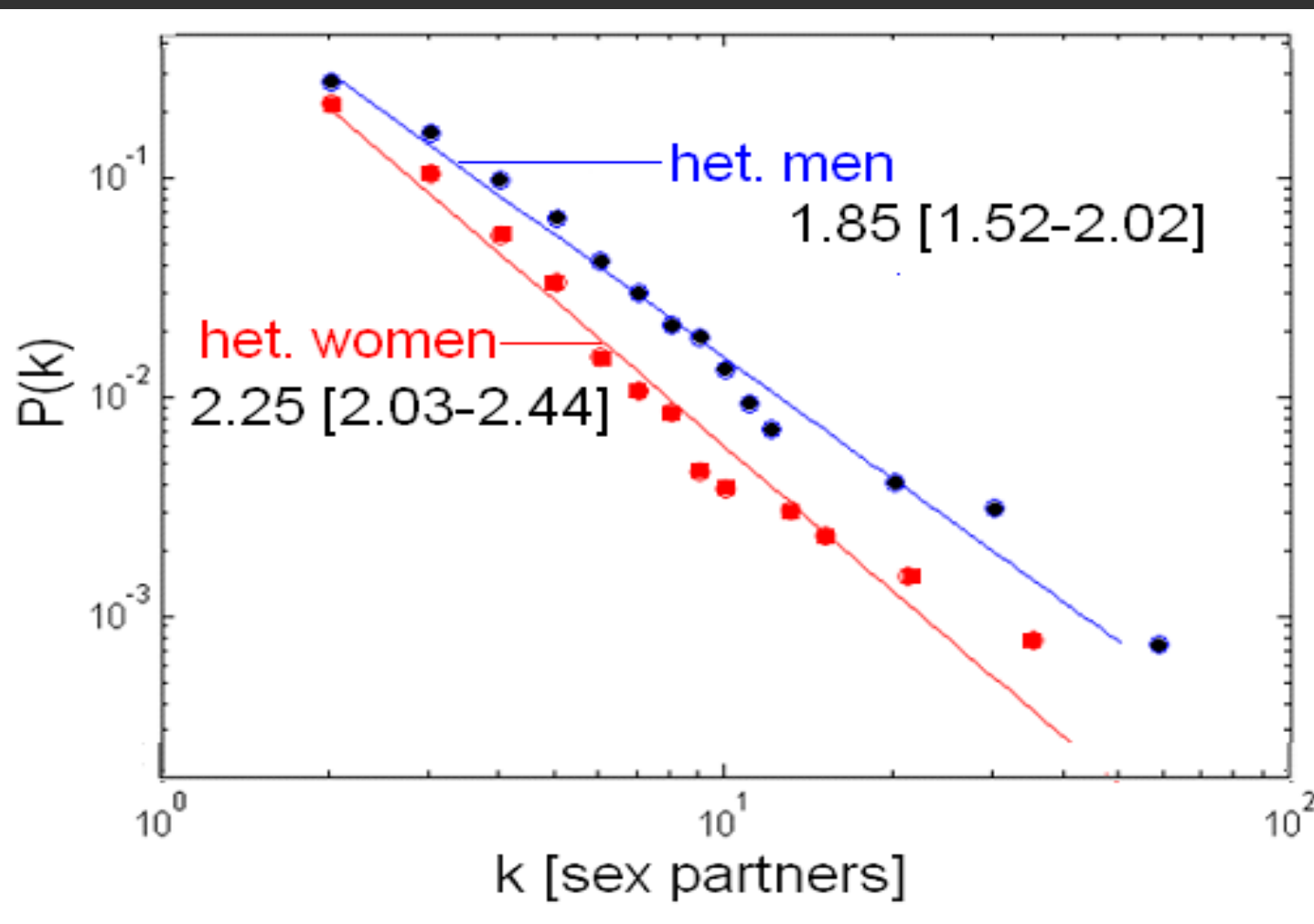


# Small world experiment



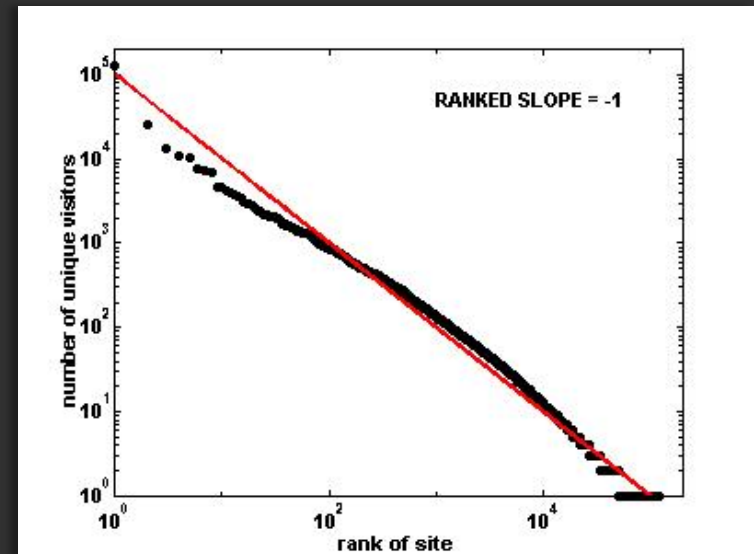
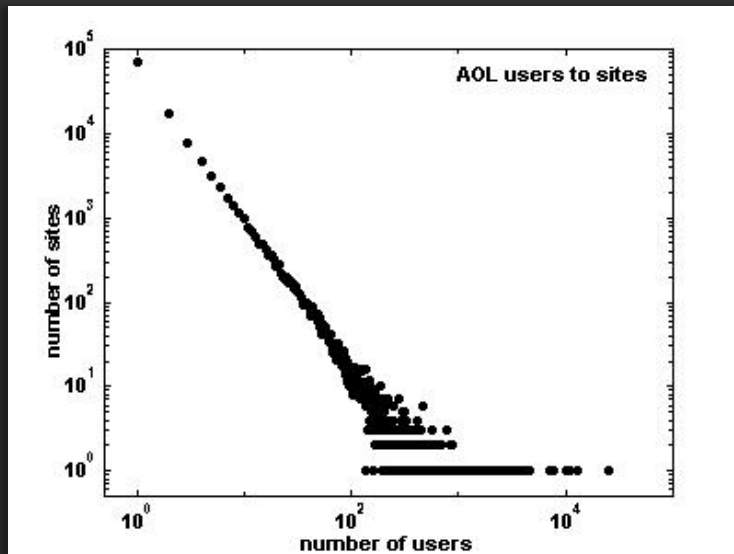
## Milgram's experiment (1960's):

- Given a target individual and a particular property, pass the message to a person you correspond with who is “closest” to the target.
- “Six degrees of separation”



# Two more examples of power laws

Distribution of users among web sites



Sites ranked by popularity

