Note Title 11/6/2013

System of 40 linear equations
42 variables.

$\Rightarrow$ 2 solutions to homogeneous system
NOT multiples of each other.

Any solution to the homogeneous system
is a linear comb of these two solutions.

$= 0$
$= 0$
$\vdots$
$= 0$

Can we be certain that any associated
non-homogeneous system has solution?

$A\vec{x}_1 = \vec{0}$ $\qquad$ $A\vec{x}_2 = \vec{0}$

$\{\vec{x}_1, \vec{x}_2\}$ linearly indep.

any $\vec{x}$ $\quad A\vec{x} = \vec{0}$ $\quad \vec{x} \in \text{Span}\{\vec{x}_1, \vec{x}_2\}$.

$\dim(\text{nul } A) = 2$.
$\dim(\text{col } A) = 40$

col A is subspace of $\mathbb{R}^{40}$
dim is 40.

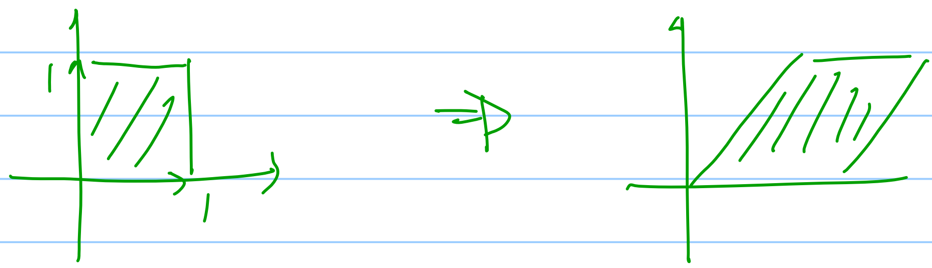
col of A span $\mathbb{R}^{40}$

For any $\vec{b}$ $\quad A\vec{x} = \vec{b}$ will have a solution.

Determinants:

Determinant assigns a real # to every matrix.

$\Rightarrow$ [$\det(A) \neq 0$ iff A is invertible.]



A

For 2×2 matrix $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$

A is invertible iff $ad - bc \neq 0$.

A is in row ech form. $\rightsquigarrow$ is the product of diagonal entries $\overset{?}{=} 0$

$\swarrow$ Yes: not inv $\qquad \searrow$ no: inv.

$$\begin{bmatrix} x & & & & & & \\ & x & & & & & \\ & & x & & & & \\ & & & x & & & \\ & 0 & & & x & & \\ & & & & & x & \\ & & & & & & x \end{bmatrix} = A \text{ invertible}$$

non-zero on diag.

$$\begin{bmatrix} x & & & & & & \\ & x & & & & & \\ 0 & & x & & & & \\ 0 & 0 & & x & & & \\ 0 & 0 & 0 & & x & & \\ 0 & 0 & 0 & 0 & & x & \\ 0 & 0 & 0 & 0 & 0 & & x \end{bmatrix}$$

If A is "upper triangular" all 0's below diag.

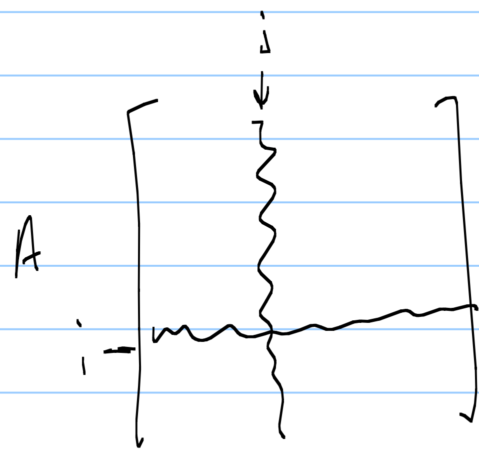
$\det(A) =$ product of diag entries.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21}\cdot a_{11} & a_{22}\cdot a_{11} & a_{23}\cdot a_{11} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \xrightarrow{-a_{21}} \Rightarrow \begin{matrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22}\cdot a_{11} - a_{12}\cdot a_{21} & a_{23}\cdot a_{11} - a_{21}a_{13} \\ a_{31} & a_{32} & a_{33} \end{matrix}$$

A is invertible iff $\underline{\hspace{6cm}} \neq 0$.

$$\left[a_{11} \det A_{11} - a_{12} \det A_{12} + a_{13} \det A_{13}\right]$$

A_{ij} = matrix A with row i + col j deleted.



