Note Title 10/9/2013

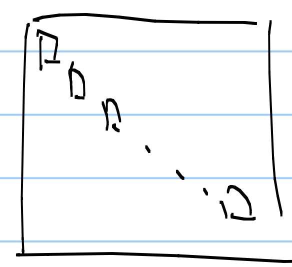
A set of vectors $\{\vec{v}_1 \cdots \vec{v}_p\}$ in $\mathbb{R}^n$

is linearly indep if the only solution to

$$x_1\vec{v}_1 + \cdots + x_p\vec{v}_p = \vec{0}$$ is trivial.

Linearly dep if ↑ has non-trivial solution.

$\{\vec{v}_1 \cdots \vec{v}_p\} \Rightarrow$ are they linearly indep??

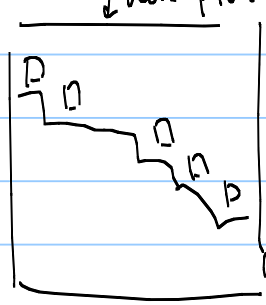
$A = [\vec{v}_1 \cdots \vec{v}_p] \rightsquigarrow$  $\vec{x} = \vec{0}$

row ech.

if every col is a pivot column

$\Rightarrow$ linearly indep

↓ non-pivot.

$\rightsquigarrow$ 

linearly dep

$\{\vec{v}_1\}$ $\quad \vec{v}_1 \neq \vec{0}$ $\quad$ lin indep.

$\{\vec{v}_1 \dots \vec{v}_p, \vec{0}\}$ $\Rightarrow$ lin dep.

$$0 \cdot \vec{v}_1 + \dots + 0\,\vec{v}_p + c\,\vec{0} = \vec{0}$$

$\{\vec{v}_1, \vec{v}_2\}$. $\quad \vec{v}_1 \neq \vec{0}$, $\vec{v}_2 \neq \vec{0}$. $\quad \vec{v}_1 \neq c\vec{v}_2$

linearly indep.

Theorem: $\{\vec{v}_1, \dots, \vec{v}_p\}$ is linearly dependent if & only if there is a vector $\vec{v}_i$ that can be expressed as a linear comb. of the others.

$\Rightarrow$ Assume $\{\vec{v}_1 \dots, \vec{v}_p\}$ lin dep

$$c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_i\vec{v}_i + \dots + c_n\vec{v}_n = \vec{0} \qquad c_i \neq 0.$$

divide by $c_i \neq 0$

$$\frac{c_1}{-c_i}\vec{v}_1 + \dots + \frac{c_{i-1}}{-c_i}\vec{v}_{i-1} + \frac{c_{i+1}}{-c_i}\vec{v}_{i+1} + \dots + \frac{c_n}{-c_i}\vec{v}_n = \cancel{c_i}\vec{v}_i = \vec{v}_i$$

$\Longleftarrow$ Assume $\overset{\vec 0}{\cancel{\vec v_i}} = c_1\vec v_1 + \cdots + c_{i-1}\vec v_{i-1} \overset{-\vec v_i}{+} c_{i+1}\vec v_{i+1} + \cdots + c_n\vec v_n$

Subtract $\vec v_i$

$$\vec 0 = c_1\vec v_1 + \cdots + c_{i-1}\vec v_{i-1} - \underbrace{1}_{\text{non-zero coeff.}}\cdot \vec v_i + c_{i+1}\vec v_{i+1} + \cdots + c_n\vec v_n$$

A $m\times n$ matrix columns $\vec a_1 + \cdots + \vec a_n \in \mathbb{R}^m$

System of lin eq $\Longleftrightarrow$ vector equation. $\Longleftrightarrow$ $A\vec x = \vec b$.

$$x_1\vec a_1 + \cdots x_n\vec a_n = \vec b$$

A as a function. $A\vec x = \vec b$ $\qquad \vec x \in \mathbb{R}^n$

$b \in \mathbb{R}^m$

A maps vectors $\mathbb{R}^n$ to vectors in $\mathbb{R}^m$

$$\begin{bmatrix} 1 & 3 & 4 & 1 \\ 2 & -1 & 0 & 5 \end{bmatrix} \underbrace{\begin{bmatrix} 1 \\ 0 \\ 1 \\ -2 \end{bmatrix}}_{\text{input}} = \underbrace{\begin{bmatrix} 3 \\ -8 \end{bmatrix}}_{\text{output.}}$$

$$\begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ 12 \end{bmatrix}$$

input output.

