Predicates $P(x)$ Domain

$\Rightarrow$ $\forall x \, P(x)$

$\exists x \, P(x)$

Domain: Students in a study hall

$H(x)$: x is doing his/her homework.

$\exists x \, H(x)$ There is a student doing her HW.

$\neg \exists x \, H(x)$ It is not the case that...

$\equiv$

$\forall x \, \neg H(x)$ Every student is not doing her HW

De Morgan's Law for quantified statements:

P's turn to H's.

$P \equiv H$

$$\neg \exists x \, H(x) \equiv \forall x \, \neg H(x)$$

Finite domain $s_1, \ldots, s_n$

$$\neg \left(H(s_1) \vee H(s_2) \; \ldots \; \vee H(s_n) \right)$$

$$\equiv \neg H(s_1) \wedge \neg H(s_2) \; \ldots \; \wedge \neg H(s_n).$$

$\forall x\, H(x)$ Every student is doing her HW.

$\neg \forall x\, H(x)$ At least one student is not doing her HW

$\equiv$

$\exists x\, \neg H(x)$

De Morgan: $\neg \forall x\, H(x) \equiv \exists x\, \neg H(x)$

$\neg \big(H(s_1) \wedge H(s_2) \ldots \wedge H(s_n) \big)$

$\equiv \neg H(s_1) \vee \neg H(s_2) \ldots \vee \neg H(s_n).$

Negation of $\forall x\, (x^2 > x)$.

$\neg \forall x\, (x^2 > x) \equiv \exists x\, \neg (x^2 > x)$

$\equiv \exists x\, (x^2 \leq x)$

Negation of $\exists x\, (x^2 = 2)$

$\neg \exists x\, (x^2 = 2) \equiv \forall x\, \neg (x^2 = 2)$

$\equiv \forall x\, x^2 \neq 2.$

Precedence of ops

$\big(\underbrace{\exists x\, P(x)}_{\text{scope of } \exists} \big) \wedge \underline{Q(x)}$

can override w/ parens.

$\exists x\, \underbrace{\big(P(x) \wedge Q(x) \big)}_{\text{scope of } \exists.}$

D : UCI students.

$C(x)$: x is in my class.
$Y(x)$: x is younger than 18

There is a student in my class who is younger than 18.

$$\exists x \, (C(x) \wedge Y(x))$$

$T(x)$: x is a transfer student
$F(x)$: x is a freshman.

Every student in my class is either a transfer student or a freshman

$$\forall x \, (C(x) \rightarrow (T(x) \vee F(x)))$$

Show: $\neg \forall x \, (P(x) \rightarrow Q(x)) \equiv \exists x \, (P(x) \wedge \neg Q(x))$

DM $\quad \exists x \, \neg (P(x) \rightarrow Q(x))$

Cond ID $\quad \exists x \, (\neg (\neg P(x) \vee Q(x)))$

DM $\quad \exists x \, (\neg\neg P(x) \wedge \neg Q(x))$

Involution. $\quad \exists x \, (P(x) \wedge \neg Q(x))$

Predicates w/ Multiple variables.

$P(x,y)$

$P(x,y)$: $x+y=3$ x,y domain: $\mathbb{R}$.

$C(x,y)$: x is enrolled in y. domain for x set students at school.
y set of courses.

Nested quantifiers: if it is inside the scope of another variable.

$\forall x \ \exists y \ (x+y=0)$ Domain for x,y is $\mathbb{R}$.

For every x, there is a y s.t. $x+y=0$. True.

Universal player picks value for x.
Existential player picks $y=-x$
$\} \ x+(-x)=0$ ✓

$\forall x \forall y \ x+y=y+x$ $x,y \in \mathbb{R}$.

true.

$\exists x \ \forall y \ (x+y=0)$.

Existential picks x: x,
Universal player $y=-x+1$

False.

$x+y=x+(-x)+1=1\neq 0$

The Sum of two positive integers is positive.

Domain: integers.

$$\forall x\, \forall y\, [((x>0) \wedge (y>0)) \rightarrow (x+y>0)]$$

Every non-zero real number has a multiplicative inverse

Domain: $\mathbb{R}$.

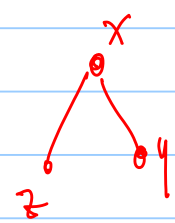
$$\forall x\, ((x \neq 0) \rightarrow \exists y\, (xy = 1))$$

$$\equiv\ \forall x\, \exists y\, ((x \neq 0) \rightarrow (xy = 1))$$

$A \rightarrow B$

Domain of discourse: Kids.

$F(x,y)$: x is friends with y.



$$\exists x\, \forall y\, \forall z\, \left(\left[\underbrace{(F(x,y) \wedge F(x,z) \wedge y \neq z)} \rightarrow \underbrace{\neg F(y,z)}\right]\right)$$

There is a kid none of whose friends are friends with each other