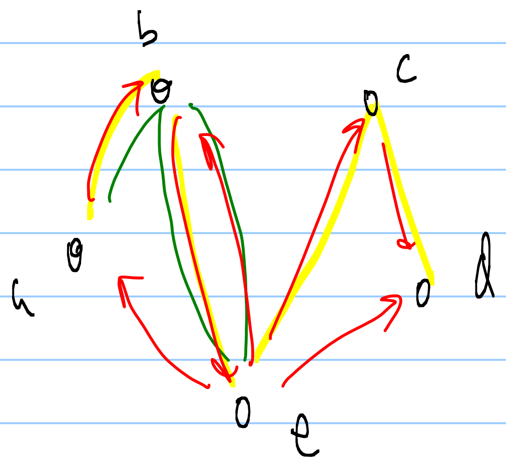
Directed graph $G = (V, E)$

$E \subseteq V \times V$



A path in G is a sequence of edges the head of each edge is the tail of the next one.

Simple.

4 $(a,b), (b,e), (e,c), (c,d)$.

$\langle a, b, e, c, d \rangle$

not simple

3 $((a,b), (b,e), (e,b))$ $\langle a, b, e, b \rangle$

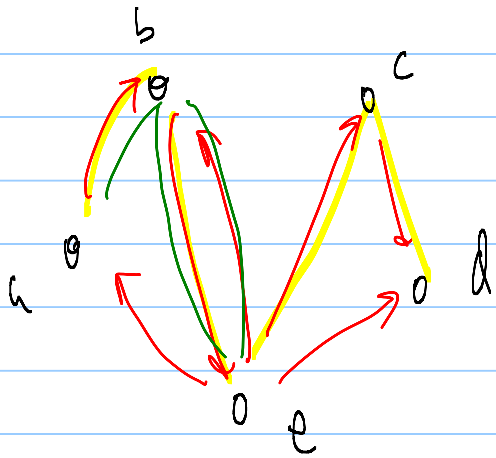
In general

$(v_0, v_1), (v_1, v_2), (v_2, v_3), \ldots, (v_{k-1}, v_k)$ k.

$\langle v_0, v_1, v_2, \ldots, v_k \rangle$

length of the path = # edges.

A cycle is a path whose first & last vertices are the same.



Cycles:

$\langle a, b, e, a \rangle$ Simple

$\langle a, b, e, b, e, a \rangle$ Not simple

A path is simple if there are no repeated vertices.

A cycle is simple if there are no repeated vertices, except for the first & the last.

Not simple: $\langle e, b, e, c, d, e \rangle$. Not simple.

Digraph $\longleftrightarrow$ Relation

paths
cycles

??

Composition of two relations:

functions

$x \longrightarrow f(x) \longrightarrow g(f(x))$

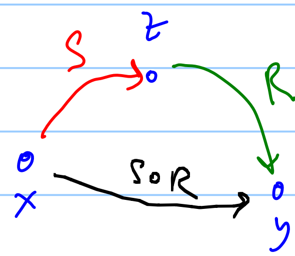
Two relations S & R defined on a set V.

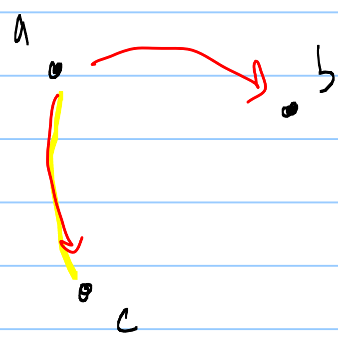
New relation $S \circ R$

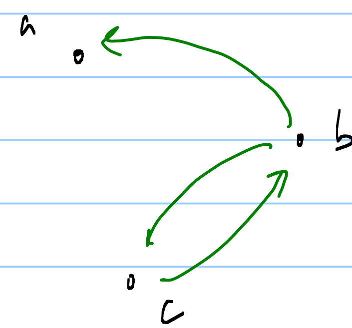
$(x,y) \in S \circ R$ iff there is some $z \in V$.

$(x,z) \in S$

$(z,y) \in R$.

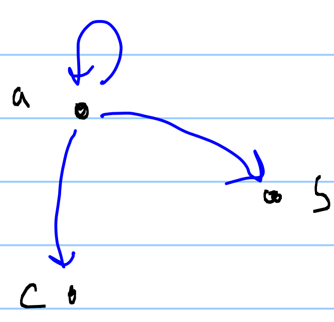






$S = \{(a,b), (a,c)\}$

$R = \{(b,a), (b,c), (c,b)\}$



$S \circ R$.

