Note Title 11/27/2013

Functionally Complete Gate Sets: $\{+ \cdot -\}$

$\{+ -\}$

$\to \{\cdot -\}$

mult can be "simulated" by $+$ $-$

NAND $x \uparrow y$

Gen expns $-$ & $\cdot$ using only NAND.

x	y	$x \uparrow y$	$(x\uparrow y) \uparrow (x \uparrow y)$
0	0	1	0
0	1	1	0
1	0	1	0
1	1	0	1

$x \cdot y$

$x \uparrow x$

x	$x \uparrow x$
0	1
1	0

$$x \cdot y = \overline{x \uparrow y} = (x\uparrow y) \uparrow (x \uparrow y)$$

Conjunctive Normal Form. : AND of OR's

* Disjunctive Normal Form OR of AND's.

DNF Boolean mult: only applied to literals: x $\bar{x}$

DNF: $\quad x\bar{y} + z + \bar{z}\bar{x}y$

$$zx\bar{y} + \bar{x}yw + \bar{w}xz$$

$$zxy$$

$$x + y + z$$

Not DNF $\quad (x+y)z$

Any boolean function has an equivalent Boolean expression in DNF form.

1/0 table $\longrightarrow$ Bool exp
gave an expression in DNF form.

$$f(x,y,z) = xyz + \bar{x}yz$$
$$= (x+\bar{x})yz$$
$$= 1\cdot yz = \underline{\underline{yz}}$$

Conjunctive Normal Form: And of Or's

$+$

OR only applied to literals

$(x+y)(\bar{x}+\bar{z})(x+w+y+z)$

$x(y+z)w$

$xy\bar{z}$

$(x+y+z)$

NOT CNF $\quad x+y+z\bar{y}$

Both CNF + DNF $\quad xy\bar{z}$

$x+\bar{y}+z$

Boolean Satisfiability (SAT)

Input: Boolean expression w/ n boolean variables → CNF

Output: Is there a setting for input values that causes the function to evaluate to 1?

$$f(x,y,z) = (\overset{0}{x}+\overset{1}{y})(\overset{1}{\bar{x}}+\overset{0}{\bar{y}}+\overset{0}{\bar{z}})\overset{1}{z}$$

