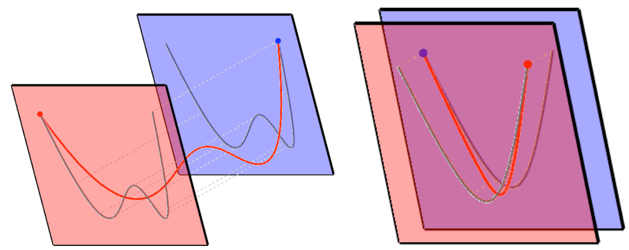
Curvature Minimizing Depth Interpolation for Intuitive and Interactive Space Curve Sketching

Gautam Chaudhary Koel Das M. Gopi

Computer Graphics Lab
University of California, Irvine
{gautam, kdas, gopi}@uci.edu

Abstract

Space curve sketching using 2D user interface is a challenging task and forms the foundation for almost all sketch based modeling systems. Since the inverse projection from 2D to 3D is a one to many function, there is no unique solution. In this paper, we propose to interpret the given 2D curve to be the projection of the 3D curve that has minimum curvature among all the candidates in 3D. We present an elegant algorithm to efficiently find a close approximation of this minimum curvature 3D space curve. Fixing the space curve to be the minimum curvature curve does not restrict the user from getting high-curvature space curve, if desired. We analyze the complete behavior of our space curve generation algorithm in order to provide simple sketching rules to achieve the curve that the user wants, even the high-curvature space curve, with no change in our algorithm.



(a) (b)

Figure 1. Given an orthographic projection of a 2D curve (gray curves on the planes) and the depth range (red and blue transparent planes), our algorithm computes the 3D space curve (red curve) that minimizes the maximum curvature and projects onto the given 2D curve. Though our method generates a low curvature curve, by reducing the range of depth values, space curves of arbitrarily high curvatures (b) can be generated.

1. Introduction

Space curve sketching using 2D user interfaces is a challenging and active research problem in the field of interactive sketch based modeling. These non-planar 3D curves are fundamental component of surface generation [2, 3] for 3D models. Space curves are also used as input for specifying implicit surfaces and also for manipulating and modifying object deformations [1]. The problem of finding the 3D space curve from a 2D sketch is mathematically indeterminable and has many possible solutions; it is well known that the fundamental difficulty is in choosing the appropriate one. Most of the sketch-based modeling tools try to avoid addressing this fundamental problem directly due to its complexity. In spite of all the successes that have been achieved in the field of sketch based modeling, the fundamental problem of space curve determination still remains to be a challenging one.

In our paper, we attempt to provide an intuitive and mathematically elegant insight into this problem of 3D space curve determination from a 2D curve. We propose that the 3D curve corresponding to the given 2D curve to be the one that minimizes the maximum normal curvature in 3D. In this paper we address the following problem: given a 2D curve, the projection parameters, and the depth range, find the 3D space curve within the given depth range that has the minimum curvature and matches with the 2D curve under the given projection.

Specifically, following are the main contributions of this paper: (i) We show the existence of the curvature minimizing space curve for an orthographic 2D projection of a polynomial curve (Fig. 1). (ii) Given only a sequence of 2D curve points as input, we present an efficient algorithm to best approximate the curvature minimizing 3D space curve, from that viewpoint. (iii) To the best of our knowledge, this is one of the earliest works on providing mathematical insight to the domain of generating intuitive 3D *free-form* space curves from 2D points. (iv) We use Laplacian filter effectively along with our approach to achieve the desired result of curvature minimizing depth interpolation.

2. Curvature Minimizing Curve

The given 2D curve is assumed to be a polynomial curve. Since any polynomial basis can be converted to and from a

Bernstein basis, we can convert the given polynomial curve into a 2D Bezier curve. We state and use a theorem involving the control points of Bezier curves to come up with a solution for the stated problem.

Theorem 1 Given a 2D Bezier curve with a known depth range, the depth of the control points of curvature minimizing 3D Bezier curve is monotonically and equally spaced within the given depth range.

