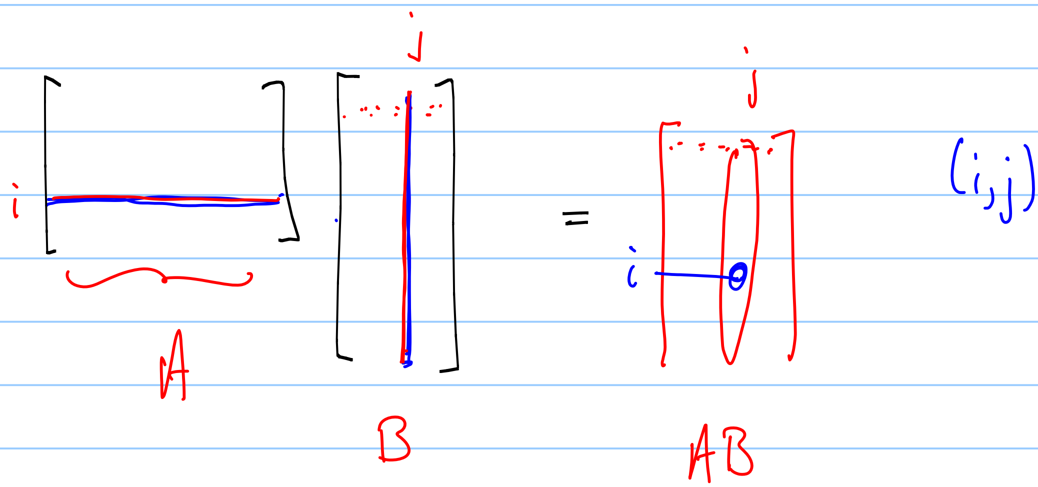
Section 2.1, cont. Matrix Operations.



$$a_{i1}\cdot b_{1j} + a_{i2}\cdot b_{2j} + \dots + a_{in} b_{nj} = \cancel{(ab)_{ij}} \quad (i,j)\text{-entry of } AB.$$

Warning! $\quad AB \neq BA$ in general.

$AB = AC$ does not imply $B = C$.

$$\underset{A}{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}} \underset{B}{\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}} = \underset{A}{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}} \underset{C}{\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}}$$

$AB = 0$ does not imply $A = 0$ or $B = 0$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

A $m \times n$ matrix

Transpose of A : A^T $n \times m$ $\qquad a_{ij} = (a^T)_{ji}$.

$$(A^T)^T = A$$

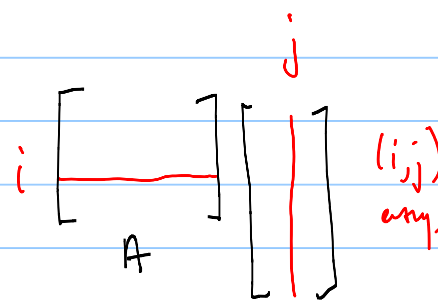
$$(A+B)^T = A^T + B^T$$

$$(rA)^T = rA^T$$

$$(AB)^T = B^T A^T$$

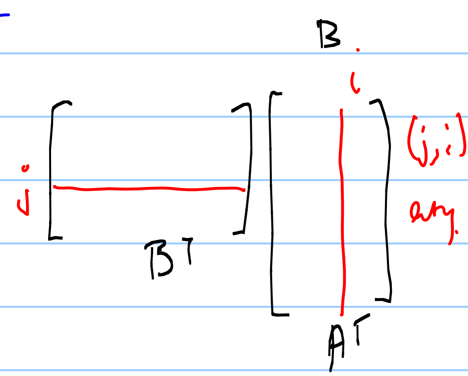
A $\quad B \quad \longrightarrow \quad AB$

$m \times n \quad n \times l \qquad\quad m \times l$.

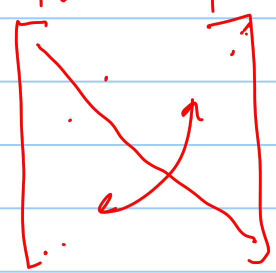


$B^T \quad A^T \quad \longrightarrow \quad (AB)^T$

$l \times n \quad n \times m \qquad\quad l \times m$



For a square matrix:



A^T

Section 2.2. Inverse of a Matrix

For real number x, there is some y $\quad x \cdot y = 1 = y \cdot x$

Inverse of 5 is $1/5$, 5^{-1}

$$5 \cdot 5^{-1} = 5^{-1} \cdot 5 = 1.$$

A matrix A is **invertible** if it is a square matrix and if there is a matrix C such that

$$AC = CA = I$$

If A has an inverse, then it is unique.

$$AC = CA = I$$
$$AB = BA = I$$

$$B = B \cdot I = B \cdot A \cdot C = I \cdot C = C$$

The inverse of A is denoted A^{-1}

Theorem If A is a $n\times n$ invertible matrix for every $\vec{b} \in \mathbb{R}^n$ there is a unique solution to $A\vec{x} = \vec{b}$.

$$\vec{x} = A^{-1}\vec{b}$$

$$\underbrace{A \cdot A^{-1}}\vec{b} = I\vec{b} = \vec{b}.$$

$$\vec{x} = A^{-1}\vec{b}$$

Some x satisfies $A\vec{x} = \vec{b}$

$$A^{-1}A\vec{x} = A^{-1}\vec{b}$$

$$\vec{x} = A^{-1}\vec{b}.$$

For 2×2 matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = A$

A is invertible iff $ad - bc \neq 0$.

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\left.\begin{array}{l} 2x_1 - x_2 = 4 \\ 3x_1 \quad\quad = 6 \end{array}\right\}$$

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix} \qquad 2\cdot 0 - (-1)\cdot 3 = 3 \neq 0,$$

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 0 & 1 \\ -3 & 2 \end{bmatrix}$$

$$\frac{1}{3}\begin{bmatrix} 0 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 6 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \qquad \begin{array}{l} x_1 = 2 \\ x_2 = 0. \end{array}$$