

$F(x,y)$: x can fool y .

Everyone can fool Fred.

$$\forall x F(x, \text{Fred}).$$

Sam can fool everyone

$$\forall x F(\text{Sam}, x).$$

Everyone can fool somebody.

$$\forall x \exists y F(x, y). \quad *$$

There is someone that everyone can fool

$$\exists y \forall x F(x, y) \quad \text{or} \quad \exists x \forall y F(y, x). \quad \Leftarrow$$

No one can fool everyone

$$\neg \exists x \forall y F(x, y).$$

$$\forall x \neg \forall y F(x, y) \\ \forall x \exists y \neg F(x, y).$$

No one can fool Fred and Sue.

$$\neg \exists x (F(x, \text{Fred}) \wedge F(x, \text{Sue}))$$

Nancy can fool exactly two people.

$$\exists x \exists y [\underline{F(\text{Nancy}, x)} \wedge \underline{F(\text{Nancy}, y)} \wedge \forall z [((z \neq x) \wedge (z \neq y)) \rightarrow \neg F(\text{Nancy}, z)]]$$

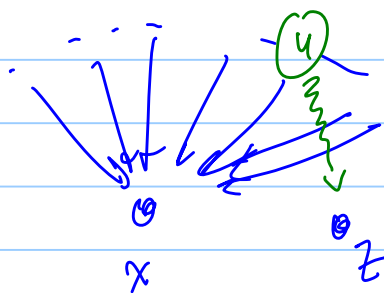
$\wedge (x \neq y)$
 $\vee$
 $\rightarrow \neg F(\text{Nancy}, z)$
 $\neg F(\text{Nancy}, z)$

There is exactly one person who everyone can fool.

$$\exists x \forall y \left[\underline{F(y, x)} \wedge \underline{\forall z [(z \neq x) \rightarrow \exists u \neg F(u, z)]} \right]$$

No one can fool him or herself.

$$\neg \exists x F(x, x).$$



$$Q(x, y)$$

$$x + y = x - y.$$

$$Q(1, 1) \text{ F}$$

$$\forall y Q(1, y) \text{ F}$$

$$\exists x Q(x, 2) \text{ F}$$

$$\Rightarrow \exists x Q(2, x) = \exists y Q(2, y)$$

$$\exists x \exists y Q(x, y) \text{ T}$$

$$\forall y \exists x Q(x, y) \text{ F}$$

$$\exists x \forall y Q(x, y) \text{ F}$$

$$\exists y \forall x Q(x, y) \text{ T}$$

$$\forall x \exists y Q(x, y) \text{ T}$$

$$x + 2 = x - 2.$$

Argument set of propositions hypotheses followed by a prop conclusion.

P_1
 P_2
 $\vdots$
 P_n
 $\hline$
 $\therefore C$

An argument is valid if the conclusion is true whenever the hypotheses are true.

An argument not valid, it is fallacy.

$(P_1 \wedge P_2 \dots \wedge P_n) \rightarrow C$
 valid Tautology

Modus Ponens:

P
 $P \rightarrow q$
 $\hline$
 $\therefore q$

valid.

P	q	$P \rightarrow q$	hyp true.
T	T	T	←
T	F	F	
F	T	T	
F	F	T	

Modus Tollens

✓

valid.

$\neg q$
 $P \rightarrow q$
 $\hline$
 $\therefore \neg P$

$\neg P = T$

P	q	$P \rightarrow q$	$\neg q$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	T

$$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p.$$

If I had a hard workout, I am sure.

If I am sure, I take aspirin.

I am not taking aspirin

$\therefore$ I did not have a hard workout.

$$w \rightarrow s$$

$$s \rightarrow a$$

$$\neg a$$

$$\neg w.$$

$$s \rightarrow a$$

$$\neg a$$

$$\neg s$$

$$w \rightarrow s$$

$$\neg w$$

(hyp)

(hyp)

M.T.

(hyp)

M.T.