If we use this theorem to construct the 3D curve from the 2D sketch, it will have seemingly two drawbacks: it can produce only monotonic curves and low curvature curves. In the problem of going from 2D to 3D, it is obvious that we have to constrain the problem to fix one solution from infinite number of solutions. Monotonicity from a given view point is a natural constraint that is required by a mathematically convincing choice of the solution – minimum curvature curve. At the same time, the generated curve can be non-monotonic in every other viewing direction and hence is not a limitation. Second, the curvature is minimized within certain constraints – the given depth range and the 2D projection. Smaller depth ranges provide less space to reduce the curvature. Hence arbitrary high curvature curves can be created by using appropriate small depth ranges (Fig. 1(b)).

3. Our Algorithm

The 2D curves drawn in sketching applications are not usually Bezier curves, but a sequence of edge connected points. Hence, we have to adapt our stated theorem for the depth extraction for a generic 2D curve.

In this section we provide a very fast and simple method for finding the depth of the sketched 2D curve points that approximates the minimum curvature 3D curve. We assume that the depth values of the control points are equally spaced and the closest point on the curve from any control point has the same depth value as the control point. Hence if we identify these *key points* on the curve that are closest to the control points and space their depth values equally, we will have the first approximation of the space curve. Since we do not have these 3D Bezier curve control points and hence of its 2D projection, identifying even the correct *number of key points* is difficult, let alone the actual key points.

Even though we do not have explicit Bezier representation of the 2D sketched curve, the number of critical points in the curve is the lower bound on the number of control points of the hypothetical Bezier representation of the same curve. All the required control points cannot be found by just one sequence of critical points. We solve this issue by subdividing the original 2D curve at the initial critical point (*key point*) locations. In this process, we increase the total number of control points (sum of the control points of the subdivided curve segments) and hence the possibility of more critical points. Using this observation, we repeat the process of finding the critical points recursively within each 2D segment between the initial sequence of critical points up till a threshold in terms of linearity of the curve segment. We show empirically that we can extract the required number of critical points for faithful representation of the sketched curve. Since we assume that all these critical points are representatives of *key points*, we assign equally spaced depth values within the range to the critical points. The depth of all other points on the 2D curve is computed by linearly interpolating the depth between the critical points based on the curve length parameterization.

The resulting 3D space curve, though a very close approximation of the curvature minimizing curve, has non-smooth, high frequency transitions of the depth values across the critical points. We chose the Laplacian low-pass filter as it is extremely fast and simple, as required by interactive sketching applications. It is a known fact that this filter produces shrinkage effect for geometric primitives but in our algorithm, the shrinking of curves is prevented by keeping the depth at the curve end points constant. We apply a few iterations of this smoothening filter *only* to the interpolated depth values of all the interior points on the curve, as $z_i\prime = z_i + \lambda(\Delta z_i)$ where $\Delta z_i = ((z_{i-1} + z_{i+1})/2) - z_i$ and $\lambda < 1.0$ for a low-pass filter. This produces an excellent approximation of the curvature minimizing curve.

4. Conclusion

In this paper we have addressed the difficult problem of generating an intuitive and aesthetic space curve from a single view of a 2D curve without using any additional cues. We believe that the mathematical rigor used to explain the intuitiveness would find its use in perceptual studies. Further, the results of this work can be used to effectively quantify the quality of curve and surface generation in free-form modeling and used extensively for a better surface generation for 3D models in sketch based modeling applications. The proposed method in this paper for generating intuitive 3D curves from 2D input is being utilized in a sketch-based free-form modeling system that we are currently developing.

References

[1] T. Igarashi and J. F. Hughes. Smooth meshes for sketch-based freeform modeling. In *Proceedings of the Symposium on Interactive 3D graphics*, pages 139–142. ACM Press, 2003.

[2] T. Igarashi, S. Matsuoka, and H. Tanaka. Teddy: A sketching interface for 3d freeform design. In *Proceedings of ACM SIGGRAPH*, pages 409–416. ACM Press, August 1999.

[3] R. C. Zeleznik, K. P. Herndon, and J. F. Hughes. Sketch: An interface for sketching 3d scenes. In *Proceedings of ACM SIGGRAPH*, pages 163–170. ACM Press, August 1996.