A $n \times n$ matrix

A_{ij} $(n-1) \times (n-1)$ matrix

$$\left[a_{11} \det A_{11} - a_{12} \det A_{12} + a_{13} \det A_{13}\right]$$

$$3(-2) - 1(-3) - 2(-3)$$

$$= -15$$

$$\begin{bmatrix} 3 & 1 & -2 \\ 2 & 1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$$

A $n \times n$ matrix

$$\det A = a_{11} \det A_{11} - a_{12} \det A_{12} + a_{13} \det A_{13} \cdots + \underset{\uparrow (-1)^{1+n}}{a_{1n}} \det A_{1n}$$

↪ expand $n!$

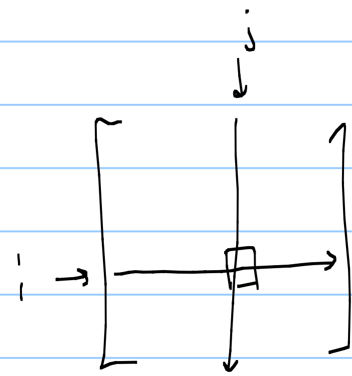
(i,j)-cofactor: C_{ij}

$(-1)^{i+j} \det A_{ij}.$

$$\det A = \sum_{j=1}^{n} \overset{(-1)^{1+j}}{\downarrow} a_{1j} \det A_{1j}$$

$$= \sum_{j=1}^{n} a_{1j} C_{1j}.$$

row i



$$\det A = \sum_{j=1}^{n} (-1)^{i+j} a_{ij} \det A_{ij}$$

$$= \sum_{i=1}^{n} (-1)^{i+j} a_{ij} \det A_{ij}.$$

$$\det \begin{bmatrix} 3 & & & & \\ 0 & 2 & & & \\ 0 & & 5 & & \\ 0 & 0 & & -1 & \\ 0 & & & & 3 \end{bmatrix}$$

Theorem: A is $n \times n$ matrix

$A \rightsquigarrow B$ by one row repl op then $\det A = \det B$.

$A \rightsquigarrow B$ by one row swap then $\det A = -\det B$.

$A \rightsquigarrow B$ by scaling a row by k $\det B = k \det A$,

$$A \xrightarrow{\text{row ops.}} U \quad \text{row ech form.}$$

$$\det A = 0 \iff \det U = 0.$$

$$\Updownarrow$$

$$\text{prod of diagonals} = 0.$$

$$\begin{bmatrix} 1 & 3 & 0 & 2 \\ -2 & -5 & 7 & 4 \\ 3 & 5 & 2 & 1 \\ 1 & -1 & 2 & -3 \end{bmatrix} \xrightarrow{\text{row rpl}} \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 7 & 8 \\ 0 & -4 & 2 & -5 \\ 0 & -4 & 2 & -5 \end{bmatrix} \Rightarrow \det = 0.$$

$$\det \begin{bmatrix} 2 & -8 & 6 & 8 \\ 3 & -9 & 5 & 10 \\ -3 & 0 & 1 & -2 \\ 1 & -4 & 0 & 6 \end{bmatrix} = 2 \cdot \det \begin{bmatrix} 1 & -4 & 3 & 4 \\ 3 & -9 & 5 & 10 \\ -3 & 6 & 1 & -2 \\ 1 & -4 & 0 & 6 \end{bmatrix}$$

$$2 \det \begin{bmatrix} 1 & -4 & 3 & 4 \\ 0 & 3 & -4 & -2 \\ 0 & -12 & 10 & 10 \\ 0 & 0 & -3 & 2 \end{bmatrix} = 2 \det \begin{bmatrix} 1 & -4 & 3 & 4 \\ 0 & 3 & -4 & -2 \\ 0 & 0 & -6 & -2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$$= 2 \cdot 1 \cdot 3 \left(-6 \cdot 2 - (-3)(-2) \right)$$

$$= -36.$$

$$\begin{bmatrix} 2 & 5 & -3 & -1 \\ 3 & 0 & 1 & -3 \\ -6 & 0 & -4 & 9 \\ 4 & 10 & -4 & -1 \end{bmatrix} \qquad \begin{bmatrix} 2 & 5 & -3 & -1 \\ 3 & 0 & 1 & -3 \\ -6 & 0 & -4 & 9 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

$$5 \cdot \left[3(-4 - 18) - -6(1 - -6) \right]$$