Note Title 11/4/2013

$\exists x \, (P(x) \rightarrow \_\_)$

$\neg \, (\underline{\underline{x \geq 0 \text{ and } x \leq 3}})$

$0 \leq x \leq 3$

$\neg (x \geq 0) \vee \neg (x \leq 3)$

$x < 0 \vee x > 3$

G directed graph $\longleftrightarrow$ E relation on set V.

$G = (V, E)$

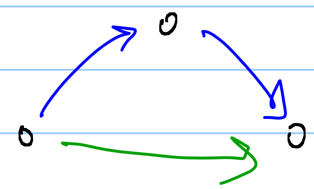
$G^k = (V, E^k)$

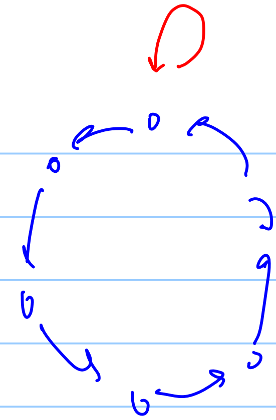
an edge in G^k iff there is a path of length k in G.

$G^+ = G \cup G^2 \cup \ldots$

$\hookrightarrow$ (u,v) an edge in G^+ if there is any path from u to v. in G

$G^+ = G \cup G^2 \cdots \cup G^n$





transitive closure.

Partial Order

A relation is a partial order if reflexive, transitive, anti-symmetric.

aRb $\qquad$ $a \preceq b$

"a is at most b".

Set A along w/ a partial order defined on it

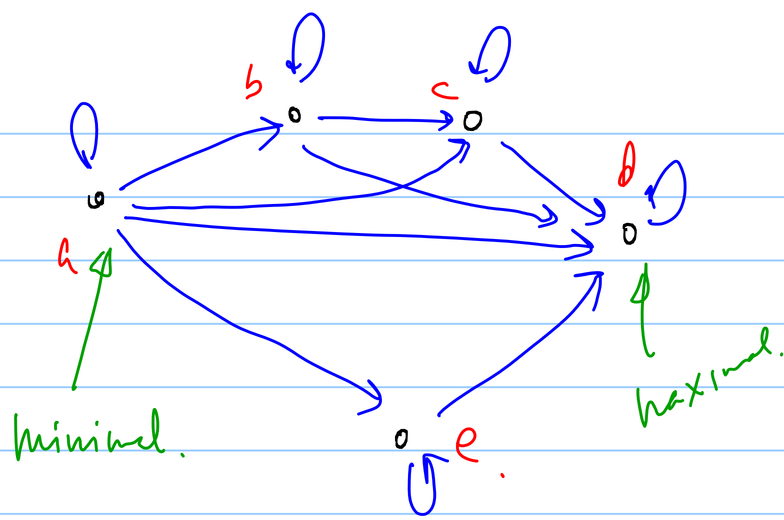
$(A, \preceq)$ is called a partially ordered set.

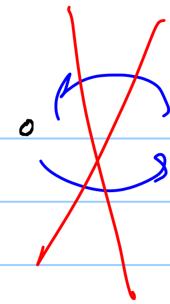
"poset".

$(\mathbb{Z}, \leq)$ refl: $x \leq x$

transitive: $x \leq y$ & $y \leq z \Rightarrow x \leq z$

anti-symmetric: $x \leq y$ and $y \leq x \Rightarrow x = y$.





if $x \preceq y$ or $y \preceq x$
then x & y are comparable

Otherwise they are
incomparable.

a & c are comparable ($a \preceq c$)

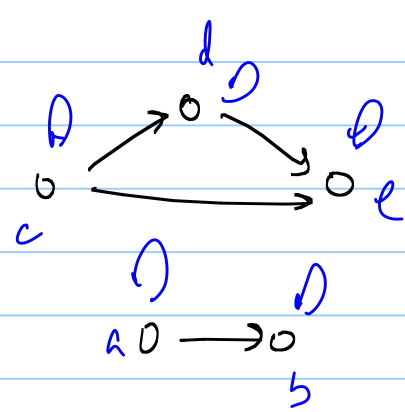
c & e are not comparable.

~~In a~~ total order is poset in which
every two elements are comparable,
Comparable.

$(\mathbb{Z}, \leq)$ is a total order.

An element x is minimal if there is no y s.t. $y \preceq x$.

" maximal " $y \succeq x$.



minimal: c & a.
maximal: e & b.

Domain $\mathbb{Z}$ $\quad x \preceq y$ if $y = x^n \quad n \in \mathbb{Z}^+$

$\Rightarrow$ reflexive $\quad x \preceq x \quad x = x^1$

$\Rightarrow$ transitive (shown before)

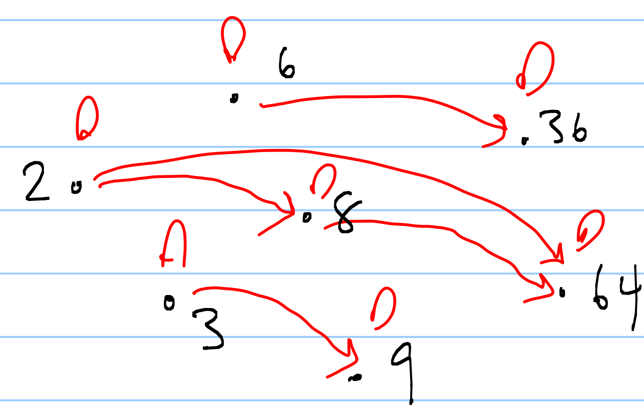
$\Rightarrow$ anti-symmetric.