$$R_1 = \{(x,y) \in \mathbb{R}^2 : x \le y\}$$

$$R_2 = \{(x,y) \in \mathbb{R}^2 : x < y\}.$$

$\underline{R_1 \circ R_2}$ $\qquad (3,5) \overset{?}{\in} R_1 \circ R_2.$

$$3 \xrightarrow{3 \le 4} 4 \xrightarrow{4 < 5} 5$$

$$3 \le \cancel{x} < 3$$

$x < y \qquad (x,y) \in R_1 \circ R_2$

$(x,x) \notin R_1 \circ R_2$

$x > y \qquad (x,y) \notin R_1 \circ R_2$

$(5,3) \overset{?}{\in} R_1 \circ R_2 \qquad 5 \le \cancel{z} < 3$

$$R_1 \circ R_2 = R_2$$

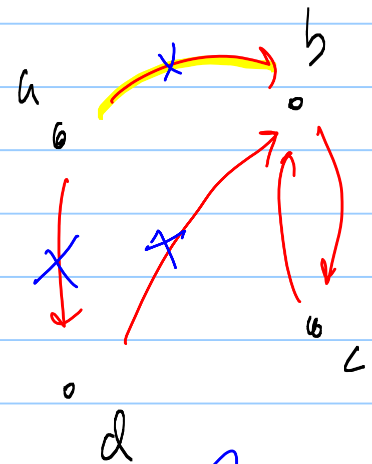
A relation can be composed with itself.

grandparent $\quad P \circ P \qquad$ Domain: set of people

$x P y$ if x is Parent of y.

$(x,y) \in \underline{P \circ P}$? $\qquad x \to z \to y$

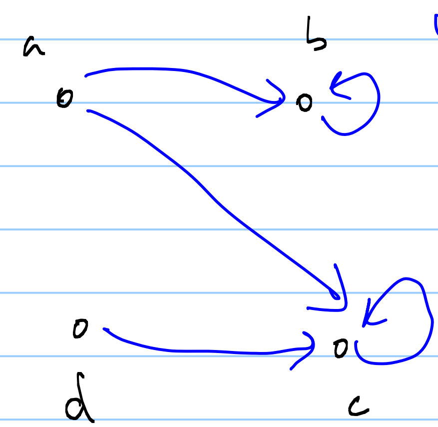
G



E edge set.
relation V.

$E \circ E = E^2$

G^2



path of length 2

edge here

$$G^2 = (V, E^2)$$

(x, y) is an edge of G^2
iff there is a path of length 2 from x to y in G.

R

$R \circ R = R^2$

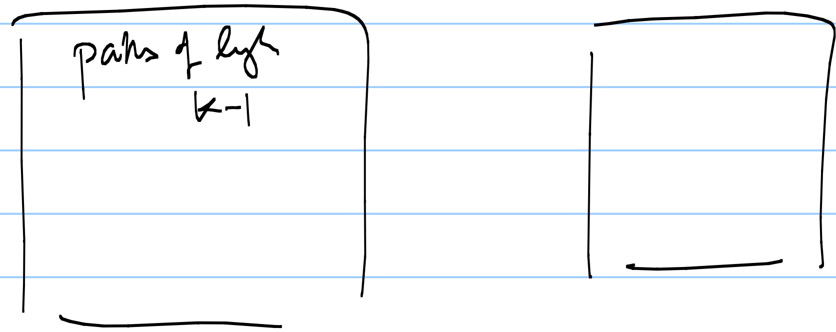
$R \circ R^2 = R \circ R \circ R = R^3$

$\vdots$

$R \circ R^{k-1} = R^k$

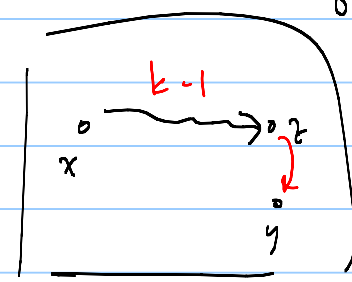
$$G^k = (V, E^k)$$

G^{k-1} $\qquad\qquad$ G



$E^{k-1} \circ E.$

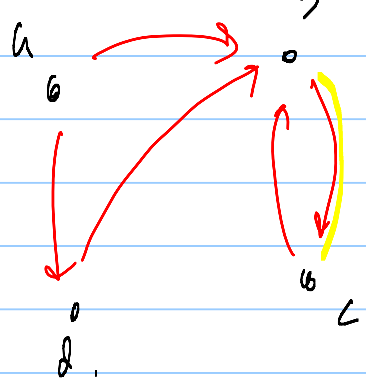
G^k



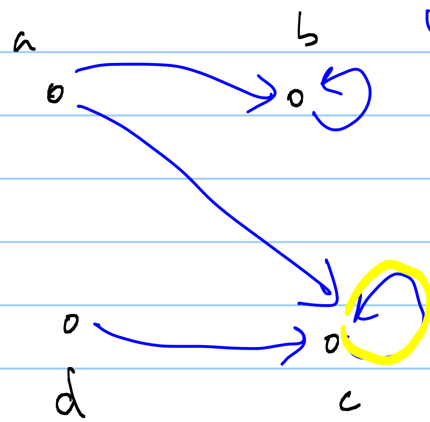
$G^k = (V, E^k)$

paths of length k in G.

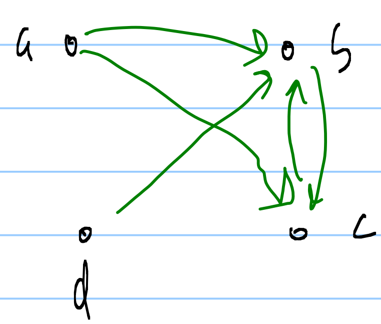
G



G^2



G^3



$G \circ G^{k-1} = G^{k-1} \circ G = G^k$

$$\boxed{G^+} = \underbrace{G \cup G^2 \cup G^3 \cup \cdots}_{} G^{|V|-1}$$

Same copy of V
Union of edges.