Note Title 10/14/2013

$T: \mathbb{R}^n \to \mathbb{R}^m$

$T(\vec{x}) = A\vec{x}$ $\longleftrightarrow$ linear: $T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v})$, $T(c\vec{u}) = cT(\vec{u})$

(A: m x n matrix)

$A = [\,T(\vec{e}_1) \;\cdots\; T(\vec{e}_n)\,]$ $\qquad \vec{e}_j = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ ← jth place

A function is <u>onto</u> if for every $\vec{b} \in \mathbb{R}^m$ there exist $\vec{x} \in \mathbb{R}^n$ $T(\vec{x}) = \vec{b}$.

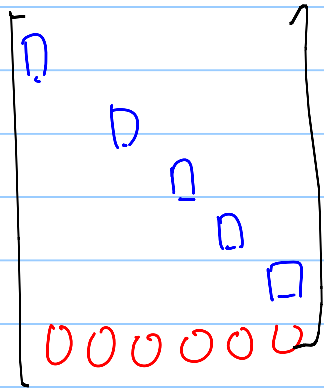
A function is 1-to-1 if:

$\vec{x} \neq \vec{y} \implies T(\vec{x}) \neq T(\vec{y})$.

T linear defined m x n matrix A.

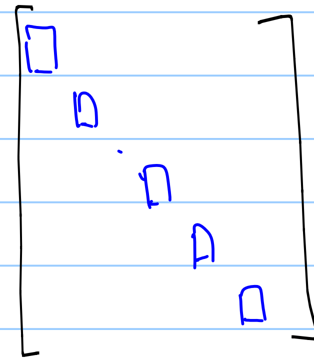
Look at row echelon form of A:

Is T onto? $\qquad A\vec{x} = \vec{b}$ consistent for every $\vec{b}$?



There is a row with no pivot (all 0's).

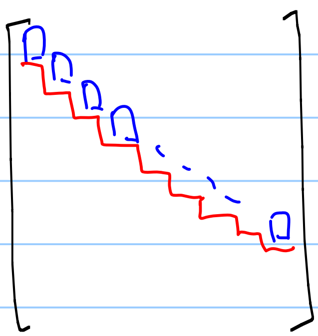
T is not onto.



pivot in every row.

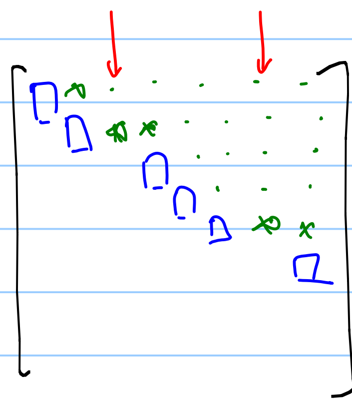
T is onto.

Is T one-to-one?



Every column is a pivot col.
No free variables.

1-1



There are cols that are not pivot cols.

A has free variables
not 1-1.

⇒ Every matrix is row equiv to a unique matrix in row ech form. False.

⇒ Any system of n linear eq w/ n variables has n solutions. False

⇒ If a system of lin eq has one solution, it has an inf # of solutions. False

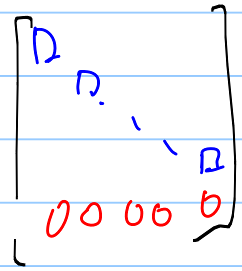
⇒ If a system of lin eq has no free variables it has a unique solution.

⇒ Any matrix $[A\ b]$ by a series of row ops $\leadsto$ $[C\ d]$.

Then $[A\ b]$ & $[C\ d]$ have exactly the same set of sols. True

If a system $A\vec{x} = \vec{b}$ has more than one solution then so does $A\vec{x} = \vec{0}$ True.

If A is an $m \times n$ matrix and $A\vec{x} = \vec{b}$ is consistent then the cols of A span $\mathbb{R}^m$.



$\exists\, b$ $A\vec{x} = \vec{b}$ has a solution

but cols do not span $\mathbb{R}^n$.

The equation $A\vec{x} = \vec{0}$ has only the trivial solution

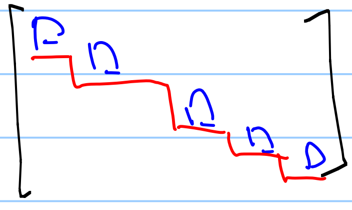
if and only if there are no free variables.

False.   W/ the "only" it's true.

If A is an $m \times n$ matrix and $A\vec{x} = \vec{b}$

consistent for every $\vec{b}$

then A has m pivot columns.



$m < n$

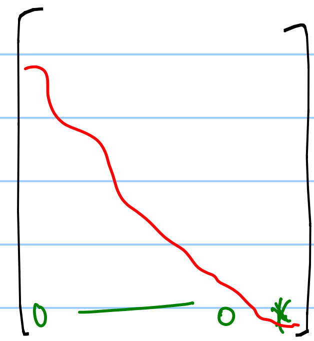
True

$A : m \times n$.

$T(\vec{x}) = A\vec{x}$

$A\vec{x} = \vec{0}$

Is there a pivot in every row?



$A\vec{x} = \vec{b}$ consistent for every $\vec{b}$?

NO   YES

Do the cols of A span $\mathbb{R}^m$?

NO   YES

Is T onto?

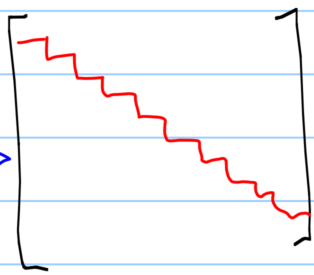
NO   YES.

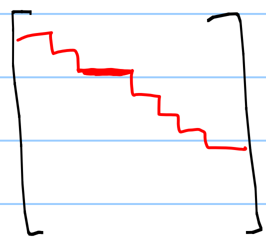
Is every $\vec{b}$ in the span of the cols of A?

NO   YES.

Is there a pivot in every column?

$m \geq n$





YES   NO.

Are there free vars?

NO   YES

If $A\vec{x} = \vec{b}$ is consistent, is $\vec{x}$ unique?

YES NO

Does $A\vec{x} = \vec{0}$ have a non-trivial solution?

NO YES.

Are cols of A linearly dep?

NO YES.

$T(\vec{x})$ is one to-one?

YES NO