Logical equivalence, cont.

$$\underbrace{(\neg p \wedge t)}_{q} \vee \underbrace{(\neg p \wedge t)}_{q} \equiv \neg p \vee t$$

$$q \vee q \equiv q.$$

$$\neg(p \vee t) \wedge (p \vee t) \equiv F$$

$$\neg q \wedge q \equiv F.$$

Show: $\neg(p \to q) \equiv p \wedge \neg q$

$\neg(p \to q)$

$\neg(\neg p \vee q)$ Cond Id.

$\neg\neg p \wedge \neg q$ De Morgan.

$p \wedge \neg q$ Involution.

An expression is a tautology iff it is logically equivalent to T.

$(p \wedge q) \to (p \vee q)$ Show $\equiv T$.

$\neg(p \wedge q) \vee (p \vee q)$ Cond Id.

$(\neg p \vee \neg q) \vee (p \vee q)$

$\neg p \vee \neg q \vee p \vee q$ Associative.

$\neg p \vee p \vee \neg q \vee q$ Commutative

$T \vee T \equiv T$ Complement

Idempotent.

Show $(p \wedge q) \to p \equiv T$

$(p \wedge q) \to p$

$(\neg (p \wedge q) \vee p)$ Cond Id.

$(\neg p \vee \neg q \vee p)$ De Morgan.

$\neg p \vee p \vee \neg q$ Com

$T \vee \neg q$ Complement. $(\underline{p \vee \neg p} \equiv T)$

T Identity.

$T \vee p \equiv T$.

Show: $(\overset{F}{p} \wedge \overset{T}{q}) \to \overset{F}{r}$ and $\overset{T}{(p \to r)} \wedge \overset{F}{(q \to r)}$ are <u>not</u> logically equiv.

$p \equiv F$
$q \equiv T$
$r \equiv F$

proves 2 expressions are <u>not</u> logically equiv.

$(\neg p \wedge \underline{(p \to q)}) \to \neg p$.

p	q	$p \to q$	$\neg p \wedge (p \to q)$	
T	T	T	F	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

<u>taut</u>

$(\neg q \wedge (p \to q)) \to \neg p$.

tautology

p	q	$p \to q$	$\neg q \wedge (p \to q)$	
T	T	T	F	T
T	F	F	F	T
F	T	T	F	T
F	F	T	T	T

Predicate: Logical statements that contain variables.

$P(x)$: $x > 3$

Domain of discourse: set of all possible values for x.

$P(4)$ true

$P(2)$ false.

Domain is $\mathbb{R}$ (real numbers)

$Q(x)$: x is a perfect square

Domain: integers.

Propositions

$Q(49)$ is true.

$Q(48)$ is false.

P(student): The student is a transfer student.

Domain: students in ICS 6B.

P(11236742) is true or false.

proposition.

in the domain.

Universal quantifier: For all x, $P(x)$

$\forall x\, P(x)$

For every x, $P(x)$

$P(x): x > 3$ Domain is $\mathbb{R}$.

$\forall x\, P(x)$ False counter-example: 2

$P(2)$ is false.

$Q(x)$: Student x showed up for the final exam.

Domain set of students ICS 6B.

$\forall x\, Q(x)$ "Every student showed up for the final."

Domain: $x_1, x_2, \ldots, x_n$ $\equiv Q(x_1) \wedge Q(x_2) \wedge \cdots \wedge Q(x_n)$

Show: $P(x): x+1 > x$ Domain $\mathbb{R}$.

$\forall x\, P(x)$ $\qquad \forall x,\ x+1 > x$ ✓

Proof statement is true:

$$\begin{array}{r} x+1 > x \\ -x \quad -x \\ \hline 1 > 0 \end{array}$$ ✓

$P(x): x^2 < 20$ Domain: positive integers less than 5

$\forall x\, P(x) \equiv P(1) \wedge P(2) \wedge P(3) \wedge P(4)$.

$\forall x,\ x^2 \geq x$ Domain is $\mathbb{R}$.

$x = 1/2$ Counterexample

$1/2 = 1/4 \not\geq 1/2$.

Domain is set of integers.

$\forall x,\ x^2 \geq x$. Proof statement is true

$x < 0 \qquad x^2 \geq 0 > x$

$x = 0 \qquad x^2 = 0 \geq x$.

$x \geq 1$ $\quad x \cdot x \geq 1 \cdot x$

$x^2 \geq x$. ✓

Existential quantification of $P(x)$

There exists an x (in the domain) such that $P(x)$ is true.

$\exists x\, P(x)$.

If Domain is finite: $x_1, \ldots, x_n$

$\exists x\, P(x) \equiv P(x_1) \vee P(x_2) \vee \cdots \vee P(x_n)$.

$\exists x \quad x^2 \geq 0$ $\quad$ Domain is $\mathbb{R}$. $\quad$ True.

$\exists x \quad x^2 \leq 0$ $\quad$ Dom: $\mathbb{R}$. $\quad$ True. $\quad x = 0$.

$\exists x \quad x+1 > 2x$ $\quad$ Domain set of integers. $\quad x = 0$.

$\exists x \quad x > 2x$ $\quad$ x = neg.

$\exists x \quad x = x+1$ $\quad$ Domain $\mathbb{R}$.

$$0 \neq 1$$
$$+x \quad\quad +x$$
$$x \neq x+1.$$

	True	False
$\forall x\, P(x)$	Show for all x	Show a counter example
$\exists x\, P(x)$	Show an example	Prove $P(x)$ false for all x

one x s.t. $\neg P(x)$

one x s.t. $P(x)$ is true.