Note Title 10/11/2013

7a) Everyone beats ~~beats~~ Sam $B(x,y)$.

$\neg B(x,x)$.

$$\forall y\, B(y, Sam)$$

$$\forall y\, [\,(y \neq Sam) \rightarrow B(y, Sam)\,]$$

Modus Ponens

$$\begin{array}{l} p \\ \underline{p \rightarrow q} \\ \therefore q \end{array}$$

Modus Tollens

$$\begin{array}{l} \neg q \\ \underline{p \rightarrow q} \\ \therefore \neg p. \end{array}$$

f r

→ If it is not foggy or it doesn't rain then the race will be held.

c

t

→ If the race is held the there will be a trophy ceremony.

→ There will be no trophy ceremony

∴ It rained.

$$\begin{array}{l} (\neg f \vee \neg r) \rightarrow c \\ c \rightarrow t \\ \underline{\neg t} \\ \therefore r \end{array}$$

$$\begin{array}{l} (\neg f \vee \neg r) \rightarrow e \\ c \rightarrow t \\ \neg t \\ \hline \therefore r \end{array}$$

$c \rightarrow t$	(hyp)
$\neg t$	(hyp)
$\neg e$	(MT).
$(\neg f \vee \neg r) \rightarrow e$	(hyp)
$\neg(\neg f \vee \neg r)$	(MT).
$\neg\neg f \wedge \neg\neg r$	(De Morgan)
$f \wedge r$	(Inv).
r	(Simplification). con

All birds can fly.
A penguin is a bird.
A penguin can fly.

$\forall x\ (B(x) \rightarrow F(x))$
$B(\text{penguin})$.
$F(\text{penguin})$.

$B(\text{pengn}) \rightarrow F(\text{pengn})$

$$\frac{\exists x\, P(x)}{\therefore P(c)} \quad \text{for some } c.$$

$$\frac{\forall x\, P(x)}{\therefore P(c)} \quad \text{for } \cancel{\text{some}} \text{ any } c \text{ in universe}$$

$$\frac{P(c)}{\therefore \exists x\, P(x).} \quad \text{for some } c.$$

$$\frac{P(c)}{\therefore \forall x\, P(x).} \quad \text{for an arbitrary } c.$$

$F(x)$

$S(x)$

Linda, an employee in the Company, owns a Ferrari.
Everyone who owns a Ferrari has gotten a speeding ticket.
$\therefore$ Some employee in company has gotten a speeding ticket.

$$\begin{array}{l} F(\text{Linda}) \\ \forall x\,[F(x) \to S(x)] \\ \hline \therefore\ \exists x\, S(x) \end{array}$$

$F(\text{Linda})$	Hyp.
$\forall x\,[F(x) \to S(x)]$	(hyp).
$F(\text{Linda}) \to S(\text{Linda})$	Univ Inst.
$S(\text{Linda})$	MP
$\exists x\, S(x)$.	

Univer set of all people $E(x)$.

$$\begin{array}{l} E(\text{Linda}) \wedge F(\text{Linda}) \\ \forall x\,[F(x) \to S(x)] \\ \hline \therefore\ \exists x\,[E(x) \wedge S(x)] \end{array}$$

$E(\text{Linda}) \wedge F(\text{Linda})$.	
$F(\text{Linda})$	(Simpl)
$\forall x\,[F(x) \to S(x)]$	(hyp).
$F(\text{Linda}) \to S(\text{Linda})$	Univ Inst.
$S(\text{Linda})$	MP.
$E(\text{Linda})$	Simpl
$S(\text{Linda}) \wedge E(\text{Linda})$	Conj.
$\therefore\ \exists x\,[S(x) \wedge E(x)]$.	

Common Pitfall: $\exists x\, P(x)$
$P(\text{Sam})$.

$\exists x\, P(x) \wedge \exists x\, Q(x)$

$\exists x\, P(x)$

$P(c)$ $\leftarrow$ c generic

$\exists x\, Q(x)$

$Q(c')$ $\leftarrow$

$P(c) \wedge Q(c)$

$\exists x\,[P(c) \wedge Q(c)]$

Theorem: a statement that can be proven true or false.

Proofs: Assume a set of axioms

Series of statements each of which follows logically from previous statements in proof or axioms or previously proved theorems.

Ends with a statement of the theorem.

Theorems of form: $p \to q$

Assume p, then establish q. Direct Proof.

Assume $\neg q$, then establish $\neg p$ Proof by contrapositive.

Integer x is even if can expressed as $x = 2k$ for some integer k.

Integer x is odd, it can expressed as $2k+1$ for some int k.

Fact: If n is an even integer, then $5n^3$ is an even integer.

Since n is even $n = 2k$ for some int k.

Then $$5n^3 = 5(2k)^3 = 2\underbrace{\left[5 \cdot 2 \cdot 2k^3\right]}_{\text{an integer}}$$

so $5n^3$ is even.

Fact: If a and b are positive integers then

$$\frac{a}{b} + \frac{b}{a} \geq 2. \longrightarrow \Delta = \frac{a}{b}. \qquad \Delta > 0$$

$$\left(\frac{b}{a} - 1\right)^2 \geq 0. \qquad \Delta \cdot \quad \Delta + \frac{1}{\Delta} \geq 2. \cdot \Delta$$

$$\left[\left(\frac{b}{a}\right)^2 - 2\frac{b}{a} + 1 \geq 0.\right] \cdot \frac{a}{b}. \qquad \Delta^2 + 1 \geq 2\Delta$$

$$\frac{b}{a} - 2 + \frac{a}{b} \geq 0. \qquad \Delta^2 - 2\Delta + 1 \geq 0.$$

$$(\Delta - 1)^2 \geq 0.$$

$$\frac{b}{a} + \frac{a}{b} \geq 2.$$