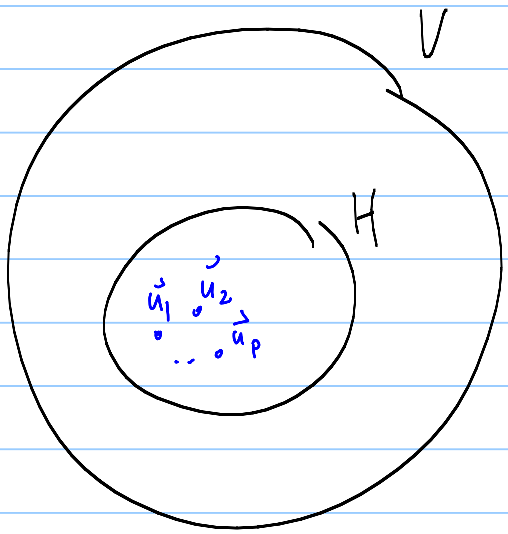
Note Title 11/4/2013

Section 4.5 (Dimension of a Vector Space).



H subspace of vector space V.

$\vec{u}_1 \ldots \vec{u}_p \in H$
linearly indep.

Can add in vectors from H
to get a basis for H.

$\Downarrow$

$\dim H \leq \dim V.$

V p-dimensional vector space:

p lin indep vectors from V $\Rightarrow$ basis.
p vectors whose span is V $\Rightarrow$ basis.

Matrix A $\quad m \times n \quad$ col A
nul A

col A $\quad$ Span of columns. $\quad$ Subspace of $\mathbb{R}^m$ — dim is # pivot colums.
nul A $\quad A\vec{x} = \vec{0} \quad$ Subspace of $\mathbb{R}^n$ — dim is # free variables.

Section 4.6 $\Rightarrow$ col A.

A $m \times n$ matrix. Each row is an element of $(\mathbb{R}^n)^T$.

Row space of A : Space spanned by the rows of A.

Will see dim col A = dim row A.

row A = Col (A^T)

__Theorem__: If two $m \times n$ matrices A + B are ~~rows~~ row equivalent then row space A = row space of B.
If B is in row echelon form the non-zero rows form a basis of row(B).

rows of A: $\vec{r}_1 \cdots \vec{r}_m$ rows of B: $\vec{s}_1 \cdots \vec{s}_m$.

row space of A
$= \text{Span}\{\vec{r}_1, \dots, \vec{r}_m\}$.

$$\left.\begin{matrix} \blacksquare & * & * & * & * & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \blacksquare & * & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \blacksquare & * & * & \cdot \\ 0 & 0 & 0 & 0 & 0 & 0 & \blacksquare & * \end{matrix}\right\} \text{lin indep.}$$
$$\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

Col A = row A   nul A

= dimension.

A $m \times n$ matrix

What dim row A? # pivot rows

= # pivot cols.

= # pivots.

4×5 = dim col A

$$\begin{array}{c} \vec{s}_1 \\ \vec{s}_2 \\ \vec{s}_3 \\ \vec{s}_4 \end{array} \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix} \sim \begin{array}{c} \vec{r}_1 \to \\ \vec{r}_2 \to \\ \vec{r}_3 \to \\ \\ \end{array} \begin{bmatrix} \boxed{1} & 3 & -5 & 1 & 5 \\ 0 & \boxed{1} & -2 & 2 & -7 \\ 0 & 0 & 0 & \boxed{-4} & 20 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ basis of row space.}$$

$$\operatorname{Span}\{\vec{r}_1, \vec{r}_2, \vec{r}_3\} = \operatorname{Span}\{\vec{s}_1, \vec{s}_2, \vec{s}_3, \vec{s}_4\}$$

Basis of col space.

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -2 & 0 & 3 \\ 0 & 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Nul (A)

$$\begin{bmatrix} -x_3 - x_5 \\ 2x_3 - 3x_5 \\ x_3 \\ 5x_5 \\ x_5 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -1 \\ -3 \\ 0 \\ 5 \\ 1 \end{bmatrix}$$

dim Nul = # free variables.

dim Col A = # pivot columns

dim Nul A = # free variables

Def Rank of a matrix A = $\dim(\text{col } A)$
$= \dim(\text{row } A)$.

Thm A $m \times n$ matrix

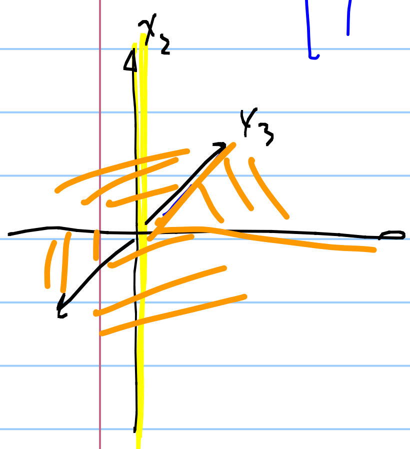
$$\dim(\text{row } A) = \dim(\text{col } A)$$

$$\underbrace{\text{rank of } A}_{\#\text{ pivot cols}} + \underbrace{\dim \text{Nul } A}_{\#\text{ free vars } = \#\text{ non-pivot cols.}} = n.$$

$$\begin{bmatrix} 3 & 0 & 1 \\ 2 & 0 & 2 \\ 1 & 0 & -1 \end{bmatrix}$$

Null Space: $\begin{bmatrix} 0 \\ x \\ 0 \end{bmatrix}$

Row Space: $\begin{bmatrix} y \\ 0 \\ z \end{bmatrix}$



Addendum to Invertible Matrix Theorem.

A is an $n \times n$ matrix.

A is invertible iff

$\rightarrow$ Col of A are a basis of $\mathbb{R}^n$

$\rightarrow$ Col A $= \mathbb{R}^n$

$\rightarrow$ dim of Col A $= n$.

$\rightarrow$ rank of A $= n$.

$\rightarrow$ nul A $= \{\vec{0}\}$.

$\rightarrow$ dim nul A $= 0$.

A is 6×9 matrix Can it have 2-D null space?

$$\underbrace{\text{dim nul } A}_{2} + \text{dim row } A = 9$$

$\Downarrow$

7.