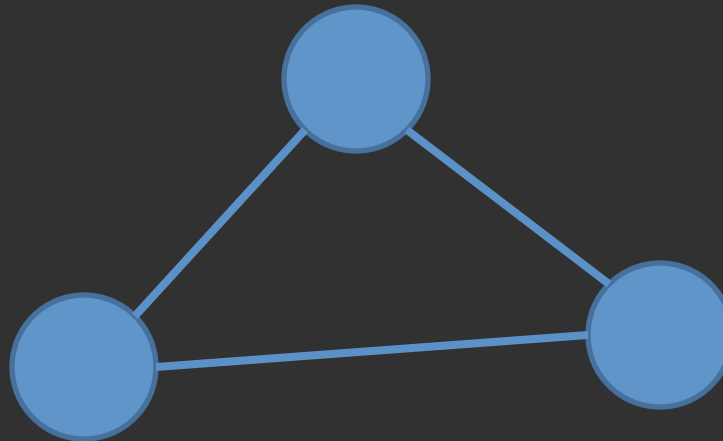
# Power Laws (Scale-Free Networks)

- Power-law
  - A scale-free network is a network whose degree distribution follows a power law, at least asymptotically.
  - That is, the fraction  $P(k)$  of nodes in the network having  $k$  connections to other nodes goes for large values of  $k$  as
$$P(k) \sim k^{-\gamma}$$
  - Typically  $\gamma$  is in the range from 2 to 3.
  - Many networks have been reported to be scale-free.



