Note Title 11/1/2013

Domain: people at a school.

$P(x)$: x is a student

$T(x)$: x is a teacher

$E(x,y)$ x has sent an email to y.

Every student has sent ~~at least~~ an email to at least one teacher.

$$\forall x \, (P(x) \to \exists y \, (T(y) \wedge E(x,y)))$$

$$\forall x \, \exists y \, ((P(x) \wedge T(y)) \to E(x,y))$$

$$E(P(x), \cancel{T}(y))$$

There is a teacher who has received an email from every student.

$$\exists y \, (T(y) \wedge \forall x \, (P(x) \to E(x,y)))$$

Binary relations on set $\longleftrightarrow$ digraphs

paths

cycles.

$G^k = (V, E^k)$

$(a,b) \in E^k$

if there is a path from a to b of length exactly k.

$E^1 = E$

$E^k = E \circ E^{k-1}$

$$G^+ = \underbrace{G \cup G^2 \cup G^3 \cup \cdots}_{\text{Vertex set same adding in edges.}}$$

(a,b) is an edge in G^+ if there is a path from a to b in G of any length.

$G = (V, E)$ $\qquad |V| = n.$

Theorem

$$G^+ = \underbrace{G^1 \cup G^2 \cup G^3 \cdots \cup G^{n-1}}_{G'}$$

any edge in G' is also an edge in G^+

$\rightarrow G' = G^1 \cup G^2 \cup \cdots \cup G^{n-1}$

$\rightarrow G^+ = G^1 \cup G^2 \cup \cdots \cup G^{n-1} \cup G^n \cup G^{n+1} \cdots$

$E' \subseteq E^+$

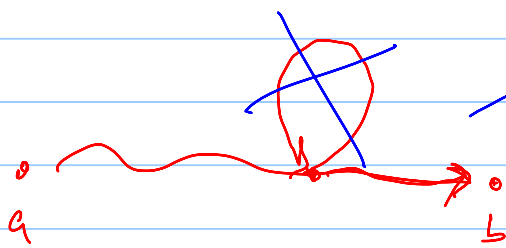
Proof by contradiction.

Suppose (a,b) in E^+ that is not in E'

(a,b) is an edge in G^{n+c}

but not in G or $G^2 \cdots G^{n-1}$

Path



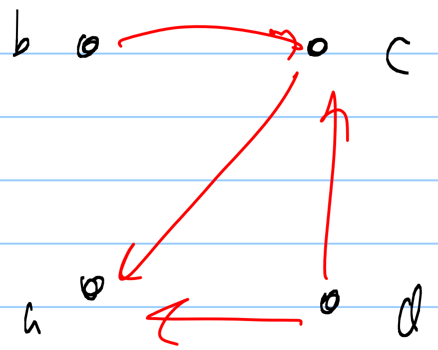
of length $n+c$.

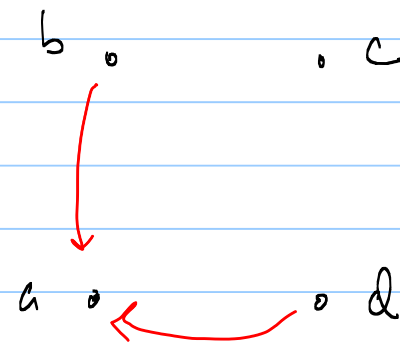
not shortest path for a to b.

$\langle a, v_1, v_2, \ldots, v_{n+c}, b \rangle$

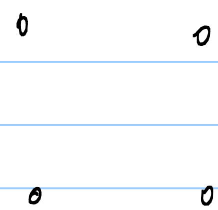
vertices in path is $n+c+1 > n$.

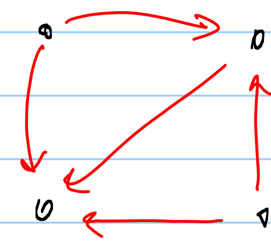
G $\qquad\qquad$ G^2



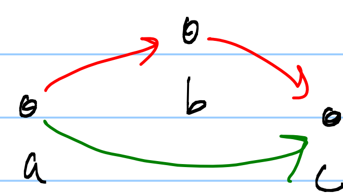


G^3 $\qquad\qquad$ G^+





G^+ is transitive:



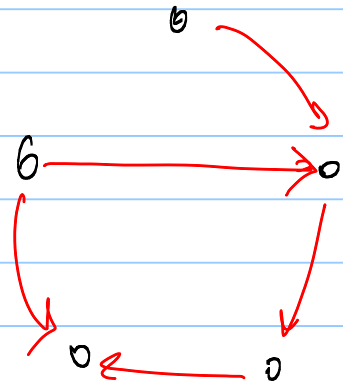
(a,b) in G^+ path from a to b in G

(b,c) in G^+ $\Leftrightarrow$ path from b to c in G.

(a,c) edge in G^+ $\Leftarrow$ there is a path from a to c in G

G^+ transitive closure of G.

G:



G^+