ICS 6B	**Due: Wed, November 27, 2011**

Homework 8

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1. Which of the following mathematical expressions are functions from $\mathbb{R}$ to $\mathbb{R}$? If f is a function, give its range.
   (a) $f(x) = \sqrt{x}$
   (b) $f(x) = 1/(x^2 - 4)$
   (c) $f(x) = \sqrt{x^2}$.

2. Consider the following functions from $\mathbb{Z} \times \mathbb{Z}$ to $\mathbb{Z}$. Which ones are onto?
   (a) $f(x, y) = x^2 - y^2$.
   (b) $f(x, y) = |x| - |y|$.
   (c) $f(x, y) = x + y - 2$.

3. Find the following values:
   (a) $\lceil \frac{5}{7} \rceil$
   (b) $\lceil -\pi \rceil$
   (c) $\lceil 4.000001 \rceil$
   (d) $\lfloor \frac{5}{7} \rfloor$
   (e) $\lfloor -3 \rfloor$
   (f) $\lfloor -4.2 \rfloor$
   (g) $\lfloor \frac{1}{2} + \lfloor \frac{1}{2} \rfloor \rfloor$

4. Define $f(x) = \lceil x + \frac{1}{2} \rceil$.
   (a) Evaluate $f(2.3)$, $f(3.8)$ and $f(2.5)$.
   (b) Describe in words what the functin f does.

5. Suppose that two functions f and g are both bijections and the target of f is the domain of g. Express the inverse of $f \circ g$ in terms of f^{-1} and g^{-1}.

6. Consider three functions, f, g, and h whose domain and target are $\mathbb{Z}$. Let $f(x) = x^2$, $g(x) = \lceil \log_2(x) \rceil$, and $h(x) = \lceil \frac{x}{5} \rceil$
   (a) Evaluate $f \circ g(4)$.
   (b) Evaluate $g \circ h(117)$.
   (c) Give a mathematical expression for $h \circ f$.
   (d) Give a mathematical expression for $f \circ g$.
   (e) What is $h \circ g \circ f(12)$?

7. Determine which of the following functions are bijections. If it is not a bijection, indicate whether it is fails to be one-to-one, onto or both. If it is a bijection, give its inverse.

(a) $f : \mathbb{R} \to \mathbb{R}$. $f(x) = \lceil \frac{x}{5} \rceil$.

(b) $f : \mathbb{R} \to \mathbb{R}$. $f(x) = x^2 + 1$.

(c) $f : \mathbb{R} \to \mathbb{R}$. $f(x) = 3x - 5$.

(d) Let $A = \{a_1, a_2, \ldots, a_n\}$. $f : P(A) \to P(A)$. For $X \subseteq A$, $f(X) = \overline{X}$.

(e) Let $A = \{a_1, a_2, \ldots, a_n\}$. $f : P(A) \to \{0, 1, \ldots, |A|\}$. For $X \subseteq A$, $f(X) = |A|$.

8. Give the first ten terms of the following sequences. You can assume that the sequences start with an index of 1.

   (a) The n^{th} term is $\lceil \sqrt{n} \rceil$.

   (b) The first two terms in the sequence are 1. The rest of the terms are the sum of the two preceding terms.

   (c) The n^{th} term is the largest integer k such that $k! \leq n$.

9. Use the ceiling and floor functions to give a mathematical expression for the following values:

   (a) There are x children in the first grade at Lee Elementary school. Each child will be given five crayons to do an art project. Crayons come on boxes of 24. How many boxes need to be purchased for the art poject?

   (b) A baker is packagaing cookies for sale in boxes of 8. He has y cookies to put into boxes. How many boxes can he sell?

10. Evaluate the following summations:

    (a) $\sum_{j=-1}^{4} j^2$

    (b) $\sum_{k=0}^{4} 2^k$

    (c) $\sum_{k=0}^{100} 3 + 5k$

    (d) $\sum_{k=0}^{100} 3 \cdot (1.1)^k$

11. A silicon valley billionaire purchases 3 new cars for his collection at the end of every month. Let a_n denote the number of cars he has after n months. Let $a_0 = 23$.

    (a) What is a_8?

    (b) If he pays \$50 each month to have a car maintained, how much does he pay for maintenance after 2 years? No need to calculate the actual number. Instead give a closed form (without the summation) mathematical expression for the number. Note that he purchases the new car at the end of each month, so during the first month, he is only maintaining 23 cars.

12. A population of rabbits on a farm grows by 12% each year. Define a sequence $\{r_n\}$ describing the rabbit population at the end of each year. Suppose that the sequence starts with $r_0 = 30$.

    (a) Give a mathematical expression for r_{12}. (You don't have to actually compute the number).

    (b) If each rabbit consumes 10 pounds of rabbit food each year, then how much rabbit food is consumed in 10 years? For simplicity, you can omit the food consumed by the baby rabbits born in a given year. For example, suppose the farm starts tabulating rabbit food on January 1, 2012 at which time the rabbit population is 30. You will count the food consumed by those 30 rabbits during 2012. You won't count the food consumed by the rabbits born in 2012 until after January 1, 2013. Again, you don't have to compute the number, but you do have to give a closed form (without the summation) mathematical expression for the number.