# Barabási & Albert (BA) Random Graph Model

- Very simple algorithm to implement
  - start with an initial set of  $m_0$  fully connected nodes
    - e.g.  $m_0 = 3$



- now add new vertices one by one, each one with exactly  $m$  edges
- each new edge connects to an existing vertex in proportion to the number of edges that vertex already has  
→ ***preferential attachment***

# Properties of a BA graph

- The degree distribution is scale free with exponent  $k = 3$   
 $P(k) = 2 m^2/k^3$
- The graph is connected
  - Every new vertex is born with a link or several links It then connects to  $m$  'older' vertices
  - Probability  $p_i$  of connecting to node  $i$ :
    - $k_i$  is the degree of node  $i$
- The older get richer
  - Nodes accumulate links as time goes on, which gives older nodes an advantage since newer nodes are going to attach preferentially – and older nodes have a higher degree to tempt them with than some new kid on the block

$$p_i = \frac{k_i}{\sum_j k_j}$$

# Common Tasks

- Measuring “importance”
  - Centrality, prestige
- Diffusion modeling
  - Epidemiological
- Clustering
  - Clustering coefficients
- Structure analysis
  - Subgraph isomorphisms, etc.
- Visualization/Privacy/etc.

# Centrality Measures

- Degree centrality
  - Edges per node (the more, the more important the node)
- Closeness centrality
  - How close the node is to every other node
- Betweenness centrality
  - How many shortest paths go through the edge node (communication metaphor)

# Common Tasks

- Measuring “importance”
  - Centrality, prestige (incoming links)
- Diffusion modeling
  - Epidemiological
- Clustering
  - Clustering coefficients
- Structure analysis
  - Subgraph Isomorphisms, etc.
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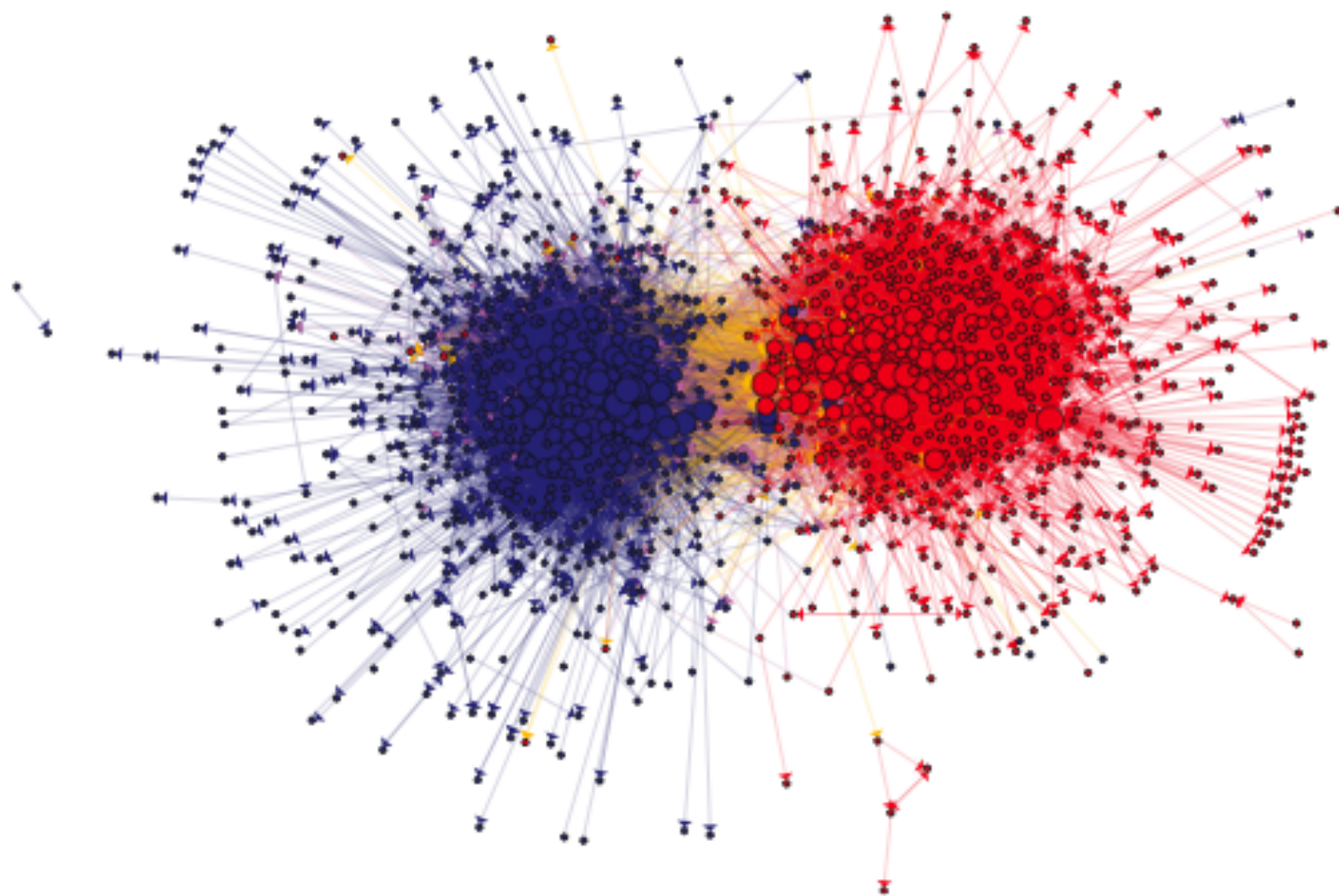


