Note Title 11/25/2013

$\cdot$	$\wedge$
$+$	$\vee$
$\overline{x}$	$\neg$

Boolean function:

One or more boolean variables.
to $\{0,1\}$.

$B = \{0,1\}$

$f: B^n \to B$.

$\rightarrow 2^n$ rows in an input/output table.

$f: B^2 \to B$

x	y	$f(x,y)$
0	0	1
0	1	1
1	0	0
1	1	1

$\rightarrow f(x,y,z) = \bar{z} + xyz = 1$.

$1 + \cancel{1 \cdot 0 \cdot 0}$.

$x = 1$
$y = 0$
$z = 0$

Boolean function defined by a Boolean expression $\Longrightarrow$ input/output table.

$\Longleftarrow$??

[Given an input/output table defining a Boolean fcn is there a boolean expression that defines the same function?]

$f(x_1, \ldots, x_n)$ n Boolean variables.

Literal: variable x_i or $\overline{x_i}$

Minterm: Product of literals (one literal for each input variable).

→ $y_1 y_2 \cdots y_n$ y_i is either x_i or $\overline{x_i}$

$x_1 \overline{x_2}\, \overline{x_3} \cdots x_n$

$f(x, y, z)$: $x\bar{y}z$ $\bar{x}\bar{y}\bar{z}$ xyz

for a n variables: # minterms $= 2^n$.

Minterm corresponds setting of input variables.

$x\bar{y}\bar{z} \iff x=1 \quad y=0 \quad z=0.$

$\bar{x}y\bar{z} \iff x=0 \quad y=1 \quad z=0$

↓ only one that makes the minterm = 1.

	x	y	z	f(x,y,z)	
$\bar{x}\bar{y}z$	0	0	1	1	—
$\bar{x}y\bar{z}$	0	1	0	1	—
	0	1	1	0	
	1	0	0	0	
$x\bar{y}z$	1	0	1	1	—
	1	1	0	0	
xyz	1	1	1	1	—

$f(x,y,z) =$

$\bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}z + xyz$

Every boolean fcn can be expressed using

$+ \cdot -$

The set $\{+, \cdot, -\}$ is functionally complete

Is the set $\{\cdot, -\}$ functionally complete?

$x + y = \overline{\overline{x+y}} = \overline{\bar{x}\bar{y}} = \bar{\bar{x}}\bar{\bar{y}}$

$\overline{x+y} = \bar{x}\bar{y}$

$\neg(x \vee y) = \neg x \wedge \neg y$

$$z(x+y)+\bar{z} = z(\bar{x}\bar{y})+\bar{z}$$

$$\overline{\overline{z(\bar{x}\bar{y})}\cdot\bar{z}} = \overline{\overline{z(\bar{x}\bar{y})}\cdot z}$$

The set $\{\cdot, -\}$ is functionally complete.

What about $\{+, -\}$? Yes.

$$xy = \overline{\overline{xy}} = \overline{\bar{x}+\bar{y}}$$

$$zy(x+\bar{z}) = (\bar{z}+\bar{y})(x+\bar{z})$$

$$= \overline{\overline{\bar{z}+\bar{y}} + \overline{x+\bar{z}}}$$

$$= \overline{\bar{z}+\bar{y} + x+\bar{z}}$$

What about $\{+, \cdot\}$?

$f(0) = 1$
$f(1) = 0.$

Is there any single Boolean operation that is functionally complete by itself?

$x \uparrow y$ NAND

x	y	$x \uparrow y$
0	0	1
0	1	1
1	0	1
1	1	0

$\Rightarrow (x \uparrow x) = \overline{x}$

x	$x \uparrow x$
0	1
1	0

Show can compute $\overline{\ }$ ✓.
u/ only $\uparrow$.

$x \cdot y = \overline{x \uparrow y}$

$= (x \uparrow y) \uparrow (x \uparrow y)$

x	y	xy	$\overline{x \uparrow y}$
0	0	0	0
0	1	0	0
1	0	0	0
1	1	1	1