Note Title 10/16/2013

If a^2 is the square of an integer and even then a^2 is evenly divisible by 4.

$$\left\{\begin{array}{l}\to \text{If } a^2 \text{ is the } \overset{s}{\text{sq}} \text{ of an int} \overset{e}{\text{ \& even}} \\ \text{then } \underset{v}{a \text{ is even}}.\end{array}\right.$$

if $(s + e)$ then v.

→ If a is even then a^2 is divisible by 4

Assumed $\neg v$ and prove $\neg(s \wedge e)$.

Assume $\neg v$ and s, prove $\neg e$.

If p_1 and p_2 and p_3 then e.

Assume $\neg e$ and p_1 and p_3 → prove $\neg p_2$

Proof by Contradiction

T : theorem.

We use a proof by contradiction.

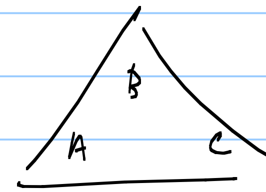
Suppose T is false

Deduce something known to be false.

This is a contradiction, therefore T is ~~true~~ true.

Theorem: A triangle has at most one obtuse angle.

obtuse: $> 90°$.



Proof by contr.

Suppose a triangle has two obtuse angles.

$A > 90$ and $B > 90$.

$A + B + C = 180$

$A, B, C > 0$.

$A + B + C = 180$

$$180 = A + B + C > 90 + B + C$$
$$> 90 + 90 + C$$

$$180 > 180 + C$$

$0 > C$. Contradiction. Therefore any triangle has at most one obtuse angle.

Claim: Among any set of at least 22 days there must be at least four that fall on the same day of the week.

Proof by contradiction:

Assume there is a set of 22 days
such that each day of the week appears ~~at~~ at
most 3 times.

Seven days of week, each appears ≤ 3 times.

Number of days $\leq (7) \cdot 3 = 21$

Contradicts the fact that the set of days has
22 days. Therefore ....

If a^2 is the sq of
an int + a^2 is even
then a^2 is divisible
by 4

Theorem $\sqrt{2}$ is irrational

Proof by contr.

Assume $\sqrt{2} = \frac{a}{b}$ a, b are integers.

reduced fraction
a + b have no common
factors.

Square both sides:

$b^2 \cdot 2 = \frac{a^2}{b^2} \cdot b^2$

$2b^2 = a^2$

a^2 is even $\Rightarrow$ a^2 is divisible by 4. $\Rightarrow$ a is even.

$2b^2 = a^2 = 4 \cdot c$ c integer.

$\frac{1}{2} \cdot \quad 2b^2 = 4 \cdot c \quad \cdot \frac{1}{2}$

$b^2 = 2 \cdot c$. b^2 divisible by 4. $\Rightarrow$ b is even.

$\frac{a^2}{b^2}$ a^2 & b^2 are divisible by 4.

a & b are even, they have a common factor of 2. Contradiction. Therefore $\sqrt{2}$ is irrational.

Theorem: There are an infinite number of prime numbers.

Proof by contradiction.

Suppose there is a finite # of prime numbers:

$P_1 \; P_2 \ldots \; P_n$.

$x = (P_1 \cdot P_2 \cdot P_3 \cdot \; \cdots \; \cdot P_n) + 1$.

x is larger than all of $P_1 \ldots P_n$ $\therefore$ is not prime.

x is composite : $x = P_k \cdot c$ for some prime P_k.

$$x = \overbrace{(p_1 \cdot p_2 \cdots p_n)}^{} + 1 = p_k \cdot c$$

$$1 = \underline{p_k \cdot c} - \underline{p_1 p_2 \cdots p_n}$$

$$/p_k \quad = p_k \underbrace{(c - p_1 p_2 \cdots p_{k-1} p_{k+1} \cdots p_n)}_{\text{integer.}} \; /p_k$$

$$\underbrace{1/p_k}_{\text{not an int.}} = \underbrace{(c - p_1 \cdots p_{k-1} p_{k+1} \cdots p_n)}_{\text{int}}$$

Contradiction. There is an infinite # of primes.

Theorem: For every real number $x \in [0, \pi/2]$.

$$\sin x + \cos x \geq 1.$$

$$\Downarrow$$

$$0 \leq \sin x \leq 1$$
$$0 \leq \cos x \leq 1$$

$$\sin^2 x + \cos^2 x = 1$$

Proof by contradiction.

Assume

$$0 \leq \sin x + \cos x < 1.$$

because $\sin x \geq 0$, $\cos x \geq 0$.

$$0 \leq (\sin x + \cos x)^2 < 1$$

$$0 \leq \sin^2 x + 2 \sin x \cos x + \cos^2 x < 1$$

$$-\sin^2 x - \cos^2 x \qquad -1$$

$$\underline{2\sin x \cdot \cos x < 0}$$

but $\sin x \geq 0$ & $\cos x \geq 0$

true for $\underline{2\sin x \cdot \cos x \geq 0}$

Contradiction.