$$\left.\begin{array}{l} y = x^n \\ x = y^m \end{array}\right\} \Rightarrow x = y.$$

$$\left(\text{if } x \preceq y \text{ then } x \le y\right).$$

$$8 \not\preceq 9$$
$$9 \not\preceq 8$$

$A = \{2, 8, 64, 3, 9, 6, 36\}$



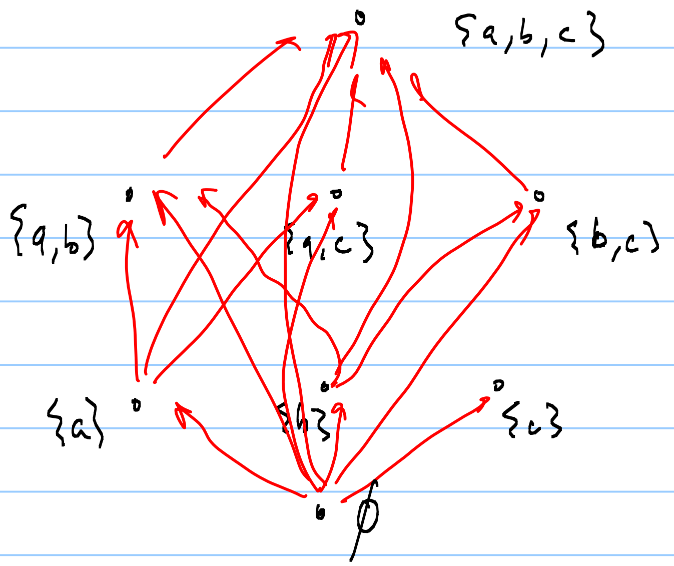
$A = \{a, b, c\} \qquad (P(A), \subseteq)$

$X \subseteq A \quad X \in P(A) \quad X \preceq Y$ if $X \subseteq Y$

$Y \subseteq A \quad Y \in P(A)$

A.

P(4)

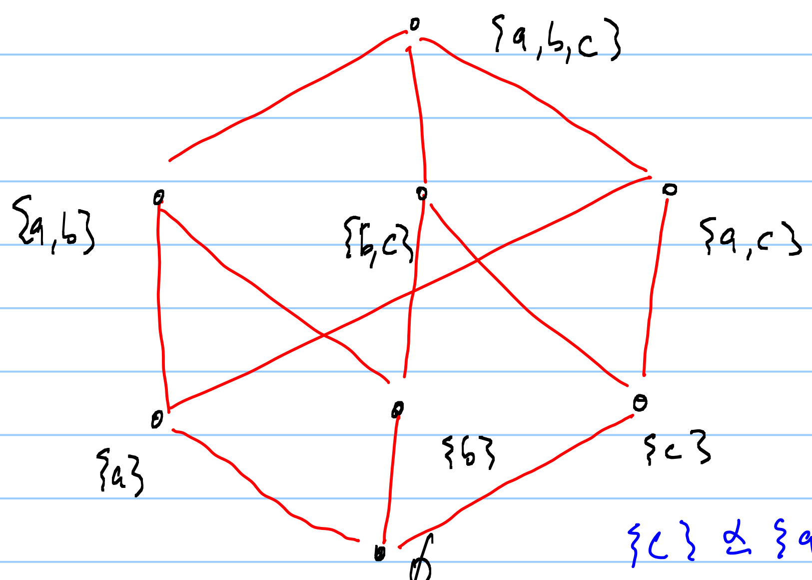


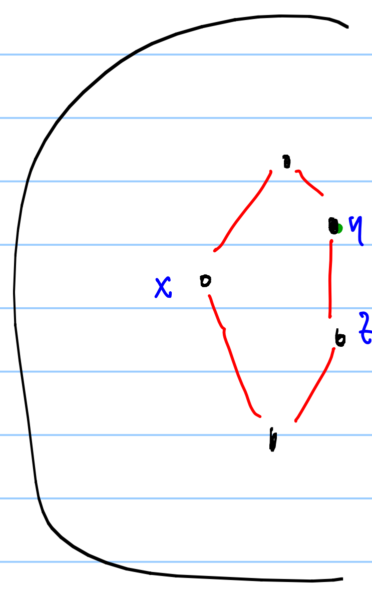
+ self-loops.

Hasse Diagram

if $x \preceq y$ then x appears lower than y.

if $\underline{x \preceq y}$ and there is no z $x \preceq z \preceq y$ then draw an edge.

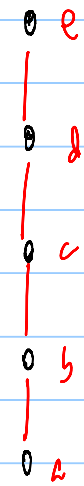




$\{c\} \preceq \{a,c\}$

Hasse diagram for a total order

$\{a, b, c, d, e\}$.



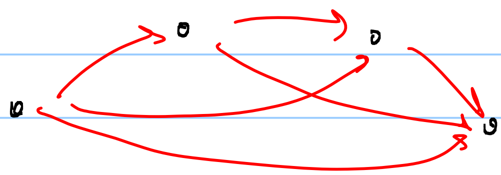
Partial Order: $\leq$ relation on elements.

Strict Order $<$

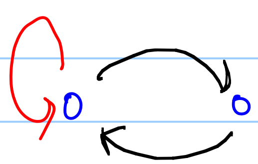
A relation R on A is a strict order if transitive and anti-reflexive.

$x < y$ x is less than y.

example



If R is transitive & anti-reflexive $\Rightarrow$ it is also anti symmetric



Assume not anti symmetric

Prove: not transitive or not anti-reflexive.