# Epidemiological

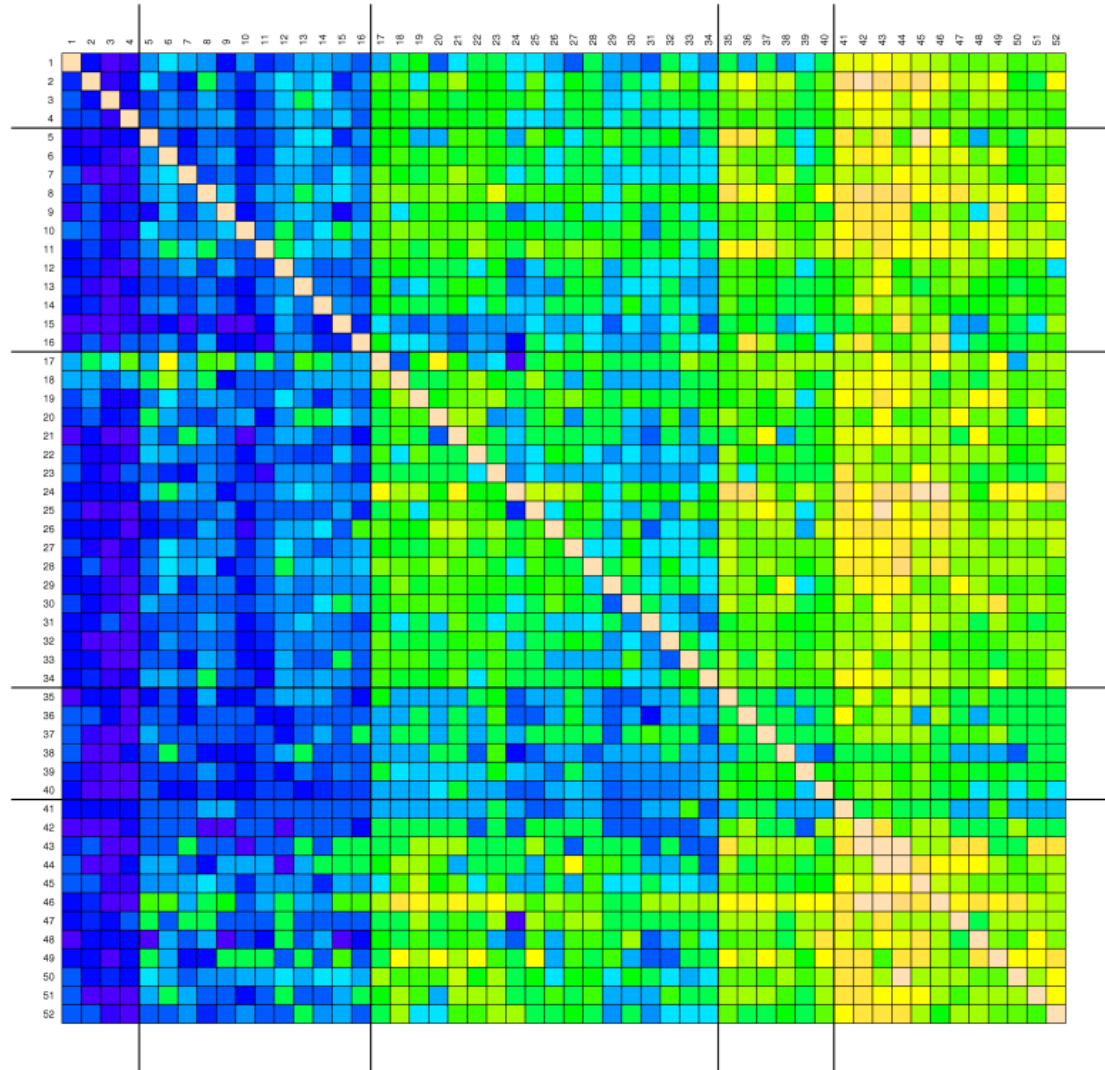
- Viruses
  - Biological, computational
  - STDs, needle sharing, etc.
  - Mark Handcock at UW
- Blog networks
  - Applying SIR models (Info Diffusion Through Blogspace, Gruhl et al.)
    - Induce transmission graph, cascade models, simulation
  - Link prediction (Tracking Information Epidemics in Blogspace, Adar et al.)
    - Find repeated “likely” infections
  - Outbreak detection (Cost-effective Outbreak Detection in Networks, Leskovec et al.)
    - Submodularity