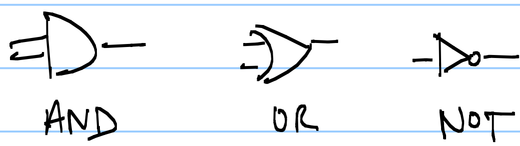
$z=1$

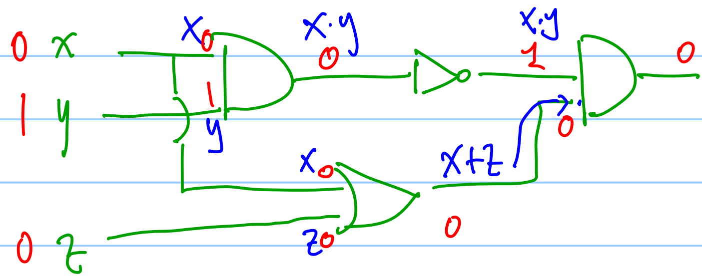
$x=0$

$y=1$

$f(x,y) = (x+y)(x+\bar{y})(\bar{x}+y)(\bar{x}+\bar{y})$ Not Satisfiable

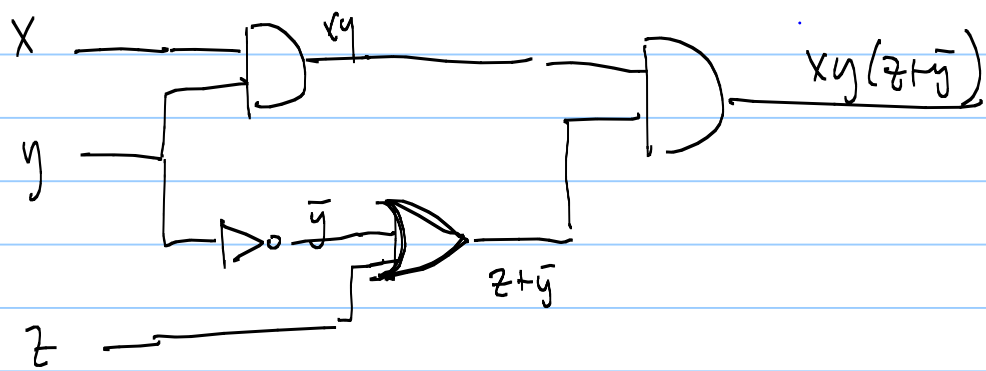
0





$\overline{x \cdot y}\,(x+z)$

$x y (z+\bar{y})$



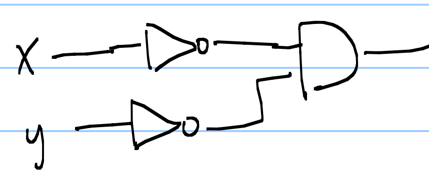
NOR Gate!

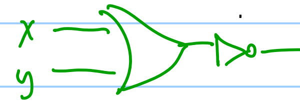
x	y	$\overline{x+y}$
0	0	1
0	1	0
1	0	0
1	1	0

$\overline{(x+y)}$

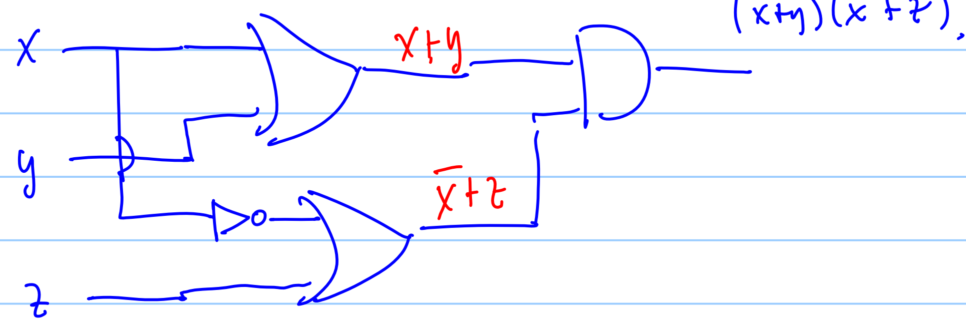
//

$f(x,y) = \bar{x}\bar{y}$

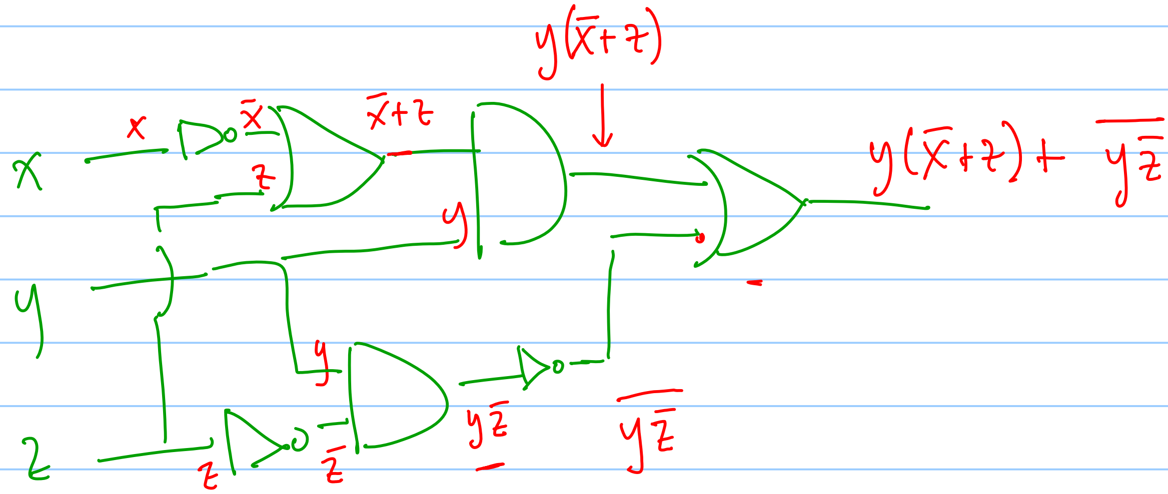




$(x+y)(\bar{x}+z)$



x
y
z
x+y
$\bar{x}+z$
$(x+y)(\bar{x}+z)$.



x
y
z
x
$\bar{x}$
z
$\bar{x}+z$
y
$y(\bar{x}+z)$
$y(\bar{x}+z)+\overline{y\bar{z}}$
y
z
$\bar{z}$
$y\bar{z}$
$\overline{y\bar{z}}$