A transformation (or function or mapping) T from $\mathbb{R}^n$ to $\mathbb{R}^m$ is a rule that assigns a vector from $\mathbb{R}^m$ to every vector in $\mathbb{R}^n$.

$$\vec{x} \in \mathbb{R}^n \qquad \underline{\underline{T(\vec{x})}} \in \mathbb{R}^m$$

$$T : \mathbb{R}^n \to \mathbb{R}^m$$

domain co-domain target.

The set of all $T(\vec{x})$ is called the range of T.

We're interested in functions defined by matrix mult.

A is $m \times n$ matrix $\qquad T(\vec{x}) \overset{\Delta}{=} A\vec{x}$

$T: \mathbb{R}^2 \to \mathbb{R}^3$ defined by matrix $[A] = \begin{bmatrix} 1 & 1 \\ 2 & 0 \\ -1 & 3 \end{bmatrix}$

① Given $\vec{u} \in \mathbb{R}^2$ compute $T(\vec{u})$.

$$\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad T(\vec{u}) = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

② Given $\vec{b} \in \mathbb{R}^3$ find an $\vec{x} \in \mathbb{R}^2$ s.t. $T(\vec{x}) = \vec{b}$

$$A\vec{x} = \vec{b}$$

$$\vec{b} = \begin{bmatrix} -1 \\ 3 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 3 \\ 3 & 0 & -3 \end{bmatrix} \rightsquigarrow \begin{array}{c} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{array}{ccc} \uparrow & \uparrow & \\ x_1 & x_2 & \end{array} \end{array}$$

$x_2 = 2$

$x_1 = -1$

$$\begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -3 \end{bmatrix}$$

③ Given $\vec{c}$ is $\vec{c}$ in the range of T?

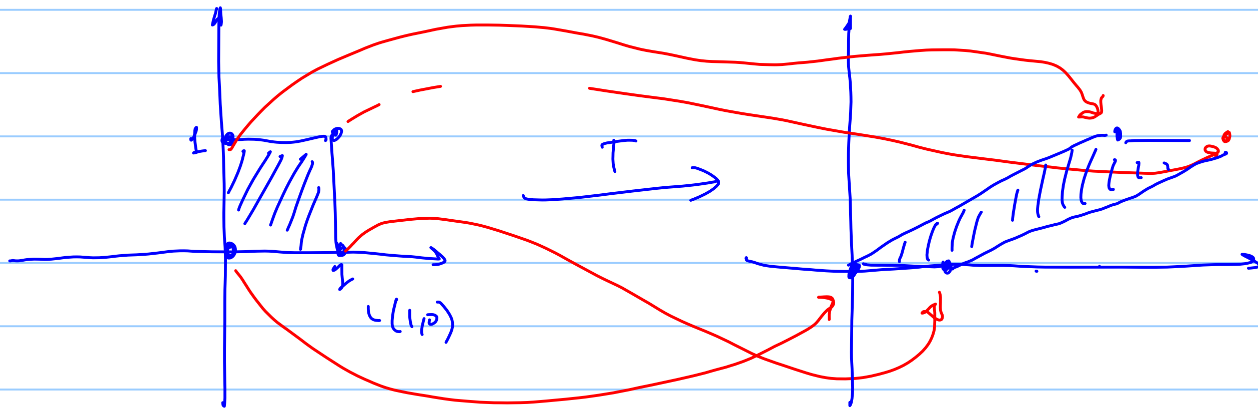
④ Is there more than one $\vec{x}$ that maps on to a particular c?

$T: \mathbb{R}^3 \to \mathbb{R}^3$ $\qquad A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$A\vec{x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix}$$

example of a projection.

$T: \mathbb{R}^2 \to \mathbb{R}^2$ $\qquad A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$



$A\vec{0} = \vec{0}$ $\qquad \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\qquad \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\qquad \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$