# Common Tasks

- Measuring “importance”
  - Centrality, prestige (incoming links)
- Diffusion modeling
  - Epidemiological
- Clustering
  - Clustering coefficients
- Structure analysis
  - Subgraph Isomorphisms, etc.
- Visualization/Privacy/etc.



Blockmodel of U.S. Philosophy Departments. Note that row/column numbers do not correspond to PGR rankings.





[illegible]

# Global Clustering Coefficient

- The global clustering coefficient  $C$  is defined as:

$$C = \frac{3 \times \text{number of triangles}}{\text{number of connected triplets of vertices}} = \frac{\text{number of closed triplets}}{\text{number of connected triplets of vertices}}.$$

- In this formula, a connected triplet is defined to be a connected subgraph consisting of three vertices and two edges. Thus, each triangle forms three connected triplets, explaining the factor of three in the formula.

# Local Clustering Coefficient

- The local clustering coefficient of a vertex (node) in a graph quantifies how close its neighbors are to being a clique (i.e., complete graph).
- The number of possible connections for the neighbors of a node  $i$  of degree  $k_i$  is, of course,  $k_i(k_i-1)/2$ .
- The local clustering coefficient  $C_i$  of node  $i$  is defined as:

$$C_i = \frac{2|\{e_{jk} : v_j, v_k \in N_i, e_{jk} \in E\}|}{k_i(k_i - 1)}.$$

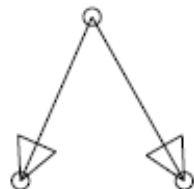
- We will discuss later how to compute these values.

# Common Tasks

- Measuring “importance”
  - Centrality, prestige (incoming links)
- Diffusion modeling
  - Epidemiological
- Clustering
  - Blockmodeling, Girvan-Newman
- **Structure analysis**
  - **Subgraph Isomorphisms, etc.**
- Visualization/Privacy/etc.



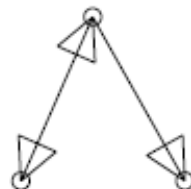
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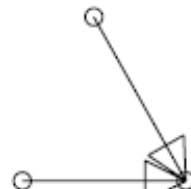
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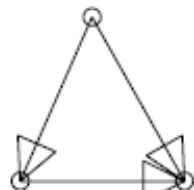
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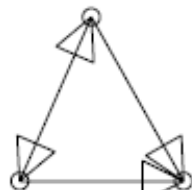
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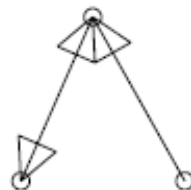
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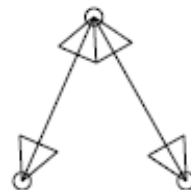
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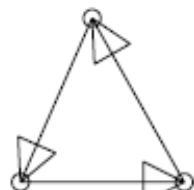
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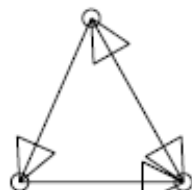
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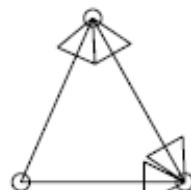
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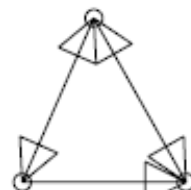
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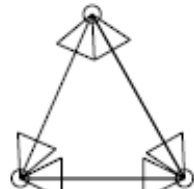
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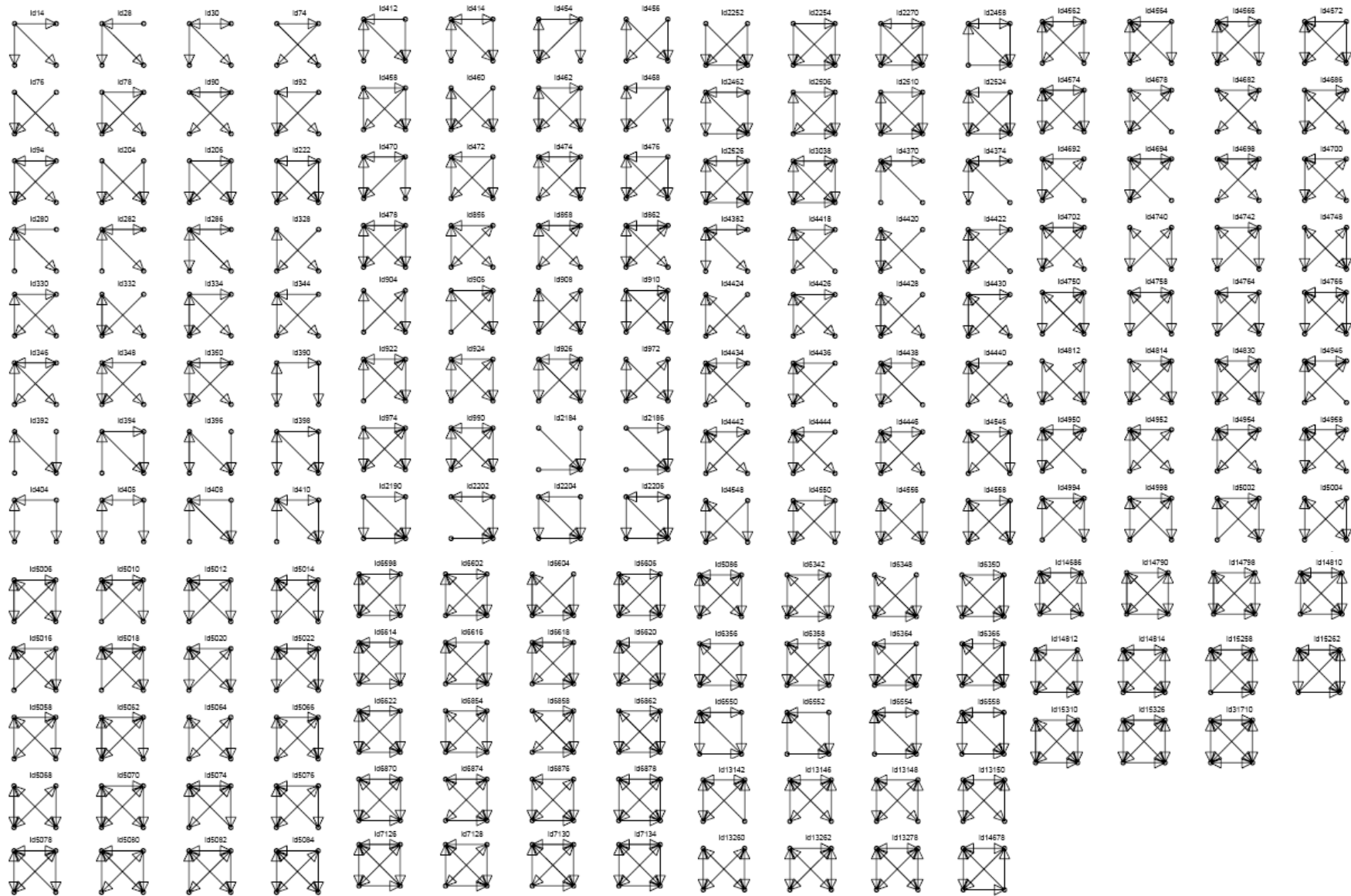


id110



id238





# Common Tasks

- Measuring “importance”
  - Centrality, prestige (incoming links)
- Diffusion modeling
  - Epidemiological
- Clustering
  - Clustering coefficients
- Structure analysis
  - Motifs, Isomorphisms, etc.
- Visualization/Privacy/etc.

# Privacy

- Emerging interest in anonymizing networks
  - Lars Backstrom (WWW'07) demonstrated one of the first attacks
- How to remove labels while preserving graph properties?
  - While ensuring that labels cannot be reapplied